RK Vision Academy

Grade 12

Applied Maths

Previous Year Questions With Solutions

2023 - 25

PH NO: 99626 87633
Rkvisionacademy.com
rkvisionacademyjeeneet@gmail.com

YouTube
RK VISION ACADEMY FOR IIT-JEE & NEET

Sl. No	Set	Page No.
Year 2022		
1	Set 1	01
Year 2023		
2	Set 1	15
3	Set 2	49
Year 2024		
4	Set 1	80
Year 2025		
5	Set 1	113
5	Compartment exam	140

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***three*** *sections –* ***Section A, B*** *and* ***C****.*

(ii) *Each section is* ***compulsory****.*

(iii) ***Section – A*** *has* ***6*** *short answer type-I questions of* ***2*** *marks each.*

(iv) ***Section – B*** *has* ***4*** *short answer type-II questions of* ***3*** *marks each.*

(v) ***Section – C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

SECTION A

Questions number ***1*** *to* ***6*** *carry* ***2*** *marks each.*

1. (a) Evaluate : *2*

$$\int_0^1 \frac{xe^x}{(x+1)^2}\,dx$$

OR

(b) Solve the following differential equation : *2*

$$\frac{dy}{dx} = e^{x+y} + x^2e^y$$

2. Find the present value of a perpetuity of ₹ 18,000 payable at the end of 6 months, if the money is worth 8% p.a. compounded semi-annually. *2*

3. (a) Find the effective rate which is equivalent to nominal rate of 10% p.a. compounded monthly. *2*

[Given that : $(1{\cdot}00833)^{12} = 1{\cdot}1047$]

OR

(b) Abhay bought a mobile phone for ₹ 30,000. The mobile phone is estimated to have a scrap value of ₹ 3,000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years. *2*

4. Consider the following hypothesis :

$H_0 : \mu = 35$

$H_1 : \mu \neq 35$

A sample of 81 items is taken whose mean is 37·5 and the standard deviation is 5. Test the hypothesis at 5% level of significance. *2*

[Given : Critical value of Z for a two-tailed test at 5% level of significance is 1·96]

5. The following table shows the annual rainfall (in mm) recorded for Cherrapunji, Meghalaya :

Year	Rainfall (in mm)
2001	1·2
2002	1·9
2003	2
2004	1·4
2005	2·1
2006	1·3
2007	1·8
2008	1·1
2009	1·3

Determine the trend of rainfall by 3-year moving average. *2*

6. Maximize $z = 3x + 4y$, if possible,

subject to the constraints :

$$x - y \leq -1$$
$$-x + y \leq 0$$
$$x, y \geq 0$$

2

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. (a) The supply function of a commodity is $100p = (x + 20)^2$. Find the Producer's Surplus (PS), when the market price is ₹ 25. *3*

OR

(b) Find : *3*

$$\int \frac{2x^2 + 1}{x^2 - 3x + 2} dx$$

8. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 for the following data : *3*

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40
2007	36

P.T.O.

9. Ten cartons are taken at random from an automatic packing machine. The mean net weight of the ten cartons is 11·8 kg and standard deviation is 0·15 kg. Does the sample mean differ significantly from the intended mean of 12 kg ? *3*

[Given that for d.f. = 9, $t_{0 \cdot 05} = 2 \cdot 26$]

10. Madhu exchanged her old car valued at ₹ 1,50,000 with a new one priced at ₹ 6,50,000. She paid ₹ x as down payment and the balance in 20 monthly equal instalments of ₹ 21,000 each. The rate of interest offered to her is 9% p.a. Find the value of x. *3*

[Given that : $(1 \cdot 0075)^{-20} = 0 \cdot 86118985$]

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. In a certain culture of bacteria, the rate of increase of bacteria is proportional to the number present. It is found that there are 10,000 bacteria at the end of 3 hours and 40,000 bacteria at the end of 5 hours. Determine the number of bacteria present in the beginning. *4*

12. (a) Calculate the EMI under 'Flat Rate System' for a loan of ₹ 5,00,000 with 10% annual interest rate for 5 years. *4*

OR

(b) A machine costing ₹ 2,00,000 has effective life of 7 years and its scrap value is ₹ 30,000. What amount should the company put into a sinking fund earning 5% p.a., so that it can replace the machine after its usual life ? Assume that a new machine will cost ₹ 3,00,000 after 7 years. *4*

[Given that : $(1 \cdot 05)^7 = 1 \cdot 407$]

13. A start-up company invested ₹ 3,00,000 in shares for 5 years. The value of this investment was ₹ 3,50,000 at the end of second year, ₹ 3,80,000 at the end of third year and on maturity, the final value stood at ₹ 4,50,000. Calculate the Compound Annual Growth Rate (CAGR) on the investment. *4*

[Given that : $(1 \cdot 5)^{1/5} = 1 \cdot 084$]

14. A dietician wishes to mix two types of foods F_1 and F_2 in such a way that the vitamin content of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food F_1 contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C, while Food F_2 contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 5 per kg to purchase Food F_1 and ₹ 7 per kg to purchase Food F_2.

Based on the above information, answer the following questions : *4*

(a) To find out the minimum cost of such a mixture, formulate the above problem as a LPP.

(b) Determine the minimum cost of the mixture.

MARKING SCHEME

Senior Secondary School Examination TERM–II, 2022

MATHEMATICS (Subject Code–241)

[Paper Code: 465]

Maximum Marks: 40

Section – A

(Questions number **1** to **6** carry **2** marks each)

1. (a) Evaluate:

$$\int_0^1 \frac{xe^x}{(x+1)^2}dx$$

Solution:

(a)

$$\int_0^1 \frac{xe^x}{(x+1)^2}dx = \int_0^1 \frac{(x+1)-1}{(x+1)^2}e^x dx = \int_0^1 \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right]dx \qquad [1]$$

$$= \left[\frac{1}{x+1}e^x\right]_0^1 = \frac{e-2}{2} \qquad \left[\frac{1}{2}+\frac{1}{2}\right]$$

OR

1. (b) Solve the following differential equation:

$$\frac{dy}{dx} = e^{x+y} + x^2e^y$$

Solution:

(b)

$$\frac{dy}{dx} = e^y(e^x + x^2)$$

$$\Rightarrow \qquad \frac{dy}{e^y} = (e^x + x^2)dx \qquad [1]$$

$$\Rightarrow \qquad -e^{-y} = e^x + \frac{1}{3}x^3 + C \qquad [1]$$

2. Find the present value of a perpetuity of ₹18,000 payable at the end of 6 months, if the money is worth 8% p.a. compounded semi-annually.

Solution:

Let P be the present value of the perpetuity.

Here, R = ₹18000 and $i = \frac{8}{100\times2} = 0.04$ **[1]**

$\therefore\ P = \frac{R}{i} = \frac{18000}{0.4} = ₹4{,}50{,}000$ **[1]**

3. (a) Find the effective rate which is equivalent to nominal rate of 10% p.a. compounded monthly.
[Given that: $(1\cdot00833)^{12} = 1\cdot1047$]

Solution:

(a) Here, nominal rate = 10% and k =12

$\therefore$ Effective rate of interest = $\left(1+\frac{r}{100k}\right)^{k} - 1$

$$= \left(1+\frac{10}{1200}\right)^{12} - 1 = (1.00833)^{12} - 1 = 0.1047 \qquad \left[\mathbf{1\frac{1}{2}}\right]$$

Hence, the effective rate of interest is 10.47% $\left[\frac{1}{2}\right]$

OR

3. (b) Abhay bought a mobile phone for ₹30,000. The mobile phone is estimated to have a scrap value of ₹3,000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years.

Solution:

(b) Here, original value of mobile phone (C) = ₹30,000

Scrap value of the phone (S) = ₹3,000

Useful life (n) = 3 years

Annual depreciation $\frac{\text{C}-\text{S}}{\text{n}} = \frac{30{,}000-3000}{3} = 9000$ **[1]**

$\therefore$ Book value of the mobile at the end of 2 years = ₹30000 − 2 × ₹9000 = ₹12000 **[1]**

4. Consider the following hypothesis:

$$H_0: \mu = 35$$
$$H_1: \mu \neq 35$$

A sample of 81 items is taken whose mean is 37.5 and the standard deviation is 5. Test the hypothesis at 5% level of significance.

[*Given*: Critical value of Z for a two-tailed test at 5% level of significance is 1·96]

Solution:

The question is not in conformity with the prescribed syllabus. Thus, 2 marks be given to each examinee.

5. The following table shows the annual rainfall (in mm) recorded for Cherrapunji, Meghalaya:

Year	**Rainfall (in mm)**

2001	**1.2**
2002	**1.9**
2003	**2**
2004	**1.4**
2005	**2.1**
2006	**1.3**
2007	**1.8**
2008	**1.1**
2009	**1.3**

Solution:

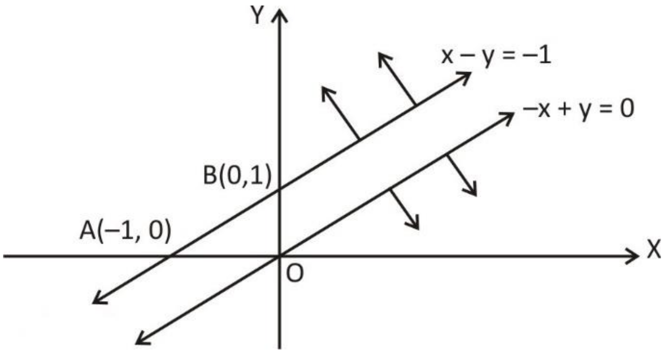
Year	Rain fall (in mm)	3 year moving total	3 year moving average	**1** mark for 3-year moving totals + **1** mark for 3-year moving averages
2001	1.2	---	---	
2002	1.9	5.1	1.7	
2003	2	5.3	1.77	
2004	1.4	5.5	1.83	
2005	2.1	4.8	1.6	
2006	1.3	5.2	1.73	
2007	1.8	4.2	1.4	
2008	1.1	4.2	1.4	
2009	1.3	---		

6. **Maximize z = 3x + 4y, if possible,**
subject to the constraints:

$$x - y \leq -1$$
$$-x + y \leq 0$$
$$x, y \geq 0$$

Solution:

The graph of the given constraints is



[1]

Here, the feasible region is empty.

So, there exists no solution to the given LPP. **[1]**

Section – B

(Questions number **7** to **10** carry 3 marks each)

7. (a) The supply function of a commodity is 100p = (x + 20) ². Find the Producer's Surplus (PS), when the market price is ₹25.

Solution:

(a) Here, $p_0 = 25$

Putting $p = p_0$ and $x = x_0$ in the given supply function $100p = (x + 20)^2$.

We have $x_0 = 30$ **[1]**

Thus,

Producer's surplus $= p_0x_0 - \int_0^{x_0} p\,dx = 750 - \frac{1}{100}\int_0^{30}(x+20)^2 dx$ **[1]**

$=750 - \frac{1}{100}\left[\frac{1}{3}(x+20)^3\right]_0^{30}$

$= 750 - \frac{1}{300}[(50)^3 - (20)^3] = ₹\,360$ **[1]**

OR

(b) Find:

$$\int \frac{2x^2+1}{x^2-3x+2}dx$$

Solution:

(b) $I = \int \frac{2x^2+1}{x^2-3x+2}dx = \int(2 + \frac{6x-3}{x^2-3x+2})dx$ **[1]**

$= \int 2dx + \int\left[\frac{-3}{x-1} + \frac{9}{x-2}\right]dx$ {Using partial fractions} **[1]**

$= 2x - 3\,log|x-1| + 9\,log|x-2| + C$ **[1]**

8. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 for the following data:

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37

2006	40
2007	36

Solution:

Year	Production (Y)	Origin = 2004 (X)	X^2	XY
2001	30	– 3	9	–90
2002	35	– 2	4	–70
2003	36	– 1	1	–36
2004	32	0	0	0
2005	37	1	1	37
2006	40	2	4	80
2007	36	3	93	108
2009	$\sum Y = 246$	$\sum XY = 0$	$\sum X^2 = 28$	$\sum XY = 29$

[1]

Trend equation is $Y_C = a + bX$; and

Normal equations are:

$$\left.\begin{aligned} a &= \frac{\Sigma Y}{n} = \frac{246}{7} = 35.14 \text{ ; and} \\ b &= \frac{\Sigma XY}{\Sigma X^2} = \frac{29}{28} = 1.03 \end{aligned}\right\}$$ **[1]**

⇒ Trend equation is $Y_C = 35.14 + 1.03\,X$ $\left[\frac{1}{2}\right]$

Thus, trend value for 2008 is [35.14 + 1.03(4)] = 39.26 $\left[\frac{1}{2}\right]$

9. Ten cartons are taken at random from an automatic packing machine. The mean net weight of the ten cartons is 11.8 kg and standard deviation is 0.15 kg. Does the sample mean differ significantly from the intended mean of 12 kg?
[Given that for d.f. = 9, $t_{0\cdot05} = 2\cdot26$]

Solution:

We are given $n = 10$, $\bar{x} = 11.8$ kg and s = 0.15 kg

Let Null hypothesis be $H_0 = \mu = 12$ kg, and

Alternate hypothesis be H_1 ; $\mu \neq 12$ kg **[1]**

Under H_0, the test statistic is

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \frac{11.8-12}{\frac{0.15}{3}} = -4$$ **[1]**

Since the tabulated value of t for *d.f.* = 9 is $t_{0.05}$ = 2.26 and the calculated |t| is much greater than the tabulated value, null hypothesis is rejected. Thus, we conclude that the sample mean differs significantly from the intended mean of 12 kg. **[1]**

10. Madhu exchanged her old car valued at ₹1,50,000 with a new one priced at ₹6,50,000. She paid ₹x as down payment and the balance in 20 monthly equal instalments of ₹21,000 each. The rate of interest offered to her is 9% p.a. Find the value of x.
[Given that: $(1\cdot0075)^{-20} = 0\cdot86118985$]

Solution:

Here, $i = \frac{9}{1200} = 0.0075$, n = 20 and E = ₹ 21,000

P = ₹ (650000 – 150000 – x) = $(500000 - x)$ **[1]**

By the reducing balance method, we have

$$E = \frac{Pi}{1-(1+i)^{-n}}$$ **[1]**

$$\Rightarrow \quad 21000 = \frac{(500000-x)(0.0075)}{1-(1.0075)^{-20}} = \frac{3750-0.0075x}{1-0.86118985} = \frac{3750-0.0075x}{0.1381015}$$ $\left[\frac{1}{2}\right]$

$\Rightarrow \quad x$ = ₹ 1,11,332 $\left[\frac{1}{2}\right]$

Section – C

(Questions number **11** to **14** carry **4** marks each)

11. In a certain culture of bacteria, the rate of increase of bacteria is proportional to the number present. It is found that there are 10,000 bacteria at the end of 3 hours and 40,000 bacteria at the end of 5 hours. Determine the number of bacteria present in the beginning.

Solution:

Let P be the number of bacteria present in the culture after t hours. Then,

$$\frac{dP}{dt} \alpha P$$ **[1]**

$$\Rightarrow \quad \frac{dP}{dt} = kP$$

$$\Rightarrow \quad \int \frac{dP}{P} = \int kdt$$

$$\Rightarrow \quad log\, P = kt + C$$

$$\Rightarrow \quad P(t) = e^{kt+C} + \lambda e^{kt} \quad \ldots. \text{(i)}$$ **[1]**

We are given that P(3) = 10000 and P(5) = 40000 we get

$$\lambda e^{k(3)} = 10000 \text{ and } \lambda e^{k(5)} = 40000 \quad \ldots \text{(ii)}$$

$\Rightarrow \quad e^{2k} = 4$, or $e^k = 2$ **[1]**

From (ii), λ = 1250

Thus, from (i), we have

$P(t) = 1250(2)^t$

⇒ $P(t) = 1250(2)^0 = 1250$

Hence, there were 1250 bacteria present in the beginning. **[1]**

12. (a) Calculate the EMI under 'Flat Rate System' for a loan of ₹5,00,000 with 10% annual interest rate for 5 years.

Solution:

(a) Here, P = ₹500000

$I = 500000 \times \frac{10}{100} \times 5 =$ ₹ 250000 **[1]**

n = 5 years = 5 × 12 months = 60 months **[1]**

EMI $= \frac{P+I}{n} = \frac{500000+250000}{60}$ **[1]**

= ₹12500 **[1]**

OR

12. (b) A machine costing ₹2,00,000 has effective life of 7 years and its scrap value is ₹30,000. What amount should the company put into a sinking fund earning 5% p.a., so that it can replace the machine after its usual life? Assume that a new machine will cost ₹ 3,00,000 after 7 years.

[Given that: $(1{\cdot}05)^7 = 1{\cdot}407$]

Solution:

(b) Cost of new machine = ₹300000; scrap value = ₹30000

⇒ Money required to buy new machine after 7 years is

₹ (300000 – 30000) = ₹ 270000 **[1]**

So, A = ₹ 270000, i = 0.05 and n = 7

Using the formula: $A = R\left[\frac{(1+i)^n-1}{i}\right]$ **[1]**

⇒ $270000 = R\left[\frac{(1.05)^7-1}{0.05}\right]$

⇒ $R = \frac{270000\times0.05}{0.407} =$ ₹33169.33 **[1+1]**

Hence, the company should put ₹ 33169.33 into sinking fund.

13. A start-up company invested ₹3,00,000 in shares for 5 years. The value of this investment was ₹3,50,000 at the end of second year, ₹3,80,000 at the end of third year and on maturity, the final value stood at ₹4,50,000. Calculate the Compound Annual Growth Rate (CAGR) on the investment.
[Given that: $(1{\cdot}5)^{1/5} = 1{\cdot}084$]

Solution:

Here, initial value of investment (V_i) = ₹ 300000

Final value of investment (V_{fi}) = ₹ 450000 **[1]**

and n = 5 years.

Now, $i = \left(\frac{V_f}{V_i}\right)^{1/n} - 1$

$\Rightarrow \quad i = \left(\frac{450000}{300000}\right)^{1/5} - 1 = (1.5)^{1/5} - 1 = 1.084 - 1 = 0.084$ **[1+1]**

$\Rightarrow$ CAGR (%) = 8.4%

Thus, the compound annual growth rate is 8.4%. **[1]**

14. A dietician wishes to mix two types of foods F_1 and F_2 in such a way that the vitamin content of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food F_1 contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C, while Food F_2 contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5 per kg to purchase Food F1 and ₹7 per kg to purchase Food F_2.

Based on the above information, answer the following questions:

(a) To find out the minimum cost of such a mixture, formulate the above problem as a LPP.

(b) Determine the minimum cost of the mixture.

Solution:

Let the mixture contain x kg of food F_1 and y kg of food F_2. Then, LPP becomes

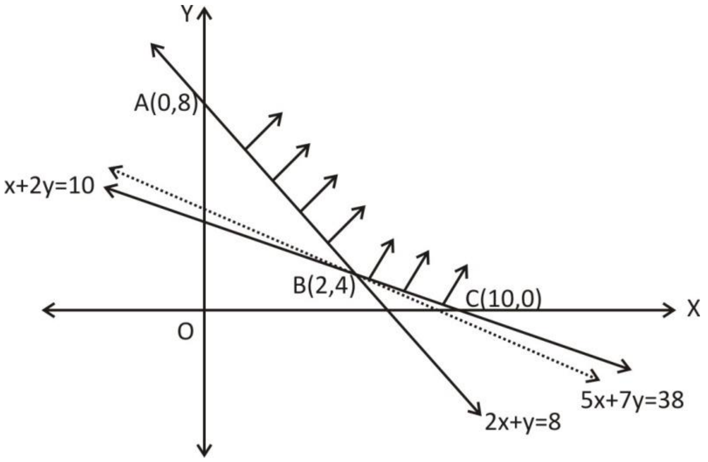
(a) Minimize Z = 5x + 7y **$\left[\frac{1}{2}\right]$**

Subject to the constraints

$2x + y \geq 8$

$x + 2y \geq 10$

$x \geq 0, y \geq 0$ **[1]**



(b) The corner points are A(0, 8), B(2,4) and C (10, 0) **[1]**

The value of Z at these corner points are

$Z_A = 56$; $Z_B = 38$; $Z_C = 50$ $\left[\frac{1}{2}\right]$

Since the feasible region is unbounded, we draw the graph of

$5x + 7y < 38$.

As the graph of $5x + 7y < 38$ does not have any point common with the Feasible region, so the minimum cost of the mixture is ₹ 38. **[1]**

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The last (unit) digit of $(22)^{12}$ is :

(a) 2 (b) 4

(c) 6 (d) 8

2. The least non-negative remainder, when 3^{15} is divided by 7 is :

(a) 1 (b) 5

(c) 6 (d) 7

3. If $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 10 \\ -10 & -5 \end{bmatrix}$, then AB is :

(a) $\begin{bmatrix} -5 & 10 \\ 0 & -5 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -5 \\ 25 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 10 & -25 \\ -5 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} -5 & 10 \\ 0 & -25 \end{bmatrix}$

4. If $\begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$, then the values of x and y are :

(a) $x = 3, y = 5$

(b) $x = 5, y = 3$

(c) $x = 2, y = 7$

(d) $x = 7, y = 2$

5. The ratio in which a grocer mixes two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg, is :

(a) 7 : 8

(b) 8 : 7

(c) 5 : 7

(d) 7 : 5

6. If $\frac{|x+1|}{x+1} > 0, x \in \mathbb{R}$, then :

(a) $x \in [-1, \infty)$

(b) $x \in (-1, \infty)$

(c) $x \in (-\infty, -1)$

(d) $x \in (-\infty, -1]$

P.T.O.

7. A and B are square matrices each of order 3 such that $|A| = -1$ and $|B| = 3$. What is the value of $|3AB|$?

(a) -9 (b) -18

(c) -27 (d) -81

8. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is :

(a) -1 (b) 0

(c) 1 (d) 3

9. The relation between 'Marginal cost' and 'Average cost' of producing 'x' units of a product is :

(a) $\frac{d(AC)}{dx} = x(MC - AC)$ (b) $\frac{d(AC)}{dx} = x(AC - MC)$

(c) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$ (d) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$

10. $\int (x-1)e^{-x}\,dx$ is equal to :

(a) $(x - 2)\,e^{-x} + C$ (b) $xe^{-x} + C$

(c) $-xe^{-x} + C$ (d) $(x + 1)\,e^{-x} + C$

11. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is :

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $xy = C$

(c) $\log x \log y = C$ (d) $x + y = C$

P.T.O.

12. If X is a Poisson variable such that $P(X = 1) = 2P(X = 2)$, then $P(X = 0)$ is :

(a) e

(b) $\frac{1}{e}$

(c) 1

(d) e^2

13. If the calculated value of $|t| < t_v(\alpha)$, then the null hypothesis is :

(a) rejected

(b) accepted

(c) cannot be determined

(d) neither accepted nor rejected

14. For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as :

(a) $n_1 - n_2 + 2$

(b) $n_1 - n_2 - 2$

(c) $n_1 + n_2 - 2$

(d) $n_1 + n_2 - 1$

15. The straight line trend is represented by the equation :

(a) $y_c = a + bx$

(b) $y_c = a - bx$

(c) $y_c = na + b\Sigma x$

(d) $y_c = na - b\Sigma x$

P.T.O.

16. The present value of a perpetuity of ₹ R payable at the end of each payment period, when the money is worth i per period, is given by :

(a)	Ri	(b)	$R + \frac{R}{i}$
(c)	$\frac{R}{i}$	(d)	R – Ri

17. The effective rate which is equivalent to nominal rate of 10% p.a. compounded quarterly is :

(a)	10·25%	(b)	10·38%
(c)	10·47%	(d)	10·53%

18. Region represented by $x \geq 0$, $y \geq 0$ lies in

(a)	I quadrant	(b)	II quadrant
(c)	III quadrant	(d)	IV quadrant

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* The function $f(x) = (x + 2)\, e^{-x}$ is increasing in the interval $(-1, \infty)$.

Reason (R) : A function f(x) is increasing, if $f'(x) > 0$.

P.T.O.

20. *Assertion (A) :* The differential equation representing the family of parabolas $y^2 = 4ax$, where 'a' is a parameter, is $x\frac{dy}{dx} - 2y = 0$.

Reason (R) : If the given family of curves has n parameters, then it is to be differentiated n times to eliminate the parameter and obtain the n^{th} order differential equation.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes ?

OR

(b) In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, then find the time taken by A to run 1 kilometre.

22. Solve for x : $\frac{x+3}{x-2} \leq 2$.

23. (a) Solve the following system of equations by Cramer's rule :

$$2x - y = 17, \ 3x + 5y = 6$$

OR

(b) Determine the integral value(s) of x for which the matrix A is singular :

$$A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$$

24. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the ordinate is changing 8 times as fast as abscissa.

25. Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items selected at random. (Given $e^{-2} = 0 \cdot 135$)

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) A bottle is full of dettol. One-third of its dettol is taken away and an equal amount of water is poured into the bottle to fill it again. This operation is repeated three times. Find the final ratio of dettol to water in the bottle.

OR

(b) A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 and 10 hours respectively. It all the three pipes are opened simultaneously, how long will it take to fill the tank ?

27. Find all the points of local maxima and local minima for the function $f(x) = x^3 - 6x^2 + 9x - 8$.

28. An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining a third six in the sixth throw of the die.

29. The mean weekly sales of a four-wheeler were 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful.
(Use $t_{0 \cdot 005} = 1 \cdot 729$ for 19 d.f.)

P.T.O.

30. (a) An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear depreciation method, find the annual depreciation of the asset and construct a yearly depreciation schedule.

OR

(b) Amrita bought a car worth ₹ 12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car.

{Given $(1{\cdot}0125)^{-48} = 0{\cdot}5508565$)}

31. Maximise $z = 300x + 190y$

subject to constraints :

$x + y \leq 24,$

$2x + y \leq 32,$

$x \geq 0, y \geq 0.$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the inverse of the matrix :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

and hence show that $AA^{-1} = I$.

OR

P.T.O.

(b) Using matrix method, solve the following system of equations for x, y and z :

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

33. (a) Divide a number 15 into two parts such that the square of one part multiplied with the cube of the other part is maximum.

OR

(b) Find a point on the curve $y^2 = 2x$ which is nearest to the point (1, 4).

34. Fit a straight line trend by method of least squares to the following data and find the trend values :

Year :	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹) :	65	68	70	72	75	67	73

35. Define Compound Annual Growth Rate (CAGR) and give the formula for calculating CAGR. Using the formula, calculate CAGR of Vikas's investment given below :

Vikas invested ₹ 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below :

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

[Use $(1{\cdot}4)^{1/6} = 1{\cdot}058$]

P.T.O.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs.

Based on the above information, answer the following questions :

(i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective. (Use : $e^{-6} = 0{\cdot}0024$) *1*

(ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs. *1*

(iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb. *2*

OR

(iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs. *2*

Case Study – 2

37. A factory manufactures tennis rackets and cricket bats. A tennis racket takes $1\frac{1}{2}$ hours of machine time and 3 hours of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively.

Based on the above information, answer the following questions :

(i) If x and y are the numbers of bats and rackets manufactured by the factory, then write the expression of total profit. *1*

(ii) Write the constraint that relates the number of craftsmanship hours. *1*

(iii) (a) Determine the maximum profit (in ₹) earned by the factory. *2*

OR

(iii) (b) How many bats and rackets respectively, are to be manufactured to earn maximum profit ? *2*

P.T.O.

Case Study – 3

38. In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7·5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions :

(i) Determine the EMI. *1*

(ii) Find the principal paid by Mr. Aggarwal in the 150th instalment. *1*

(iii) (a) Find the total interest paid by Mr. Aggarwal. *2*

OR

(iii) (b) How much was paid by Mr. Aggarwal to repay the entire amount of home loan ? *2*

[Use $(1{\cdot}00625)^{240} = 4{\cdot}4608$; $(1{\cdot}00625)^{91} = 1{\cdot}7629$]

Set 4

M.S. Code S46547A

MARKING SCHEME

APPLIED MATHEMATICS

Section A

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	The last (unit) digit of $(22)^{12}$ is : (a) 2 (b) 4 (c) 6 (d) 8	
Sol.	(c) 6	**(1)**
2.	The least non-negative remainder, when 3^{15} is divided by 7 is : (a) 1 (b) 5 (c) 6 (d) 7	
Sol.	(c) 6	**(1)**
3.	If $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 10 \\ -10 & -5 \end{bmatrix}$, then AB is : (a) $\begin{bmatrix} -5 & 10 \\ 0 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -5 \\ 25 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 10 & -25 \\ -5 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & 10 \\ 0 & -25 \end{bmatrix}$	

Sol.	(d) $\begin{bmatrix} -5 & 10 \\ 0 & -25 \end{bmatrix}$	**(1)**
4.	If $\begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$, then the values of x and y are : (a) $x = 3, y = 5$ (b) $x = 5, y = 3$ (c) $x = 2, y = 7$ (d) $x = 7, y = 2$	
Sol.	(a) $\boldsymbol{x = 3, y = 5}$	**(1)**
5.	The ratio in which a grocer mixes two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg, is : (a) 7 : 8 (b) 8 : 7 (c) 5 : 7 (d) 7 : 5	
Sol.	(b) 8 : 7	**(1)**
6.	If $\dfrac{\lvert x+1 \rvert}{x+1} > 0, x \in \mathbb{R}$, then : (a) $x \in [-1, \infty)$ (b) $x \in (-1, \infty)$ (c) $x \in (-\infty, -1)$ (d) $x \in (-\infty, -1]$	
Sol.	(b) $\boldsymbol{x \in (-1, \infty)}$	**(1)**
7.	A and B are square matrices each of order 3 such that $\lvert A \rvert = -1$ and $\lvert B \rvert = 3$. What is the value of $\lvert 3AB \rvert$? (a) -9 (b) -18 (c) -27 (d) -81	
Sol.	(d) $\boldsymbol{-81}$	**(1)**

8.	If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is : (a) -1 (b) 0 (c) 1 (d) 3	
Sol.	(a) $\mathbf{-1}$	**(1)**
9.	The relation between 'Marginal cost' and 'Average cost' of producing 'x' units of a product is : (a) $\frac{d(AC)}{dx} = x(MC - AC)$ (b) $\frac{d(AC)}{dx} = x(AC - MC)$ (c) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$ (d) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$	
Sol.	(d) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$	**(1)**
10.	$\int (x-1)e^{-x}\,dx$ is equal to : (a) $(x-2)\,e^{-x} + C$ (b) $xe^{-x} + C$ (c) $-xe^{-x} + C$ (d) $(x+1)\,e^{-x} + C$	
Sol.	(c) $\mathbf{-xe^{-x} + C}$	**(1)**
11.	The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is : (a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $xy = C$ (c) $\log x \log y = C$ (d) $x + y = C$	
Sol.	(b) $xy = C$	**(1)**

12.	If X is a Poisson variable such that P(X = 1) = 2P(X = 2), then P(X = 0) is : (a) e (b) $\frac{1}{e}$ (c) 1 (d) e^2	
Sol.	(b) $\boldsymbol{\frac{1}{e}}$	**(1)**
13.	If the calculated value of $\lvert t \rvert < t_v(\alpha)$, then the null hypothesis is : (a) rejected (b) accepted (c) cannot be determined (d) neither accepted nor rejected	
Sol.	(b) accepted	**(1)**
14.	For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as : (a) $n_1 - n_2 + 2$ (b) $n_1 - n_2 - 2$ (c) $n_1 + n_2 - 2$ (d) $n_1 + n_2 - 1$	
Sol.	(c) $\boldsymbol{n_1 + n_2 - 2}$	**(1)**
15.	The straight line trend is represented by the equation : (a) $y_c = a + bx$ (b) $y_c = a - bx$ (c) $y_c = na + b\Sigma x$ (d) $y_c = na - b\Sigma x$	
Sol.	(a) $\boldsymbol{y_c = a + bx}$	**(1)**

16.	The present value of a perpetuity of ₹ R payable at the end of each payment period, when the money is worth i per period, is given by : (a) Ri (b) $R + \frac{R}{i}$ (c) $\frac{R}{i}$ (d) $R - Ri$	
Sol.	(c) $\frac{R}{i}$	**(1)**
17.	The effective rate which is equivalent to nominal rate of 10% p.a. compounded quarterly is : (a) 10·25% (b) 10·38% (c) 10·47% (d) 10·53%	
Sol.	(b) 10.38%	**(1)**
18.	Region represented by $x \geq 0$, $y \geq 0$ lies in (a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant	
Sol.	(a) I quadrant	**(1)**
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true and Reason (R) is false. (d) Assertion (A) is false and Reason (R) is true.	
19.	*Assertion (A) :* The function $f(x) = (x + 2) e^{-x}$ is increasing in the interval $(-1, \infty)$. *Reason (R) :* A function f(x) is increasing, if $f'(x) > 0$.	
Sol.	(d) Assertion (A) is false and Reason (R) is true.	**(1)**

20.	*Assertion (A) :* The differential equation representing the family of parabolas $y^2 = 4ax$, where 'a' is a parameter, is $x\frac{dy}{dx} - 2y = 0$. *Reason (R) :* If the given family of curves has n parameters, then it is to be differentiated n times to eliminate the parameter and obtain the n^{th} order differential equation.	
Sol.	(d) Assertion (A) is false and Reason (R) is true.	**(1)**
	SECTION B This section comprises very short answer (VSA) type questions of **2 marks each.**	
21(a).	Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes ?	
Sol.	Let B be closed after n minutes. Then, pipe A runs for 18 minutes and B runs for n minutes to fill the tank. $\therefore \frac{\mathbf{18}}{\mathbf{24}} + \frac{\mathbf{n}}{\mathbf{32}} = 1$ $\Rightarrow \frac{3}{4} + \frac{n}{32} = 1 \Rightarrow n = 8.$ Hence, pipe B must be closed after 8 min	**(1)** **(1)**
	OR	
21(b).	In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, then find the time taken by A to run 1 kilometre.	
Sol.	Suppose A takes 't' seconds to run 1 km race. Then, B takes (t + 30) seconds and C takes (t + 30 + 15) seconds, i.e. (t + 45) seconds. We find A beats C by (30 + 15) seconds = 45 seconds and it is given that A beats C by 180 metres.	

	∴ C runs 180 m in 45 seconds ⇒ C runs 1000 m in $\left(\frac{45}{180} \times 1000\right)$ seconds = 250 seconds. ∴ t + 45 = 250 ⇒ t = 205 Hence, A takes 205 seconds to run 1 km	**$(\frac{1}{2})$** **(1)** **$(\frac{1}{2})$**
22.	Solve for x : $\frac{x+3}{x-2} \leq 2.$	
Sol.	$\frac{x+3}{x-2} - 2 \leq 0 \quad \Rightarrow \frac{-x+7}{x-2} \leq 0$ or $\frac{x-7}{x-2} \geq 0$ Thus, the solution set is $(-\infty, 2) \cup [7, \infty)$	**(1)** **(1)**
23(a).	Solve the following system of equations by Cramer's rule : $2x - y = 17,\ 3x + 5y = 6$	
Sol.	Here, D = $\begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 13$ $D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 91$ $D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = -39$ Thus, $x = \frac{D_1}{D} = 7; y = \frac{D_2}{D} = -3$	**$(\frac{1}{2})$** **$(\frac{1}{2})$** **$(\frac{1}{2})$** **$(\frac{1}{2})$**
	OR	
23(b).	Determine the integral value(s) of x for which the matrix A is singular : $A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$	
Sol.	A is singular gives $\begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0$ i.e. $(x + 1)[(x + 2)(x - 6) - 2] + 3[-5x + 30 - 8] + 4[-5 - 4x - 8] = 0$	**$(\frac{1}{2})$**

	i.e. $(x + 1)\ (x^2 - 4x - 14) - 15x + 66 - 52 - 16x = 0$ i.e. $x^3 - 3x^2 - 49x = 0$ $x = 0,\ \frac{3\pm\sqrt{205}}{2}$ Hence, x = 0 is the only integral value.	**(1)** $(\frac{1}{2})$
24.	A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the ordinate is changing 8 times as fast as abscissa.	
Sol.	Here, $6y = x^3 + 2$ $\Rightarrow 6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ As $\frac{dy}{dt} = 8\frac{dx}{dt}$, we have $48\cancel{\frac{dx}{dt}} = 3x^2\cancel{\frac{dx}{dt}} \Rightarrow x = 4, -4$ when $x = 4$, $y = 11$; when $x = -4$, $y = \frac{-31}{3}$. $\therefore$ Points on the curve are $(4, 11), \left(-\ 4,\ \frac{-31}{3}\right)$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
25.	Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items selected at random. (Given $e^{-2} = 0{\cdot}135$)	
Sol.	Let p be the probability that an item is defective so, $p = \frac{2}{100} = 0{\cdot}02$. Here $n = 100 \therefore m = np = 2$ $P(X = r) = \frac{m^r}{r!}e^{-m} = \frac{2^r e^{-2}}{r!}$ $\Rightarrow \mathbf{P(X = 3) = \frac{2^3 e^{-2}}{3!} = \frac{4}{3} \times 0{\cdot}135 = 0{\cdot}18}$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
	SECTION C	

	This section comprises short answer (SA) type questions of **3 marks each.**	
26(a).	A bottle is full of dettol. One-third of its dettol is taken away and an equal amount of water is poured into the bottle to fill it again. This operation is repeated three times. Find the final ratio of dettol to water in the bottle.	
Sol.	Let the original quantity of dettol be x litres and the quantity of Dettol replaced by water be y litres. So, $y = \frac{x}{3}$. After 3 operations the quantity of dettol left = $x\left(1 - \frac{y}{x}\right)^3$. After 3 operations the quantity of water in the bottle = $x - x\left(1 - \frac{x}{3x}\right)^3$ Hence, the required ratio is $x\left(1 - \frac{x}{3x}\right)^3 : [x - x\left(1 - \frac{x}{3x}\right)^3]$ $= \left(1 - \frac{1}{3}\right)^3 : [1 - \left(1 - \frac{1}{3}\right)^3]$ $= \frac{8}{27} : \frac{19}{27}$ $= 8 : 19$	**(1)** **(1)** **(1)**
26(b).	A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 and 10 hours respectively. It all the three pipes are opened simultaneously, how long will it take to fill the tank ?	
Sol.	Here, $n_A = 3$, $n_B = 7$ and $n_C = 10$. $\frac{1}{n} = \frac{1}{n_A} - \frac{1}{n_B} - \frac{1}{n_C}$ $\Rightarrow \frac{1}{n} = \frac{1}{3} - \frac{1}{7} - \frac{1}{10}$ $\Rightarrow \frac{1}{n} = \frac{19}{210} \Rightarrow n = 11\frac{1}{19}$ Hence, the tank is filled in $11\frac{1}{19}$ hours.	**(2)** **(1)**

27.	Find all the points of local maxima and local minima for the function f(x) = $x^3 - 6x^2 + 9x - 8$.	
Sol.	$y = x^3 - 6x^2 + 9x - 8$	
	$\Rightarrow \frac{dy}{dx} = 3x^2 - 12x + 9$	**(1)**
	$\Rightarrow \frac{dy}{dx} = 3(x-1)(x-3)$	
	Critical points are 1, 3	**(1)**
	Showing, x=1 is a point of local maxima.	$\left(\frac{1}{2}\right)$
	Showing, x=3 is a point of local minima.	$\left(\frac{1}{2}\right)$
28.	An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining a third six in the sixth throw of the die.	
Sol.	Let A be the event of obtaining two sixes in the first five throws of a die. Let B be the event of obtaining a six in the sixth throw of a die. Then required probability = P(AB) = P(A) P(B)	
	Here, P(B) = $\frac{1}{6}$ and P(A) = $5_{C_2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{625}{3888}$	**(2)**
	Thus, Required probability = $\frac{625}{3888} \times \frac{1}{6} = \frac{625}{23328}$	**(1)**
29.	The mean weekly sales of a four-wheeler were 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful. (Use $t_{0\cdot005} = 1\cdot729$ for 19 d.f.)	

Sol. We are given

$\mu = 50$, $\bar{x} = 55$, SD = 10, $n = 20$ **(1)**

$H_0 : \mu = 50$

$H_1 : \mu > 50$

$$t = \frac{\bar{x}-\mu}{\frac{SD}{\sqrt{n}}} = \frac{55 - 50}{\frac{10}{\sqrt{20}}} = 2{\cdot}236$$ **(2)**

$$t_{\text{cal value}} > t_{\text{tab value}}$$

Hence H_0 is rejected.
So, Advertising Campaign was successful.

30(a). An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear depreciation method, find the annual depreciation of the asset and construct a yearly depreciation schedule.

Sol. Here C = ₹ 4,50,000

S = ₹ 1,00,000

and n = 5 years.

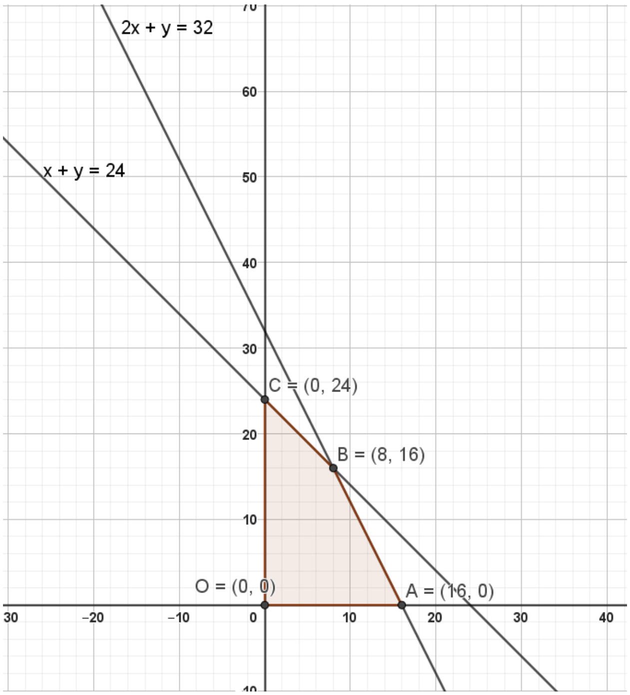
Annual depreciation $D = \frac{C - S}{n}$ = ₹ 70,000 **(2)**

Thus, yearly depreciation schedule is as follows:

Years	Book value at the beginning of the year (in ₹)	Depreciation (in ₹)	Book value at the end of the year (in ₹)
1	4,50,000	70,000	3,80,000
2	3,80,000	70,000	3,10,000

<table>
<tr>
<td></td>
<td>
<table>
<tr><td>3</td><td>3,10,000</td><td>70,000</td><td>2,40,000</td></tr>
<tr><td>4</td><td>2,40,000</td><td>70,000</td><td>1,70,000</td></tr>
<tr><td>5</td><td>1,70,000</td><td>70,000</td><td>1,00,000</td></tr>
</table>
</td>
<td>(1 for correct table)</td>
</tr>
<tr>
<td>30(b).</td>
<td>Amrita bought a car worth ₹ 12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car.
{Given $(1{\cdot}0125)^{-48} = 0{\cdot}5508565$)}</td>
<td></td>
</tr>
<tr>
<td>Sol.</td>
<td>Here P = ₹ 9,50,000, i = $\frac{15}{1200} = 0{\cdot}0125$
n = 48 months
Using the reducing balancing method,
$E = \frac{Pi}{1-(1+i)^{-n}} = \frac{9{,}5{,}0000 \times 0{\cdot}0125}{1-(1+0{\cdot}0125)^{-48}}$
$= \frac{11875}{1-(1.0125)^{-48}} = \frac{11875}{1-0.5508565}$
= ₹ 26,439·21</td>
<td>$(\frac{1}{2})$
$(\frac{1}{2})$
(1)
$(\frac{1}{2})$
$(\frac{1}{2})$</td>
</tr>
<tr>
<td>31.</td>
<td>Maximise z = 300x + 190y
subject to constraints :
$x + y \leq 24$,
$2x + y \leq 32$,
$x \geq 0, y \geq 0$.</td>
<td></td>
</tr>
</table>

Sol.



(2)

Corner Points	Value of Z
O (0,0)	0
A (16,0)	4800
B (8,16)	5440 → Max Value
C (0,24)	4560

(1)

So Z is maximum at B (8,16)

Max Value of Z = 5440

SECTION D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32(a).	Find the inverse of the matrix : $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ and hence show that $AA^{-1} = I$.	
Sol.	Here, |A| = – (– 4 – 3) – (12 + 1) + 2(9 – 1)	
	$= 7 - 13 + 16 = 10 \neq 0$	**(1)**
	$\Rightarrow \text{adj}(A) = \begin{bmatrix} -7 & -13 & 8 \\ 2 & -2 & 2 \\ 3 & 7 & -2 \end{bmatrix}^T = \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$	**($2\frac{1}{2}$)**
	Hence $A^{-1} = \frac{1}{10}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$	**($\frac{1}{2}$)**
	$AA^{-1} = \frac{1}{10}\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**(1)**
	OR	
32(b).	Using matrix method, solve the following system of equations for x, y and z : $x - y + z = 4$ $2x + y - 3z = 0$ $x + y + z = 2$	
Sol.	The matrix equation AX = B is $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$	**($\frac{1}{2}$)**

	$\lvert A\rvert = 10$ adj A = $\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ Here $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ Thus, x = 2, y = – 1, z = 1	(1) (2) ($\frac{1}{2}$) (1)
33(a).	Divide a number 15 into two parts such that the square of one part multiplied with the cube of the other part is maximum.	
Sol.	Let the two parts be x and 15 – x. Then, let y = $x^2(15 - x)^3$ $\Rightarrow \frac{dy}{dx} = x(15 - x)^2\,(-5x + 30)$ $\frac{dy}{dx} = 0$ gives x = 0, 15, 6 Rejecting x = 0, 15. Hence x = 6 Showing, $x = 6$ is a point of maxima So, y is maximum when x = 6. Hence two parts are 6 and 9	(1) (1) ($1\frac{1}{2}$) (1) ($\frac{1}{2}$)
	OR	
33(b).	Find a point on the curve $y^2 = 2x$ which is nearest to the point (1, 4).	

Sol.

Let P (x, y) be the required point which is nearest to Q (1, 4). Then distance PQ should be minimum and hence $(PQ)^2$ should be minimum. **$\left(\frac{1}{2}\right)$**

Now, $(PQ)^2 = (x - 1)^2 + (y - 4)^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y - 4)^2$ **(1)**

$= \frac{y^4 - 32y + 68}{4}$ **(1)**

Let $D = \frac{y^4 - 32y + 68}{4}$

$\frac{dD}{dy} = y^3 - 8$ **$\left(\frac{1}{2}\right)$**

$\frac{dD}{dy} = 0 \Rightarrow y = 2$ **(1)**

Showing, y = 2 is a point of minima **$\left(\frac{1}{2}\right)$**

Thus, the point is (2, 2) **$\left(\frac{1}{2}\right)$**

34. Fit a straight line trend by method of least squares to the following data and find the trend values :

Year :	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹) :	65	68	70	72	75	67	73

Sol.

Consider year 2014 as the year of origin. Calculation of trend values by method of least squares.

Year	Sales (in lakh ₹) y	Deviations from 2014 (x)	Squares of Deviations (x^2)	Sales deviation (xy)
2010	65	– 4	16	– 260
2012	68	– 2	4	– 136
2013	70	– 1	1	– 70

2014	72	0	0	0
2015	75	1	1	75
2016	67	2	4	134
2019	73	5	25	365
n = 7	$\sum y = 490$	$\sum x = 1$	$\sum x^2 = 51$	$\sum xy = 108$

(2 for correct table)

The equation of the straight-line trend is

$$y_c = a + bx$$

Two normal equations are

$$\sum y = na + b\sum x$$
$$\sum xy = a\sum x + b\sum x^2$$

$\Rightarrow 490 = 7a + b$ and $108 = a + 51b$

$\Rightarrow a = 69{\cdot}9$ and $b = 0{\cdot}75$ **(1)**

$y_c = 69.9 + 0.75x$ **(1)**

Thus, trend values are

$$y_{2010} = 69{\cdot}9 + 0{\cdot}75(-4) = 66{\cdot}90$$

$$y_{2012} = 69{\cdot}9 + 0{\cdot}75(-2) = 68{\cdot}40$$

$$y_{2013} = 69{\cdot}9 + 0{\cdot}75(-1) = 69{\cdot}15$$

$$y_{2014} = 69{\cdot}9 + 0{\cdot}75(0) = 69{\cdot}90$$

$$y_{2015} = 69{\cdot}9 + 0{\cdot}75(1) = 70{\cdot}65$$

$$y_{2016} = 69{\cdot}9 + 0{\cdot}75(2) = 71{\cdot}40$$

(1 for correct trend values)

$$y_{2019} = 69{\cdot}9 + 0{\cdot}75\ (5) = 73{\cdot}65$$

35. Define Compound Annual Growth Rate (CAGR) and give the formula for calculating CAGR. Using the formula, calculate CAGR of Vikas's investment given below :

Vikas invested ₹ 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below :

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

[Use $(1{\cdot}4)^{1/6} = 1{\cdot}058$]

Sol. CAGR is the mean annual growth rate of an investment over a specified period of time longer than one year. **(1)**

$$\text{CAGR} = \left[\frac{\text{Ending investment amount}}{\text{Start amount}}\right]^{\frac{1}{\text{no.of years}}} - 1$$ **(1)**

$$\left.\begin{aligned} &\text{P.V.} = ₹\ 10{,}000 \\ &\text{F.V.} = ₹\ 14{,}000 \\ &n = 6 \text{ years} \end{aligned}\right\}$$ **(1)**

$$\text{So, CAGR} = \left(\frac{14000}{10000}\right)^{1/6} - 1 = (1{\cdot}4)^{1/6} - 1$$ **$(\frac{1}{2})$**

$$= 1{\cdot}058 - 1$$ **$(\frac{1}{2})$**

$$= 0{\cdot}058$$ **$(\frac{1}{2})$**

Hence, CAGR = 5·8% **$(\frac{1}{2})$**

SECTION E

This section comprises of 3 case-study based questions of **4 marks each.**

36.	A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs. Based on the above information, answer the following questions : (i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective. (Use : $e^{-6} = 0{\cdot}0024$) (ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs. (iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb. **OR** (iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs.	
Sol.	n= 100, p = $\frac{6}{100}$, m= np Here m = $100 \times \frac{6}{100} = 6$. $P(r) = e^{-m}\frac{m^r}{r!}$ (i) $P(0) = e^{-m}\frac{m^0}{0!} = e^{-6} = 0{\cdot}0024$ (ii) $P(2) = e^{-m}\frac{m^2}{2!} = e^{-6} \times \frac{36}{2} = 0.0432$ (iii)(a) $P(0) + P(1) = e^{-6} + e^{-6}\frac{m^1}{1!} = e^{-6} + 6e^{-6} = 7e^{-6} = 0{\cdot}0168$ **OR** (iii)(b) Mean = Variance = m = np = 6	**(1)** **(1)** **(1+1)** **(1+1)**

37. A factory manufactures tennis rackets and cricket bats. A tennis racket takes $1\frac{1}{2}$ hours of machine time and 3 hours of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively.

Based on the above information, answer the following questions :

(i) If x and y are the numbers of bats and rackets manufactured by the factory, then write the expression of total profit.

(ii) Write the constraint that relates the number of craftsmanship hours.

(iii) (a) Determine the maximum profit (in ₹) earned by the factory.

OR

(iii) (b) How many bats and rackets respectively, are to be manufactured to earn maximum profit ?

Sol. (i) Z = 10x + 20y **(1)**

(1)

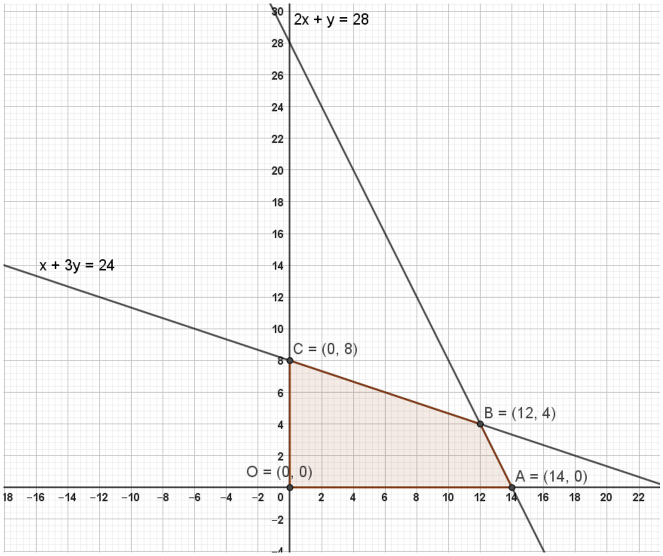
(ii) $x + 3y \leq 24$

(iii) (a) other constraints are

$2x + y \leq 28$

$x \geq 0$

$y \geq 0$



(1)

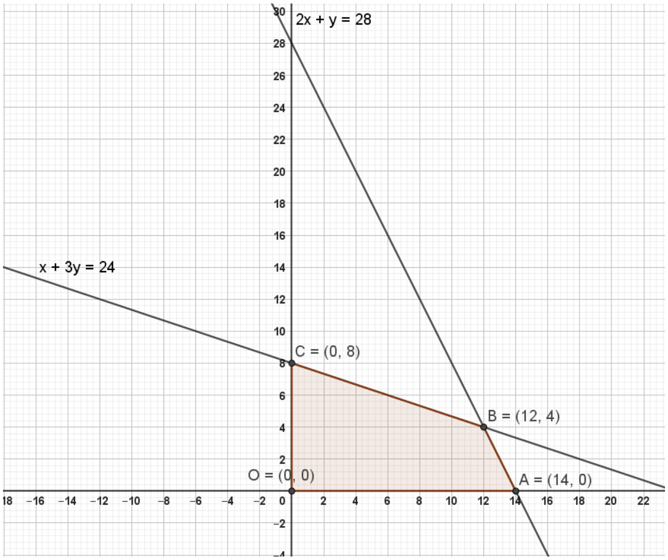
Corner Points	Value of Z
O (0,0)	0
A (14,0)	140
B (12,4)	200 → Max value
C (0,8)	160

(1)

∴ P is maximum at B (12,4); which is ₹ 200

OR

(iii) (b)



(1)

Corner Points	Value of Z
O (0,0)	0
A (14,0)	140
B (12,4)	200 → Max value
C (0,8)	160

(1)

12 bats and 4 rackets

38. In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7·5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions :

(i) Determine the EMI.

(ii) Find the principal paid by Mr. Aggarwal in the 150th instalment.

(iii) (a) Find the total interest paid by Mr. Aggarwal.

OR

(iii) (b) How much was paid by Mr. Aggarwal to repay the entire amount of home loan ?

[Use $(1\cdot00625)^{240} = 4\cdot4608$; $(1\cdot00625)^{91} = 1\cdot7629$]

Sol. Given P = ₹ 30,00,000, $i = \frac{7\cdot5}{1200} = 0\cdot00625$

and n = 12 × 20 = 240 months

(i) $\text{EMI} = \frac{P\,i}{1-(1+i)^{-n}}$

$= \frac{30,00,000 \times 0\cdot00625}{1-(1\cdot00625)^{-240} - 1}$ $\left(\frac{1}{2}\right)$

$= \frac{30,00,000 \times 0\cdot00625 \times 4\cdot4608}{3\cdot4608}$ $\left(\frac{1}{2}\right)$

= ₹ 24167.82

(ii) Interest paid on 150th instalment

$= \frac{\text{EMI} \times [(1+\text{i})^{240-150+1} - 1]}{(1+\text{i})^{240-150+1}}$

$= \frac{24167 \times [1\cdot7629 - 1]}{1\cdot7629}$ $\left(\frac{1}{2}\right)$

= ₹ 10458.70

⇒ Principal paid in 150th instalment = EMI – interest

=₹ (24167.82 – 10458.70) $\left(\frac{1}{2}\right)$

	= ₹ 13709.12	
	(iii) (a) Total Interest paid = n × EMI – P	
	= ₹ (240 × 24167.82 – 30,00,000)	**(1)**
	= ₹ 28,00,276.80	**(1)**
	OR	
	(iii) (b) Total amount paid = n x EMI	
	= 240 x 2416.81	**(1)**
	= ₹ 5800276.8	**(1)**

General Instructions :

Read the following instructions very carefully and strictly follow them :

- *(i)* *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*
- *(ii)* *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*
- *(iii)* *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*
- *(iv)* *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*
- *(v)* *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*
- *(vi)* *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*
- *(vii)* *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*
- *(viii)* *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 2 questions in Section E.*
- *(ix)* *Use of calculators is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If 100 $\equiv$ x (mod 7), then the least positive value of x is :

(a) 6 (b) 4

(c) 3 (d) 2

2. In a kilometre race, A beats B by 50 metres or 10 seconds. The time taken by A to complete the race is :

(a) 90 seconds (b) 120 seconds

(c) 190 seconds (d) 200 seconds

P.T.O.

3. If a man rows 32 km downstream and 14 km upstream in 6 hours each, then the speed of the stream is :

(a) 2 km/h
(b) 1·5 km/h
(c) 2·5 km/h
(d) 2·25 km/h

4. If $-3x + 17 < -13$, then :

(a) $x \in (10, \infty)$
(b) $x \in [10, \infty)$
(c) $x \in (-\infty, 10]$
(d) $x \in [-10, 10]$

5. If A and B are two matrices such that AB = A and BA = B, then B^2 is equal to :

(a) B
(b) A
(c) I
(d) O

6. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ is a symmetric matrix, then :

(a) $x = 0, y = 5$
(b) $x = 5, y = 0$
(c) $x = y$
(d) $x + y = 0$

7. The value of x for which the points $(2, -1)$, $(-3, 4)$ and $(x, 5)$ are collinear, is :

(a) -4
(b) 4
(c) 2
(d) -2

P.T.O.

8. If $x + y = 8$, then the maximum value of xy is :

(a) 12 (b) 16

(c) 20 (d) 24

9. The function $f(x) = x^x$, $x > 0$ is decreasing in the interval :

(a) $(-\infty, e)$ (b) $(0, e)$

(c) $\left(0, \frac{1}{e}\right)$ (d) $\left[\frac{1}{e}, \infty\right)$

10. $\int \frac{1}{x + x \log x} dx$ is equal to :

(a) $1 + \log x + C$ (b) $x + \log x + C$

(c) $x \log (1 + \log x) + C$ (d) $\log (1 + \log x) + C$

11. If the supply function is $p = 4 + x$, then the producer's surplus when 12 units are sold, is :

(a) 72 (b) 64

(c) 76 (d) 46

12. The order of the differential equation corresponding to family of curves $y = Ae^{3x} + Be^{-3x}$ is :

(a) 1 (b) 2

(c) 3 (d) 4

P.T.O.

13. If 'm' is the mean of Poisson distribution, then P(r = 0) is given by :

(a)	e^{-m}	(b)	e^{m}
(c)	e	(d)	m^{-e}

14. Normal distribution is symmetric about :

(a)	variance	(b)	standard deviation
(c)	mean	(d)	covariance

15. For a student's t-test, the test statistic t is given by :

(a)	$t = \bar{x} - \mu$	(b)	$t = \frac{\bar{x}}{s}$
(c)	$t = \frac{\bar{x} - \mu}{s}$	(d)	$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$

16. For the given values 15, 23, 28, 36, 41, 46, the 3-yearly moving averages are :

(a)	24, 29, 35, 41	(b)	22, 28, 35, 41
(c)	22, 29, 35, 41	(d)	24, 28, 35, 41

17. The effective rate of return which is equivalent to nominal rate of 8% p.a. compounded quarterly is : [Given $(1{\cdot}02)^4 = 1{\cdot}0824$]

(a)	8·16%	(b)	7·95%
(c)	8·24%	(d)	8·5%

18. The maximum value of the function z = 7x + 5y, subject to constraints $x \le 3, y \le 2, x \ge 0, y \ge 0$, is :

(a)	21	(b)	10
(c)	31	(d)	37

P.T.O.

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. *Assertion (A) :* The slope of the normal to the curve $y = 2x^2 - 5x$ at $x = -1$ is -1.

Reason (R) : The slope of the normal to the curve $y = f(x)$ at point (α, β) is given by $(x - \alpha) + \left(\frac{dy}{dx}\right)_{(\alpha, \beta)} \cdot (y - \beta) = 0$.

20. *Assertion (A) :* If A is a square matrix of order 3 such that $|\text{adj } A| = 144$, then the value of $|A|$ is ± 12.

Reason (R) : If A is an invertible matrix of order n, then $|\text{adj } A| = |A|^{n-1}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Solve the inequality :

$$\frac{3}{5}x - \frac{2x - 1}{3} > 1, x \in W$$

OR

(b) Solve the inequality :

$$-\frac{2}{3} < -\frac{x}{3} + 1 \leq \frac{2}{3}, x \in R$$

P.T.O.

22. Write the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

23. (a) It is given that 2% of screws manufactured by a company are defective. Using Poisson distribution, find the probability that a packet of 100 screws contains no defective screw.

(Given : $e^{-2} = 0 \cdot 14$)

OR

(b) If the standard deviation of a Poisson variable X is $\sqrt{3}$, then find $P(X > 0)$. [Use : $e^{-3} = 0 \cdot 05$]

24. Find the present value of a sequence of payments of ₹ 1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually.

25. The value of a machine purchased two years ago, depreciates at the annual rate of 10%. If its present value is ₹ 97,200, find.

(i) its value after 3 years;

(ii) its value when it was purchased.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Formulate the following problem as an LPP :

A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit.

P.T.O.

27. A person can row a boat at 5 km/h in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.

28. A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated four more times. What is the amount of juice left in the container after final replacement ? [Take $(0{\cdot}9)^5 = 0{\cdot}59049$]

OR

In a 1000-metre race, A, B and C get Gold, Silver and Bronze medals respectively. If A beats B by 100 metres and B beats C by 100 metres, then by how many metres does A beat C ?

29. A fair coin is tossed 9 times. Find the probability of getting.

(i) exactly 5 tails;

(ii) at least 5 tails;

(iii) at most 5 tails.

OR

Let X denote the number of hours a person watches T.V. during a randomly selected day. The probability that X can take the values x_i, has the following form, where k is some unknown constant.

$$P(X = x_i) = \begin{cases} 0{\cdot}2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.

(ii) Find : $P(X = 2)$, $P(X \geq 2)$ and $P(X \leq 2)$.

P.T.O.

30. 2000 students appeared in an examination. Distribution of marks is assumed to be normal with mean 30 and standard deviation 6·25. How many students are expected to get marks

(i) between 20 and 40 ?

(ii) less than 25 ?

[Use : $P(0 \leq z \leq 1{\cdot}6) = 0{\cdot}4452$, $P(0 \leq z \leq 0{\cdot}8) = 0{\cdot}2881$]

31. The mean of IQs of a group of 18 students from one area of a city was found to be 95 with a standard deviation of 10, while the mean of IQs of a group of 12 students from another area of the city was found to be 100 with a standard deviation of 8. Test whether there is a significant difference between the IQs of two groups of students at 1% level of significance. [Use : $t_{28}(0{\cdot}01) = 2{\cdot}763$]

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Determine for what values of x, the function $f(x) = x^4 - \frac{x^3}{3}$ is strictly increasing or strictly decreasing.

OR

(b) A firm has the following total cost and demand functions :

$$C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50 \text{ and } x = 100 - p$$

(i) Find the total revenue function in terms of x.

(ii) Find the total profit function P in terms of x.

(iii) Find the profit maximizing level of output of x.

(iv) What is the maximum profit, taking rupee as a unit of money ?

P.T.O.

33. (a) A company establishes a sinking fund to provide for the payment of ₹ 1,00,000 debt, maturing in 4 years. Contributions to the fund are to be made at the end of year. Find the amount of each annual deposit if interest is 18% per annum. [Use $(1{\cdot}18)^4 = 1{\cdot}9388$]

OR

(b) A firm bought a machinery for ₹ 7,40,000 on 1st April, 2020 and ₹ 60,000 is spent on its installation. Its useful life is estimated to be of 5 years. Its scrap value at the end of 5 years is estimated to be ₹ 40,000. Find the amount of annual depreciation and the rate of depreciation.

34. A person takes a housing loan worth ₹ 10,00,000 at an interest rate of 6% p.a compounded monthly. He decided to repay the loan by equal monthly instalments in 15 years. Calculate the EMI, using

(i) flat rate method,

(ii) reducing balance method.

[Given : $(1{\cdot}005)^{-180} = 0{\cdot}4074824$]

35. A library has to accommodate two different types of books on a shelf. The books are each 6 cm and 4 cm thick and each weighs 1 kg and $1\frac{1}{2}$ kg respectively. The shelf is 96 cm long and can support a weight of atmost 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books ? Formulate it as an L.P.P and so solve it graphically.

P.T.O.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students, and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and the second group is four times that of the third group. Assume that x, y and z denote the number of students in first, second and third group respectively.

Based on the above information, answer the following questions :

(a) Write the system of linear equations that can be formulated from the above described situation. *1*

(b) Write the coefficient matrix, say A. *1*

(c) (i) Write the matrix of cofactors of every element of matrix A. *2*

OR

(c) (ii) Determine the number of students of each group. *2*

Case Study – 2

37. A company notes that higher sales of a particular item, which it produced, is achieved by lowering the price charged. As a result, the total revenue from the sales at first rises as the number of units sold increases, reaches the highest point, and then falls off. The pattern of revenue is described by the relation $y = 40,00,000 - (x - 2000)^2$, where y is the total revenue and x is the number of units sold.

Based on the above information, answer the following questions :

(a) Find what number of units sold maximizes total revenue. *2*

(b) What is the amount of this maximum revenue ? *1*

(c) What would be the total revenue if 2500 units were sold ? *1*

P.T.O.

Case Study – 3

38. The following data shows the percentage of rural, urban and sub-urban Indians who have high speed internet connection at home.

Year	Rural	Urban	Sub-urban
2016	3	9	9
2017	6	18	17
2018	9	21	23
2019	16	29	29
2020	24	38	40

Based on the above information, answer the following questions :

(a) Derive straight-line trend by the method of least squares for the rural students. *2*

OR

(a) Derive straight-line trend by the method of least squares for the urban Indians. *2*

(b) What is the forecast for the year 2021 for urban group using trend equation ? *1*

(c) What is the forecast for the year 2021 for rural group using trend equation ? *1*

MARKING SCHEME
MATHEMATICS (Subject Code–241)
(PAPER CODE: 465)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	If $100 \equiv x \pmod 7$, then the least positive value of x is : (a) 6 (b) 4 (c) 3 (d) 2	
Ans.	(d) 2	**1**
2.	In a kilometre race, A beats B by 50 metres or 10 seconds. The time taken by A to complete the race is : (a) 90 seconds (b) 120 seconds (c) 190 seconds (d) 200 seconds	
Ans.	(c) 190 seconds	**1**
3.	If a man rows 32 km downstream and 14 km upstream in 6 hours each, then the speed of the stream is : (a) 2 km/h (b) 1·5 km/h (c) 2·5 km/h (d) 2·25 km/h	
Ans.	(b) 1.5 km/h	**1**
4.	If $-3x + 17 < -13$, then : (a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10]$	

Ans.	(a) $\boldsymbol{x \in (10, \infty)}$	**1**
5.	If A and B are two matrices such that AB = A and BA = B, then B^2 is equal to : (a) B (b) A (c) I (d) O	
Ans.	(a) B	**1**
6.	If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ is a symmetric matrix, then : (a) $x = 0, y = 5$ (b) $x = 5, y = 0$ (c) $x = y$ (d) $x + y = 0$	
Ans.	(c) $\boldsymbol{x = y}$	**1**
7.	The value of x for which the points (2, – 1), (– 3, 4) and (x, 5) are collinear, is : (a) – 4 (b) 4 (c) 2 (d) – 2	
Ans.	(a) $\boldsymbol{-4}$	**1**
8.	If $x + y = 8$, then the maximum value of xy is : (a) 12 (b) 16 (c) 20 (d) 24	
Ans.	(b) 16	**1**

9.	The function $f(x) = x^x, x > 0$ is decreasing in the interval : (a) $(-\infty, e)$ (b) $(0, e)$ (c) $\left(0, \frac{1}{e}\right)$ (d) $\left[\frac{1}{e}, \infty\right)$	
Ans.	(c) $(\mathbf{0}, \frac{1}{e})$	**1**
10.	$\int \frac{1}{x + x \log x} dx$ is equal to : (a) $1 + \log x + C$ (b) $x + \log x + C$ (c) $x \log (1 + \log x) + C$ (d) $\log (1 + \log x) + C$	
Ans.	(d) $\mathbf{\log(1 + \log x) + C}$	**1**
11.	If the supply function is $p = 4 + x$, then the producer's surplus when 12 units are sold, is : (a) 72 (b) 64 (c) 76 (d) 46	
Ans.	(a) 72	**1**
12.	The order of the differential equation corresponding to family of curves $y = Ae^{3x} + Be^{-3x}$ is : (a) 1 (b) 2 (c) 3 (d) 4	
Ans.	(b) 2	**1**
13.	If 'm' is the mean of Poisson distribution, then $P(r = 0)$ is given by : (a) e^{-m} (b) e^{m} (c) e (d) m^{-e}	

Ans.	(a) e^{-m}	**1**
14.	Normal distribution is symmetric about : (a) variance (b) standard deviation (c) mean (d) covariance	
Ans.	(c) mean	**1**
15.	For a student's t-test, the test statistic t is given by : (a) $t = \overline{x} - \mu$ (b) $t = \frac{\overline{x}}{s}$ (c) $t = \frac{\overline{x} - \mu}{s}$ (d) $t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n-1}}}$	
Ans.	(d) $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$	**1**
16.	For the given values 15, 23, 28, 36, 41, 46, the 3-yearly moving averages are : (a) 24, 29, 35, 41 (b) 22, 28, 35, 41 (c) 22, 29, 35, 41 (d) 24, 28, 35, 41	
Ans.	(c) 22, 29, 35, 41	**1**
17.	The effective rate of return which is equivalent to nominal rate of 8% p.a. compounded quarterly is : [Given $(1{\cdot}02)^4 = 1{\cdot}0824$] (a) 8·16% (b) 7·95% (c) 8·24% (d) 8·5%	
Ans.	(c) 8.24 %	**1**

18.	The maximum value of the function z = 7x + 5y, subject to constraints $x \le 3, y \le 2, x \ge 0, y \ge 0$, is : (a) 21 (b) 10 (c) 31 (d) 37	
Ans.	(c) 31	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.	
19.	*Assertion (A) :* The slope of the normal to the curve $y = 2x^2 - 5x$ at $x = -1$ is -1. *Reason (R) :* The slope of the normal to the curve y = f(x) at point (α, β) is given by $(x - \alpha) + \left(\frac{dy}{dx}\right)_{(\alpha, \beta)} \cdot (y - \beta) = 0$.	
Ans.	Both (A) and (R) are false, so 1 mark may be given to all who attempted	**1**
20.	*Assertion (A) :* If A is a square matrix of order 3 such that $\lvert \text{adj } A \rvert = 144$, then the value of $\lvert A \rvert$ is ± 12. *Reason (R) :* If A is an invertible matrix of order n, then $\lvert \text{adj } A \rvert = \lvert A \rvert^{n-1}$.	
Ans.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21(a).	Solve the inequality : $\frac{3}{5}x - \frac{2x - 1}{3} > 1, x \in W$	

Ans.	$\frac{3}{5}x-\frac{2x-1}{3}>1$ $\Rightarrow 9x-5(2x-1)>15$ Solving we get, $x<-10$ Solution set is $\emptyset$	**1** ½ ½
	OR	
21(b).	Solve the inequality : $$-\frac{2}{3}<-\frac{x}{3}+1\leq\frac{2}{3}, x\in R$$	
Ans.	$-\frac{2}{3}<-\frac{x}{3}+1\leq\frac{2}{3}$ $\Rightarrow -\frac{5}{3}<-\frac{x}{3}\leq-\frac{1}{3}$ $\Rightarrow \frac{1}{3}\leq\frac{x}{3}<\frac{5}{3}$ $\Rightarrow 1\leq x<5$ Solution set is $[1,5)$	**1** **1**
22.	Write the matrix $\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.	
Ans.	Let $A=\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\Rightarrow A'=\begin{bmatrix}7 & -1 & -1\\ -3 & 1 & 0\\ -3 & 0 & 1\end{bmatrix}$ $P=\frac{A+A'}{2}=\begin{bmatrix}7 & -2 & -2\\ -2 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}$ and $Q=\frac{A-A'}{2}=\begin{bmatrix}0 & -1 & -1\\ 1 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}$ So, $P'=P\Rightarrow$ symmetric, $Q'=-Q\Rightarrow$ skew symmetric $A=\frac{A+A'}{2}+\frac{A-A'}{2}=\begin{bmatrix}7 & -2 & -2\\ -2 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -1 & -1\\ 1 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}$	½ ½+½ ½
23(a).	It is given that 2% of screws manufactured by a company are defective. Using Poisson distribution, find the probability that a packet of 100 screws contains no defective screw. (Given : $e^{-2}=0{\cdot}14$)	
Ans.	Let p be the probability that the screw is defective	

	Then $p=\frac{2}{100}$. Here $n=100$ So, $m=np=2$ Let X denote the number of defective screws in a packet of 100 screws. Then X follows the probability distribution as $P(X=r)=e^{-m}\frac{m^r}{r!}, r=0,1,2\ldots$ P(no defective screw) $=P(X=0)=e^{-m}\frac{m^0}{0!}=e^{-2}=0.14$	1 1
	OR	
23(b).	If the standard deviation of a Poisson variable X is $\sqrt{3}$, then find P (X > 0). [Use : $e^{-3}=0{\cdot}05$]	
Ans.	Let X follows Poisson distribution with parameter m Then Variance $=m=(\sqrt{3})^2=3$ $\therefore P(X=r)=e^{-m}\frac{m^r}{r!}=e^{-3}\frac{3^r}{r!}, r=0,1,2\ldots$ Hence $P(X>0)=1-P(X=0)=1-e^{-3}=1-0.05=0.95$	½ 1½
24.	Find the present value of a sequence of payments of ₹ 1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually.	
Ans.	The given annuity is a perpetuity of first type in which $R=$ ₹ 1000 and $r=\frac{8}{2}\%=4\%$ per half year So, $i=\frac{4}{100}=0.04$ Present value $=P=\frac{R}{i}=\frac{1000}{0.04}=25000$ Hence the present value is ₹ 25000	1 1
25.	The value of a machine purchased two years ago, depreciates at the annual rate of 10%. If its present value is ₹ 97,200, find. (i) its value after 3 years; (ii) its value when it was purchased.	
Ans.	Given $P=$ ₹ $97{,}200, r=10\%$ p.a $\Longrightarrow i=\frac{10}{100}=0.1$ (i) Value after 3 years = Present value × $(1-0.1)^3$	

	$= ₹\ 97200(0.9)^3$ $= ₹\ 70858.80$ (ii) Present value = value 2 years ago × $(1-0.1)^2$ $\Longrightarrow 97200 =$Value 2 years ago $\times\ (0.9)^2$ $\Longrightarrow$ Value 2 years ago $= ₹\ \frac{97200}{(0.9)^2} = ₹\ 120000$	**1** **1**
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	Formulate the following problem as an LPP : A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit.	
Ans.	Let $\boldsymbol{x}$ and $\boldsymbol{y}$ denote the number of rings and chains respectively Maximize $\boldsymbol{Z = 300\,x + 190\,y}$ Subject to constraints $\left.\begin{array}{c} \boldsymbol{x + y \leq 24} \\ \boldsymbol{x + \frac{y}{2} \leq 16} \\ \boldsymbol{x, y \geq 0} \end{array}\right\}$	**1** **2**
27.	A person can row a boat at 5 km/h in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.	
Ans.	Let the rate at which the stream is flowing be x km/h and let the distance covered by the boat be y km According to question, $\frac{3y}{5+x} = \frac{y}{5-x}$ $\Longrightarrow 3(5-x) = 5+x$ $\Longrightarrow x = 2.5$ Thus, the stream is flowing at the rate of 2.5 km/h	**2** **1**

28(a).	A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated four more times. What is the amount of juice left in the container after final replacement ? [Take $(0{\cdot}9)^5 = 0{\cdot}59049$]	
Ans.	Total juice in the container = 50 litres Juice taken out = 5 litres No. of times process repeated = 5 Amount of juice in container after final replacement $= \mathbf{50\left(1-\frac{5}{50}\right)^{5}}$ $= \mathbf{29.52}$ litres	**2** **1**
28(b).	In a 1000-metre race, A, B and C get Gold, Silver and Bronze medals respectively. If A beats B by 100 metres and B beats C by 100 metres, then by how many metres does A beat C ?	
Ans.	A beats B by 100 metres, means A travels 1000 metres in the same time in which B travels 900 metres. B beats C by 100 metres, means B travels 1000 metres in the same time in which C travels 900 metres. $\left.\begin{array}{l} \therefore A : B = 10 : 9 \\ B : C = 10 : 9 \end{array}\right\}$ $\Rightarrow \mathrm{A : B : C} = 100 : 90 : 81$ So, A travels 100 metres and in the same time C travels 81 metres Thus, A beats C by 190 metres	**1** **1** **1**
29(a).	A fair coin is tossed 9 times. Find the probability of getting. (i) exactly 5 tails; (ii) at least 5 tails; (iii) at most 5 tails.	

Ans.

Repeated tosses of a fair coin qualify as Bernoulii's trials

Let X denote the number of trails in an experiment of such trials and hence is the binomial distribution

Here $n = 9, p = \frac{1}{2}$ and $q = 1 - \frac{1}{2} = \frac{1}{2}$

(a) $P(\text{exactly 5 success}) = P(X = 5) = 9_{C_5} p^5 q^4 = 9_{C_5} \left(\frac{1}{2}\right)^9$

$$= \frac{63}{256}$$

1

(b) $P(\text{at least 5 successes}) = P(X \geq 5)$

$$= \left(\frac{1}{2}\right)^9 \left[9_{C_5} + 9_{C_6} + 9_{C_7} + 9_{C_8} + 9_{C_9}\right]$$

$$= \frac{256}{512} = \frac{1}{2}$$

1

(c) $P(\text{at most 5 successes}) = P(X \leq 5) = 1 - P(X > 5)$

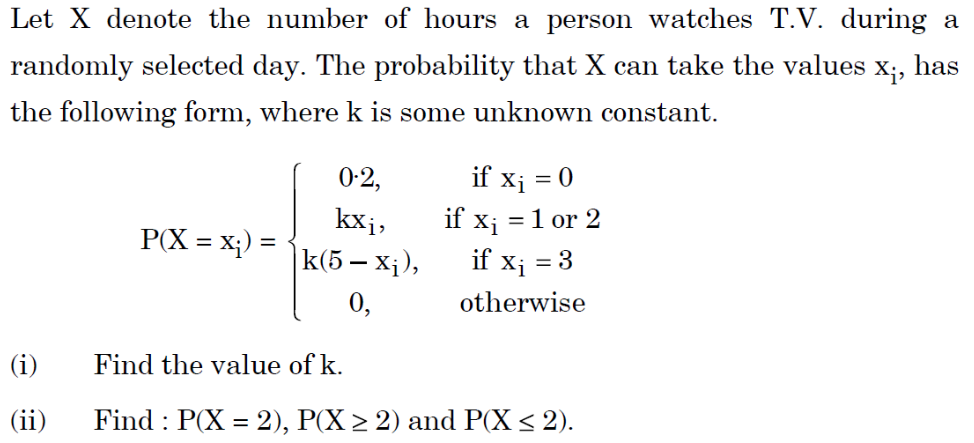
$$= 1 - \left(\frac{1}{2}\right)^9 \left[9_{C_6} + 9_{C_7} + 9_{C_8} + 9_{C_9}\right]$$

$$= 1 - \frac{130}{512} = \frac{382}{512} = \frac{191}{256}$$

1

OR

29(b).



Let X denote the number of hours a person watches T.V. during a randomly selected day. The probability that X can take the values x_i, has the following form, where k is some unknown constant.

$$P(X = x_i) = \begin{cases} 0{\cdot}2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.

(ii) Find : $P(X = 2)$, $P(X \geq 2)$ and $P(X \leq 2)$.

Ans.

x_i	0	1	2	3	otherwise
$P(x_i) = p_i$	0.2	k	$2k$	$2k$	0

½

(i) As $\sum p_i = 1$, we have

$$0.2 + k + 2k + 2k = 1$$

$$\Rightarrow 5k = 0.8 \Rightarrow k = 0.16$$

1

	(ii) $P(X=2)=2k=0.32$ $P(X\geq 2)=2k+2k=0.64$ $P(X\leq 2)=0.2+k+2k=0.68$	½ ½ ½
30.	2000 students appeared in an examination. Distribution of marks is assumed to be normal with mean 30 and standard deviation 6·25. How many students are expected to get marks (i) between 20 and 40 ? (ii) less than 25 ? [Use : P(0 ≤ z ≤ 1·6) = 0·4452, P(0 ≤ z ≤ 0·8) = 0·2881]	
Ans.	Let X denote the marks of the student. Mean $(\mu)=30$, S.D $(\sigma)=6.25$ $Z=\frac{X-\mu}{\sigma}=\frac{X-30}{6.25}$ (i) When $X=20, Z=-1.60$ When $X=40, Z=1.60$ $\therefore P(20\leq X\leq 40)=P(-1.60\leq Z\leq 1.60)$ $=P(-1.60\leq Z\leq 0)+P(0\leq Z\leq 1.60)$ $=2P(0\leq Z\leq 1.60)$ $=2\times 0.4452=0.8904$ Thus, out of 2000 students, the expected number of students getting marks between 20 and 40 = $2000\times 0.8904=1780.8$ or 1781 (ii) When $X=25, Z=-0.80$ So, $P(X<25)=P(Z<-0.8)=P(Z>0.8)$ $=P(Z\geq 0)-P(0\leq Z\leq 0.8)$ $=0.5-0.2881=0.2119$ Thus, out of 2000 students the expected number of students getting marks less than 25 = $2000\times 0.2119=423.8$ or 424	½ ½ 1 1
31.	The mean of IQs of a group of 18 students from one area of a city was found to be 95 with a standard deviation of 10, while the mean of IQs of a group of 12 students from another area of the city was found to be 100 with a standard deviation of 8. Test whether there is a significant difference between the IQs of two groups of students at 1% level of significance. [Use : $t_{28}(0{\cdot}01)=2{\cdot}763$]	
Ans.	Here $n_1=18, \overline{x}=95, s_1=10$ and $n_2=12, \overline{y}=100, s_2=8$	

	$\therefore s = \sqrt{\frac{n_1 {s_1}^2 + n_2 {s_2}^2}{n_1 + n_2 - 2}} = \sqrt{\frac{18 \times 10^2 + 12 \times 8^2}{18+12-2}}$ $= \sqrt{\frac{2568}{28}} = 9.58$ Let H_0= no significant difference between IQs of two group of students H_α= significant difference between IQs of two group of students The test statistic $'t'$ is $t = \frac{\bar{x} - \bar{y}}{s} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$ $= \frac{95-100}{9.58} \times \sqrt{\frac{18 \times 12}{18+12}} = -\frac{5}{9.58} \times 2.68 = -1.398$ $\Rightarrow \lvert t \rvert = 1.398$ The test statistics t follows student's t-distribution with $n = 18 + 12 - 2$ i.e., 28 degrees of freedom At 1% level of significance, we have $t_{28}(0.01) = 2.763$ As $1.398 < 2.763$, null hypothesis is accepted Hence, there is no significance difference between IQs of two groups	1½ 1 ½
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32(a).	Determine for what values of x, the function f(x) = $x^4 - \frac{x^3}{3}$ is strictly increasing or strictly decreasing.	
Ans.	$f(x) = x^4 - \frac{x^3}{3} \Rightarrow f'(x) = 4x^3 - x^2 = x^2(4x-1)$ $f'(x) = 0 \Rightarrow \mathrm{x} = 0, \frac{1}{4}$ Thus, critical points divide R into three parts $x < 0, 0 < x < \frac{1}{4}, x > \frac{1}{4}$ When $x < 0, f'(x)$ is $-$ve $\therefore f(x)$ is strictly decreasing for $x < 0$ When $0 < x < \frac{1}{4}, f'(x)$ is $+$ve $\therefore f(x)$ is strictly increasing for $x < 0$ When $x > \frac{1}{4}, f'(x)$ is $-$ve $\therefore f(x)$ is strictly decreasing for $x < 0$ Hence $f(x)$ is strictly increasing on $\left(\frac{1}{4}, \infty\right)$ and strictly decreasing on $(-\infty, 0) \cup \left(0, \frac{1}{4}\right)$	1 ½ 1 1 1 ½

	OR	
32(b).	A firm has the following total cost and demand functions : $C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $x = 100 - p$ (i) Find the total revenue function in terms of x. (ii) Find the total profit function P in terms of x. (iii) Find the profit maximizing level of output of x. (iv) What is the maximum profit, taking rupee as a unit of money ?	
Ans.	Here $C = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $x = 100 - p$ i.e., $p = 100 - x$ (i) R, the revenue function is $R = px = (100 - x)x = 100\,x - x^2$ (ii) Profit function $P(x) = R(x) - C(x)$ $= (100\,x - x^2) - \left(\frac{x^3}{3} - 7x^2 + 111x + 50\right)$ $= -\frac{x^3}{3} + 6x^2 - 11x - 50$ (iii) $\frac{dP}{dx} = -x^2 + 12x - 11$ For P to be maximum, $\frac{dP}{dx} = 0 \Rightarrow x = 1, 11$ $\frac{d^2P}{dx^2} = -2x + 12 > 0$ at $x = 1$ and < 0 at $x = 11$ Thus P is maximum when $x = 11$ Hence, the profit maximising level of output is 11 units (iv) Maximum profit $= [P(x)]_{x=11}$ $= -\frac{(11)^3}{3} + 6(11)^2 - 11(11) - 50$ $= 111.33\ or\ 111$	½ 1 ½ 1 1 1
33(a).	A company establishes a sinking fund to provide for the payment of ₹ 1,00,000 debt, maturing in 4 years. Contributions to the fund are to be made at the end of year. Find the amount of each annual deposit if interest is 18% per annum. [Use $(1{\cdot}18)^4 = 1{\cdot}9388$]	
Ans.	Let each annual deposit to the sinking fund be ₹ R $\therefore 100000 = R\left[\frac{(1+0.18)^4-1}{0.18}\right]$ $= R\left[\frac{(1.18)^4-1}{0.18}\right]$	2

	$= R\left[\frac{0.4388}{0.18}\right] = R(5.2186)$ $\Rightarrow \mathrm{R} = \frac{100000}{5.2186} = \text{₹}\, 19162.22$	1½ 1½
	OR	
33(b).	A firm bought a machinery for ₹ 7,40,000 on 1st April, 2020 and ₹ 60,000 is spent on its installation. Its useful life is estimated to be of 5 years. Its scrap value at the end of 5 years is estimated to be ₹ 40,000. Find the amount of annual depreciation and the rate of depreciation.	
Ans.	$C = \text{₹}\, 7,40,000 + \text{₹}\, 60,000 = \text{₹}\, 8,00,000$ And $S = \text{₹}\, 40,000$ $\therefore$ Annual depreciation $= \frac{C-S}{n} = \text{₹}\, \frac{7,60,000}{5} = \text{₹}\, 1,52,000$ Rate of depreciation $= \frac{D}{C-S} \times 100$ $= \frac{1,52,000}{8,00,000-40,000} \times 100$ $= 20\%$	1 1 1 1 1
34.	A person takes a housing loan worth ₹ 10,00,000 at an interest rate of 6% p.a compounded monthly. He decided to repay the loan by equal monthly instalments in 15 years. Calculate the EMI, using (i) flat rate method, (ii) reducing balance method. [Given : $(1{\cdot}005)^{-180} = 0{\cdot}4074824$]	
Ans.	Here, $P = \text{₹}\, 10,00,000, i = \frac{6}{12\times100} = 0.005$ $n = 15$ years $= 180$ months (i) Using flat rate method $\text{EMI} = P\left(i + \frac{1}{n}\right)$ $= 10,00,000\left(0.005 + \frac{1}{180}\right) = \text{₹}\, 10555.55$ (ii) Using reducing balancing method $\text{EMI} = \frac{Pi}{1-(1+i)^{-n}}$ $= \frac{10,00,000\times0.005}{1-(1.005)^{-180}}$	1 1+½ 1

$$= \frac{5000}{1-0.4704824} = ₹\,8438.57$$ **1½**

35. A library has to accommodate two different types of books on a shelf. The books are each 6 cm and 4 cm thick and each weighs 1 kg and $1\frac{1}{2}$ kg respectively. The shelf is 96 cm long and can support a weight of atmost 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books ? Formulate it as an L.P.P and so solve it graphically.

Ans.

Types of boxes	Thickness (in cm)	Weight (in kg)
Type 1	6	1
Type 2	4	$1\frac{1}{2}$
Max Availability	96 cm	21 kg

Let the two types of boxes be x and y respectively

Let Z denote the maximum number of books that can be accommodated in the shelf

LPP is

$\therefore Z = x + y$ **1**

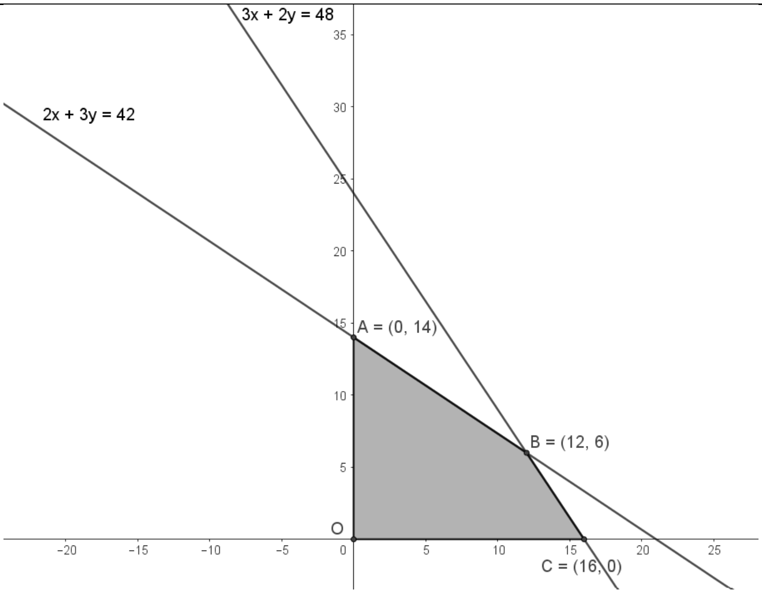
Subject to constraints

$6x + 4y \leq 96$ or $3x + 2y \leq 48$

$x + \frac{3}{2}y \leq 21$ or $2x + 3y \leq 42$

$x, y \geq 0$

1½

	 Here, $(Z)_A = 14, (Z)_B = 18, (Z)_C = 16$ So, Z is maximum at B Hence, the shelf should be filled with 12 books of type 1 and 6 books of type 2	**1½ For graph with correct region** **1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36.

Case Study – 1

10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students, and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and the second group is four times that of the third group. Assume that x, y and z denote the number of students in first, second and third group respectively.

Based on the above information, answer the following questions :

(a) Write the system of linear equations that can be formulated from the above described situation. *1*

(b) Write the coefficient matrix, say A. *1*

(c) (i) Write the matrix of cofactors of every element of matrix A. *2*

OR

(c) (ii) Determine the number of students of each group. *2*

Ans.

(a) $x + y + z = 10, 2x + y = 13, x + y = 4z$ **1**

(b) coefficient matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$ **1**

(c) (i) Cofactor matrix of $A = \begin{bmatrix} -4 & 8 & 1 \\ 5 & -5 & 0 \\ -1 & 2 & -1 \end{bmatrix}$ **2**

OR

(ii) Adj (A) $= \begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ and $|A| = 5 \neq 0$

$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ **1**

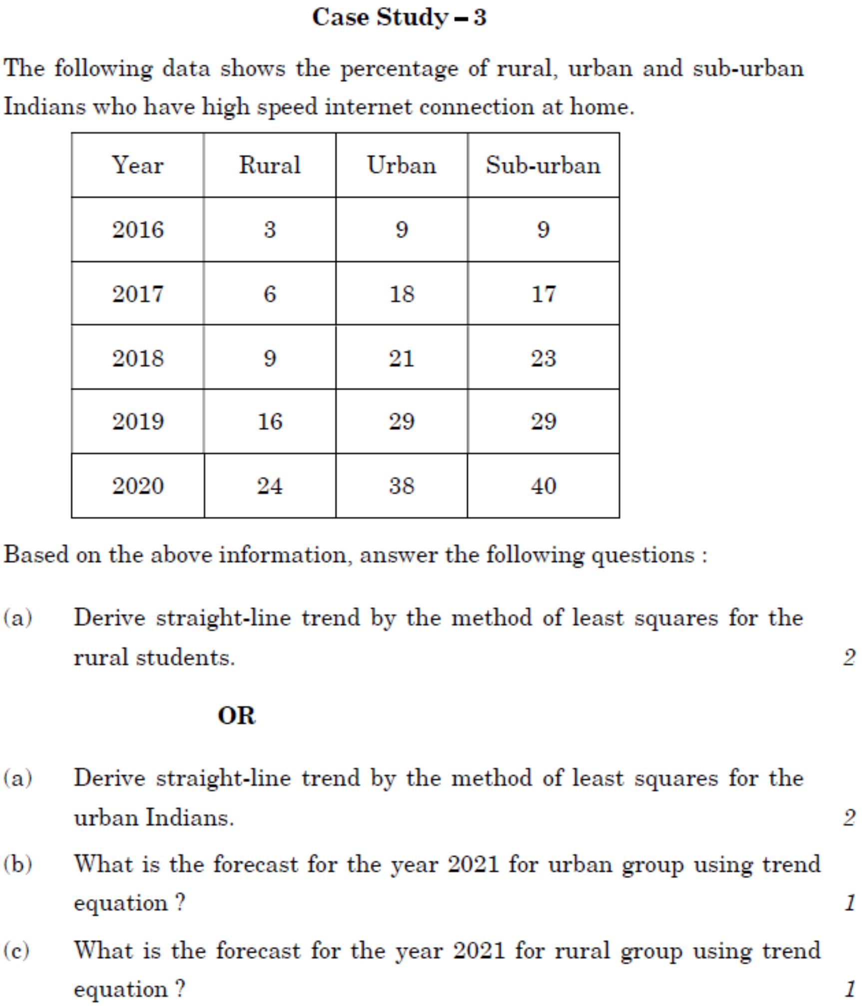
Thus, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 25 \\ 15 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$

$\therefore$ 5 students in first group, 3 students in second group and 2 students in third group **1**

37.	**Case Study – 2** A company notes that higher sales of a particular item, which it produced, is achieved by lowering the price charged. As a result, the total revenue from the sales at first rises as the number of units sold increases, reaches the highest point, and then falls off. The pattern of revenue is described by the relation y = 40,00,000 – (x – 2000)^2, where y is the total revenue and x is the number of units sold. Based on the above information, answer the following questions : (a) Find what number of units sold maximizes total revenue. *2* (b) What is the amount of this maximum revenue ? *1* (c) What would be the total revenue if 2500 units were sold ? *1*	
Ans.	(a) $y = 40,00,000 - (x - 2000)^2$ Gives $\frac{dy}{dx} = -2(x - 2000)$ So, $\frac{dy}{dx} = 0$ when $x = 2000$	**1½**
	$\frac{d^2y}{dx^2} = -2 < 0$ Hence y is max at $x = 2000$	**½**
	(b) Max Revenue $= 40,00,000 - (2000 - 2000)^2 = 40,00,000$	**1**
	(c) Total Revenue $= 40,00,000 - (2500 - 2000)^2$ $= 37,50,000$	**1**

38.



Case Study – 3

The following data shows the percentage of rural, urban and sub-urban Indians who have high speed internet connection at home.

Year	Rural	Urban	Sub-urban
2016	3	9	9
2017	6	18	17
2018	9	21	23
2019	16	29	29
2020	24	38	40

Based on the above information, answer the following questions :

(a) Derive straight-line trend by the method of least squares for the rural students. *2*

OR

(a) Derive straight-line trend by the method of least squares for the urban Indians. *2*

(b) What is the forecast for the year 2021 for urban group using trend equation ? *1*

(c) What is the forecast for the year 2021 for rural group using trend equation ? *1*

Ans.

(a)

y	x	x^2	xy
3	-2	4	-6
6	-1	1	-6
9	0	0	0
16	1	1	16
24	2	4	48
$\sum y = 58$		$\sum x^2 = 10$	$\Sigma xy = 52$

1½

Trend value is $y = \frac{\Sigma y}{5} + \frac{\Sigma xy}{\Sigma x^2} x$

$$y = 11.6 + 5.2\, x$$

½

OR

<table>
<tr>
<td></td>
<td>
<table>
<tr><th>y</th><th>x</th><th>x^2</th><th>xy</th></tr>
<tr><td>9</td><td>-2</td><td>4</td><td>-18</td></tr>
<tr><td>18</td><td>-1</td><td>1</td><td>-18</td></tr>
<tr><td>21</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>29</td><td>1</td><td>1</td><td>29</td></tr>
<tr><td>38</td><td>2</td><td>4</td><td>76</td></tr>
<tr><td>$\sum y = 115$</td><td></td><td>$\sum x^2 = 10$</td><td>$\sum xy = 69$</td></tr>
</table>
Trend value is $y = \frac{115}{5} + \frac{69}{10}x$
$y = 23 + 6.9\ x$
(b) For $x = 3$, we have
$y = 23 + 6.9\ (3) = 43.7$
(c) For $x = 3$, we have
$y = 11.6 + 5.2\ (3) = 27.2$
</td>
<td>1½
½
1
1</td>
</tr>
</table>

General Instructions :

Read the following instructions very carefully and strictly follow them :

- *(i)* *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*
- *(ii)* *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*
- *(iii)* *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*
- *(iv)* *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*
- *(v)* *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*
- *(vi)* *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*
- *(vii)* *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*
- *(viii)* *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.*
- *(ix)* *Use of calculators is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions of 1 mark each.

1. In a 1 km race, player P beats player Q by 18 metres or 9 seconds. What is P's time to complete the race ?

(A) 512 seconds (B) 502 seconds

(C) 491 seconds (D) 481 seconds

2. If $x > y$ and $z < 0$, then :

(A) $xz > yz$ (B) $xz \geq yz$

(C) $\frac{x}{z} > \frac{y}{z}$ (D) $\frac{x}{z} < \frac{y}{z}$

P.T.O.

3. If AB = A and BA = B, then $(B^2 + B)$ equals :

(A)	2A	(B)	O
(C)	2I	(D)	2B

4. The value of $\Delta = \begin{vmatrix} 42 & 2 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ is :

(A)	0	(B)	1
(C)	– 3	(D)	–15

5. If $y = e^{-2x}$, then $\frac{d^3y}{dx^3}$ is equal to :

(A)	$2\,e^{-2x}$	(B)	e^{-4x}
(C)	$4\,e^{-4x}$	(D)	$-8\,e^{-2x}$

6. The function $f(x) = x^2 - x + 1$ is :

(A) increasing in (0, 1)

(B) decreasing in (0, 1)

(C) increasing in $(0, \frac{1}{2})$ and decreasing in $(\frac{1}{2}, 1)$

(D) increasing in $(\frac{1}{2}, 1)$ and decreasing in $(0, \frac{1}{2})$

7. The **order** and the **degree** of the differential equation

$$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$$

are respectively :

(A)	1, 1	(B)	1, 2
(C)	2, 1	(D)	1, not defined

P.T.O.

8. A fair coin is tossed twice and outcomes are noted. If the random variable X represents the number of heads that appeared in the experiment, then the mathematical expectation of X is :

(A) 1
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $1\frac{1}{2}$

9. What time will it be after 1275 hours, if the present time is 9:00 p.m. ?

(A) 11 p.m.
(B) 12 p.m.
(C) 9 p.m.
(D) 9 a.m.

10. If for a Poisson variate X,

$$P(X = k) = P(X = k + 1),$$

then the variance of X is :

(A) $k - 1$
(B) k
(C) $k + 1$
(D) $k + 2$

11. If the calculated value of $|t| < t_v(\alpha)$ (critical value of t), then the null hypothesis :

(A) is rejected.
(B) is accepted.
(C) is neither accepted nor rejected.
(D) cannot be determined.

12. For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as :

(A) $n_1 - n_2 + 2$
(B) $n_1 - n_2 - 2$
(C) $n_1 + n_2 - 2$
(D) $n_1 + n_2 + 2$

P.T.O.

13. For the given values 23, 32, 40, 47, 58, 33, 42; the 5-yearly moving averages are :

(A) 38, 40, 42 (B) 40, 42, 44

(C) 40, 42, 46 (D) 42, 44, 46

14. Using flat rate method, the EMI to repay a loan of ₹ 20,000 in $2\frac{1}{2}$ years at an interest rate of 8% p.a. is :

(A) ₹ 700 (B) ₹ 800

(C) ₹ 900 (D) ₹ 100

15. A mobile phone costs ₹ 12,000 and its scrap value after a useful life of 3 years is ₹ 3,000. Then, the book value of the mobile phone at the end of 2 years is :

(A) ₹ 3,000 (B) ₹ 6,000

(C) ₹ 5,000 (D) ₹ 7,000

16. What sum of money should be deposited at the end of every 6 months to accumulate ₹ 50,000 in 8 years, if money is worth 6% p.a. compounded semi-annually ? [Given : $(1{\cdot}03)^{16} = 1{\cdot}6047$]

(A) ₹ 3,432·53 (B) ₹ 2,783·08

(C) ₹ 2,480·57 (D) ₹ 2,149·93

17. The graph of the inequation 2x + 3y > 6 is the :

(A) entire XOY-plane

(B) half-plane that contains the origin

(C) half-plane that neither contains the origin nor the points on the line 2x + 3y = 6

(D) whole XOY-plane excluding the points on the line 2x + 3y = 6

P.T.O.

18. In an LPP, if the objective function Z = ax + by has same maximum value on two corner points of the feasible region, then the number of points at which maximum value of Z occurs is :

(A) 0 (B) 2

(C) finite (D) infinite

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The function $f(x) = x^2 - x + 1$ is strictly increasing on $(-1, 1)$.

Reason (R) : If f(x) is continuous on [a, b] and derivable on (a, b), then f(x) is strictly increasing on [a, b] if $f'(x) > 0$ for all $x \in (a, b)$.

20. In a binomial distribution, n = 200 and p = 0·04. Taking Poisson distribution as an approximation to the binomial distribution :

Assertion (A) : Mean of Poisson distribution = 8.

Reason (R) : $P(X = 4) = \frac{512}{3e^8}$.

P.T.O.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find the value of k such that $A^2 - 8A + kI = 0$.

OR

(b) If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the values of x, y, z and w.

22. Using Cramer's rule, solve the following system of equations :

$$2x_1 + 3x_2 = 5$$

$$11x_1 - 5x_2 = 6$$

23. Find the solution to the following linear programming problem (if it exists) graphically :

Maximize $Z = x + y$

subject to the constraints

$$x - y \le -1$$

$$-x + y \le 0$$

$$x, y \ge 0.$$

24. At 6% p.a., compounded quarterly, find the present value of a perpetuity of ₹ 600 payable at the end of each quarter.

P.T.O.

25. (a) Assume an investment's starting value is ₹ 20,000 and it grows to ₹ 50,000 in 3 years. Calculate CAGR (Compounded Annual Growth Rate) [Use : $(2{\cdot}5)^{1/3} = 1{\cdot}355$]

OR

(b) A man bought an item for ₹ 12,000. At the end of the year, he decided to sell it for ₹ 15,000. If the inflation rate was 6%, find the nominal and real rate of return.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated 4 more times. Determine the quantity of juice in the container after final replacement.

[Use $(0{\cdot}9)^5 = 0{\cdot}59049$]

27. (a) Evaluate : $\int_0^2 x^2 \, dx$ and hence show the region on the graph whose area it represents.

OR

(b) Evaluate : $\int_0^1 \frac{e^{-x}}{1 + e^x} \, dx$

28. Find the differential equation of all circles in the first quadrant which touches both the coordinate axes.

29. Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, determine the probability that a candidate who takes the test will score between 90 and 110.

[Given $P(Z < 1) = 0{\cdot}8413$ and $P(Z < -1) = 0{\cdot}1587$]

P.T.O.

30. The mean weekly sales of a 4-wheeler was 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful.

[Given $\sqrt{5} = 2 \cdot 24$, $t_{19}(0 \cdot 05) = 1 \cdot 729$]

31. (a) A recent accounting graduate opened a new business and installed a computer system that costs ₹ 45,200. The computer system will be depreciated linearly over 3 years and will have a scrap value of ₹ 0.

(i) What is the rate of depreciation ?

(ii) Give a linear equation that describes the computer system's book value at the end of t^{th} year, where $0 \leq t \leq 3$.

(iii) What will be the computer system's book value at the end of the first year and a half ?

OR

(b) Find the effective rate which is equivalent to normal rate of 10% p.a. compounded :

(i) semi-annually.

(ii) quarterly.

[Given $(1{\cdot}05)^2 = 1{\cdot}1025$, $(1{\cdot}025)^4 = 1{\cdot}1038$]

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. A cistern has three pipes A, B and C. Pipes A and B are inlet pipes whereas C is an outlet pipe. Pipes A and B can fill the cistern separately in 3 hours and 4 hours respectively; while pipe C can empty the completely filled cistern in 1 hour. If the pipes A, B and C are opened in order at 5, 6 and 7 a.m. respectively, at what time will the cistern be empty ?

P.T.O.

33. (a) Find all the points of local maxima and local minima of the function :

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105.$$

OR

(b) Find the intervals in which the following function f is strictly increasing or strictly decreasing :

$$f(x) = 20 - 9x + 6x^2 - x^3.$$

34. (a) Let X denote the number of hours a Class 12 student studies during a randomly selected school day. The probability that X can take the values x_i, for an unknown constant 'k' :

$$P(X = k) = \begin{cases} 0 \cdot 1 & \text{if} \quad x_i = 0 \\ kx_i & \text{if} \quad x_i = 1 \text{ or } 2 \\ k(5 - x_i) & \text{if} \quad x_i = 3 \text{ or } 4 \end{cases}$$

(i) Find the value of k.

(ii) Determine the probability that the student studied for at least 2 hours.

(iii) Determine the probability that the student studied for at most 2 hours.

OR

(b) A river near a small town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation ? Also, calculate the probability of 3 or less overflows and floods in a 10-year interval. [Given $e^{-2} = 0 \cdot 13534$]

35. Amrita buys a car for which she makes a down payment of ₹ 2,50,000 and the balance is to be paid in 2 years by monthly instalments of ₹ 25,448 each. If the financer charges interest at the rate of 20% p.a, find the actual price of the car. [Given $\left(\frac{61}{60}\right)^{-24} = 0 \cdot 67253$]

P.T.O.

SECTION E

This section comprises 3 case study-based questions of 4 marks each.

Case Study – 1

36. On her birthday, Prema decides to donate some money to children of an orphanage home.

If there are 8 children less, everyone gets ₹ 10 more. However, if there are 16 children more, everyone gets ₹ 10 less.

Let the number of children in the orphanage home be x and the amount to be donated to each child be ₹ y.

Based on the above information, answer the following questions :

(i) Write the system of linear equations in x and y formed of the given situation. *1*

P.T.O.

(ii) Write the system of linear equations, obtained in (i) above, in matrix form AX = B. *1*

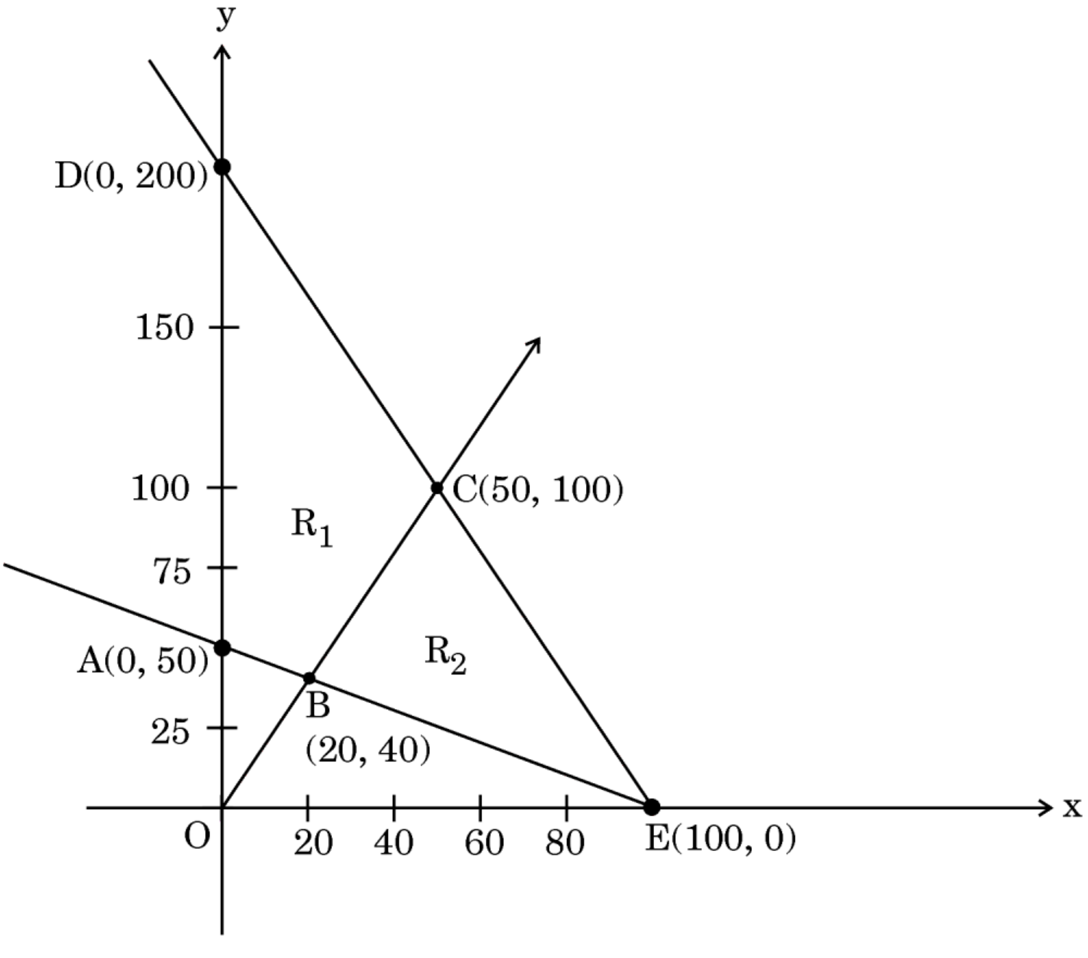
(iii) (a) Find the inverse of matrix A. *2*

OR

(b) Determine the values of x and y. *2*

Case Study – 2

37. In number theory, it is often important to find factors of an integer N. The number N has two trivial factors, namely 1 and N. Any other factor, if exists, is called non-trivial factor of N. Naresh has plotted a graph of some constraints (linear inequations) with points A (0, 50), B (20, 40), C(50, 100), D(0, 200) and E(100, 0). This graph is constructed using three non-trivial constraints and two trivial constraints. One of the non-trivial constraints is $x + 2y \geq 100$.



P.T.O.

Based on the above information, answer the following questions :

(i) What are the two trivial constraints ? *1*

(ii) (a) If R_1 is the feasible region, then what are the other two non-trivial constraints ? *2*

OR

(b) If R_2 is the feasible region, then what are the other two non-trivial constraints ? *2*

(iii) If R_1 is the feasible region, then find the maximum value of the objective function $z = 5x + 2y$. *1*

Case Study – 3

38. When observed over a long period of time, a time series data can predict trends that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

The table below shows the sale of an item in a district during 1996 – 2001 :

Year :	1996	1997	1998	1999	2000	2001
Sales (in lakh ₹) :	6·5	5·3	4·3	6·1	5·6	7·8

Based on the above information, answer the following questions :

(i) Determine the equation of the straight-line trend. *2*

P.T.O.

(ii) (a) Tabulate the trend values of the years and also compute expected sales trend for the year 2002. *2*

OR

(b) Fit a straight-line trend by the method of least squares for the following data : *2*

Year :	2004	2005	2006	2007	2008	2009	2010
Profit (₹ '000)	114	130	126	144	138	156	164

MARKING SCHEME

APPLIED MATHEMATICS (Subject Code–241)

(PAPER CODE: 465)

Section A

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	In a 1 km race, player P beats player Q by 18 metres or 9 seconds. What is P's time to complete the race ? (A) 512 seconds (B) 502 seconds (C) 491 seconds (D) 481 seconds	
Sol.	(C) 491 seconds	**(1)**
2.	If $x > y$ and $z < 0$, then : (A) $xz > yz$ (B) $xz \geq yz$ (C) $\frac{x}{z} > \frac{y}{z}$ (D) $\frac{x}{z} < \frac{y}{z}$	
Sol.	(D) $\frac{x}{z} < \frac{y}{z}$	**(1)**
3.	If $AB = A$ and $BA = B$, then $(B^2 + B)$ equals : (A) 2A (B) O (C) 2I (D) 2B	
Sol.	(D) 2B	**(1)**
4.	The value of $\Delta = \begin{vmatrix} 42 & 2 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ is : (A) 0 (B) 1 (C) – 3 (D) –15	
Sol.	(A) 0	**(1)**

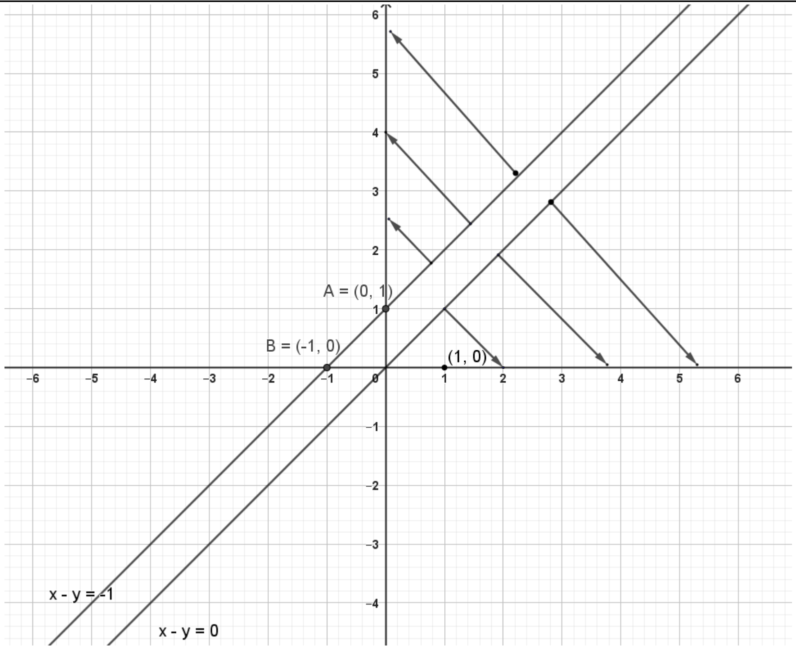
5.	If $y = e^{-2x}$, then $\frac{d^3y}{dx^3}$ is equal to : (A) $2e^{-2x}$ (B) e^{-4x} (C) $4e^{-4x}$ (D) $-8e^{-2x}$	
Sol.	(D) $\mathbf{-8e^{-2x}}$	**(1)**
6.	The function $f(x) = x^2 - x + 1$ is : (A) increasing in (0, 1) (B) decreasing in (0, 1) (C) increasing in $(0, \frac{1}{2})$ and decreasing in $(\frac{1}{2}, 1)$ (D) increasing in $(\frac{1}{2}, 1)$ and decreasing in $(0, \frac{1}{2})$	
Sol.	(D) increasing in $\left(\frac{1}{2}, 1\right)$ and decreasing in $\left(0, \frac{1}{2}\right)$	**(1)**
7.	The **order** and the **degree** of the differential equation $y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$ are respectively : (A) 1, 1 (B) 1, 2 (C) 2, 1 (D) 1, not defined	
Sol.	(A) 1,1	**(1)**
8.	A fair coin is tossed twice and outcomes are noted. If the random variable X represents the number of heads that appeared in the experiment, then the mathematical expectation of X is : (A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $1\frac{1}{2}$	
Sol.	(A) 1	**(1)**
9.	What time will it be after 1275 hours, if the present time is 9:00 p.m. ? (A) 11 p.m. (B) 12 p.m. (C) 9 p.m. (D) 9 a.m.	
Sol.	Since correct answer is not in the options given So, it is suggested that 1 mark may be given to all who attempted this question	**(1)**

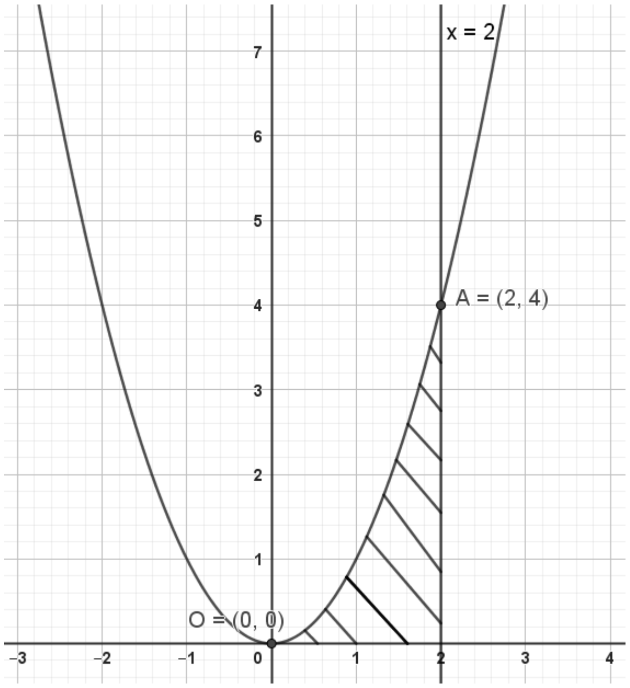
10.	If for a Poisson variate X, $P(X = k) = P(X = k + 1)$, then the variance of X is : (A) $k - 1$ (B) k (C) $k + 1$ (D) $k + 2$	
Sol.	(C) $\mathbf{k + 1}$	**(1)**
11.	If the calculated value of $\lvert t \rvert < t_v(\alpha)$ (critical value of t), then the null hypothesis : (A) is rejected. (B) is accepted. (C) is neither accepted nor rejected. (D) cannot be determined.	
Sol.	(B) is accepted	**(1)**
12.	For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as : (A) $n_1 - n_2 + 2$ (B) $n_1 - n_2 - 2$ (C) $n_1 + n_2 - 2$ (D) $n_1 + n_2 + 2$	
Sol.	(C) $\boldsymbol{n_1 + n_2 - 2}$	**(1)**
13.	For the given values 23, 32, 40, 47, 58, 33, 42; the 5-yearly moving averages are : (A) 38, 40, 42 (B) 40, 42, 44 (C) 40, 42, 46 (D) 42, 44, 46	
Sol.	(B) 40, 42, 44	**(1)**
14.	Using flat rate method, the EMI to repay a loan of ₹ 20,000 in $2\frac{1}{2}$ years at an interest rate of 8% p.a. is : (A) ₹ 700 (B) ₹ 800 (C) ₹ 900 (D) ₹ 100	
Sol.	(B) ₹ 800	**(1)**

15.	A mobile phone costs ₹ 12,000 and its scrap value after a useful life of 3 years is ₹ 3,000. Then, the book value of the mobile phone at the end of 2 years is : (A) ₹ 3,000 (B) ₹ 6,000 (C) ₹ 5,000 (D) ₹ 7,000	
Sol.	(B) ₹ 6,000	**(1)**
16.	What sum of money should be deposited at the end of every 6 months to accumulate ₹ 50,000 in 8 years, if money is worth 6% p.a. compounded semi-annually ? [Given : $(1\cdot03)^{16} = 1\cdot6047$] (A) ₹ 3,432·53 (B) ₹ 2,783·08 (C) ₹ 2,480·57 (D) ₹ 2,149·93	
Sol.	(C) ₹ 2480.57	**(1)**
17.	The graph of the inequation $2x + 3y > 6$ is the : (A) entire XOY-plane (B) half-plane that contains the origin (C) half-plane that neither contains the origin nor the points on the line $2x + 3y = 6$ (D) whole XOY-plane excluding the points on the line $2x + 3y = 6$	
Sol.	(C) half-plane that neither contains the origin nor the points on the line $\mathbf{2x + 3y = 6}$	**(1)**
18.	In an LPP, if the objective function $Z = ax + by$ has same maximum value on two corner points of the feasible region, then the number of points at which maximum value of Z occurs is : (A) 0 (B) 2 (C) finite (D) infinite	
Sol.	(D) infinite	**(1)**

	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* The function $f(x) = x^2 - x + 1$ is strictly increasing on $(-1, 1)$. *Reason (R) :* If f(x) is continuous on [a, b] and derivable on (a, b), then f(x) is strictly increasing on [a, b] if $f'(x) > 0$ for all $x \in (a, b)$.	
Sol.	(D) Assertion (A) is false, but Reason (R) is true	**(1)**
20.	In a binomial distribution, n = 200 and p = 0·04. Taking Poisson distribution as an approximation to the binomial distribution : *Assertion (A) :* Mean of Poisson distribution = 8. *Reason (R) :* $P(X = 4) = \frac{512}{3e^8}$.	
Sol.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is the not the correct explanation of the Assertion (A).	**(1)**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21(a).	If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find the value of k such that $A^2 - 8A + kI = 0$.	
Sol.	Given $\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, $\mathbf{A^2} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$ $\mathbf{A^2 - 8A + kI = 0}$ gives $\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 1 - 8 + k & 0 - 0 + 0 \\ -8 + 8 + 0 & 49 - 56 + k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\Rightarrow \mathbf{k = 7}$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$

	OR	
21(b).	If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the values of x, y, z and w.	
Sol.	Here, $\mathbf{x - y = - 1, 2x - y = 0, 2x + z = 5, 3z + w = 13}$ Solving these equations, we get $\mathbf{x = 1, y = 2, z = 3, w = 4}$	**(1)** **(1)**
22.	Using Cramer's rule, solve the following system of equations : $2x_1 + 3x_2 = 5$ $11x_1 - 5x_2 = 6$	
Sol.	Here, $\mathbf{D} = \begin{vmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{11} & \mathbf{-5} \end{vmatrix} = \mathbf{-43 \neq 0}$ $\mathbf{D_1} = \begin{vmatrix} \mathbf{5} & \mathbf{3} \\ \mathbf{6} & \mathbf{-5} \end{vmatrix} = \mathbf{-43}$ $\mathbf{D_2} = \begin{vmatrix} \mathbf{2} & \mathbf{5} \\ \mathbf{11} & \mathbf{6} \end{vmatrix} = \mathbf{-43}$ So, $\mathbf{x_1} = \frac{\mathbf{D_1}}{\mathbf{D}} = \frac{\mathbf{-43}}{\mathbf{-43}} = \mathbf{1}, \mathbf{x_2} = \frac{\mathbf{D_2}}{\mathbf{D}} = \frac{\mathbf{-43}}{\mathbf{-43}} = \mathbf{1}$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
23.	Find the solution to the following linear programming problem (if it exists) graphically : Maximize $Z = x + y$ subject to the constraints $x - y \leq - 1$ $- x + y \leq 0$ $x, y \geq 0.$	

Sol.	 Since feasible region is empty, there is no solution to the problem.	**$(1\frac{1}{2})$ for correct graph** $(\frac{1}{2})$
24.	At 6% p.a., compounded quarterly, find the present value of a perpetuity of ₹ 600 payable at the end of each quarter.	
Sol.	Here, $\mathbf{R = 600, i = \frac{0.06}{4} = 0.015}$ So, $\mathbf{PV = \frac{R}{i} = \frac{600}{0.015} =}$ ₹ $\mathbf{40,000}$	**(1)** **(1)**
25 (a).	Assume an investment's starting value is ₹ 20,000 and it grows to ₹ 50,000 in 3 years. Calculate CAGR (Compounded Annual Growth Rate) [Use : $(2\cdot5)^{1/3} = 1\cdot355$]	
Sol.	CAGR $= \mathbf{\left[\left(\frac{50000}{20000}\right)^{1/3} - 1\right] \times 100}$ $\mathbf{= \left[(2.5)^{1/3} - 1\right] \times 100 = (1.355 - 1) \times 100}$ $\mathbf{= 0.355 \times 100 = 35.5\%}$	**(1)** **(1)**
	OR	
25 (b).	A man bought an item for ₹ 12,000. At the end of the year, he decided to sell it for ₹ 15,000. If the inflation rate was 6%, find the nominal and real rate of return.	
Sol.	Nominal rate of return $= \mathbf{\frac{15000-12000}{12000} \times 100}$ $\mathbf{= \frac{3000}{12000} \times 100 = 25\%}$	$(\frac{1}{2})$ $(\frac{1}{2})$

	Real rate of return =Nominal rate of return – Inflation rate $= 25\% - 6\% = 19\%$	**(1)**
	SECTION C **This section comprises short answer (SA) type questions of 3 marks each.**	
26.	A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated 4 more times. Determine the quantity of juice in the container after final replacement. [Use $(0{\cdot}9)^5 = 0{\cdot}59049$]	
Sol.	Juice contained in the container after final replacement $= 50\left(1-\frac{5}{50}\right)^5 = 50\left(\frac{9}{10}\right)^5$ $= 50 \times 0.59049 = 29.5$ litres	**(2)** **(1)**
27(a).	Evaluate : $\int_0^2 x^2\, dx$ and hence show the region on the graph whose area it represents.	
Sol.	Required Area $= \int_0^2 x^2dx = \left\|\frac{x^3}{3}\right\|_0^2 = \frac{8}{3}$ 	**$(1\frac{1}{2})$** **$(1\frac{1}{2})$ for correct graph**
	OR	

27(b).	Evaluate : $\int\limits_0^1 \frac{e^{-x}}{1+e^x}\,dx$	
Sol.	$\mathbf{I = \int_0^1 \frac{dx}{e^x(1+e^x)}}$ $= \int_1^e \frac{dt}{t^2(1+t)}$ **(Put $e^x = t \Rightarrow e^x dx = dt$)** $= \int_1^e \left(-\frac{1}{t}+\frac{1}{t^2}+\frac{1}{1+t}\right) dt$ $= \left[-\log(t) - \frac{1}{t} + \log(1+t)\right]_1^e$ $= \left[\log\left(\frac{1+t}{t}\right) - \frac{1}{t}\right]_1^e$ $= \left[\log\left(\frac{1+e}{e}\right) - \frac{1}{e}\right] - [\log 2 - 1]$ $= \log\left(\frac{1+e}{2e}\right) - \frac{1}{e} + 1$	$(\frac{1}{2})$ **(1)** **(1)** $(\frac{1}{2})$
28.	Find the differential equation of all circles in the first quadrant which touches both the coordinate axes.	
Sol.	Here, the equation of the circle is $(x-a)^2 + (y-a)^2 = a^2 \ldots (1)$ i.e., $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ $\Rightarrow 2x + 2y\frac{dy}{dx} - 2a - 2a\frac{dy}{dx} = 0$ $\Rightarrow a = \frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}$ From (1), we have $\left(x - \frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)^2 + \left(y - \frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)^2 = \left(\frac{x+y\frac{dy}{dx}}{1+\frac{dy}{dx}}\right)^2$ or $\left(\frac{dy}{dx}\right)^2 (x^2 - 2xy) - 2xy\frac{dy}{dx} + y^2 - 2xy = 0$	**(1)** **(1)** **(1)**
29.	Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, determine the probability that a candidate who takes the test will score between 90 and 110. [Given P (Z < 1) = 0·8413 and P (Z < – 1) = 0·1587]	

Sol.	Here, $\mathbf{Z} = \frac{X-100}{10}$ $P(90 < X < 110) = P(X < 110) - P(X < 90)$ $\Rightarrow P(90 < X < 110) = P(-1 < Z < 1) = P(Z < 1) - P(Z < -1)$ $= 0.8413 - 0.1587$ $= 0.6826$	$(\frac{1}{2})$ **(2)** $(\frac{1}{2})$
30.	The mean weekly sales of a 4-wheeler was 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful. [Given $\sqrt{5} = 2\cdot 24$, $t_{19}(0\cdot 05) = 1\cdot 729$]	
Sol.	Here, $\mu_0 = 50, \bar{x} = 55, n = 20$ and $S = 10$ $H_0: \mu = 50$ (The advertisement campaign was not successful) $H_\alpha: \mu > 50$ (The advertisement campaign was successful) The test statistic t is given by $t = \frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}} = \frac{55-50}{\frac{10}{\sqrt{20}}} = \frac{2\sqrt{5}}{2} = \sqrt{5} = 2.24$ Degree of freedom $= 20 - 1 = 19$ Here, $t > t_{19}(0.05)$ as $2.24 > 1.729$ $\Longrightarrow$ null hypothesis is rejected i.e., Advertising campaign was successful	$(\frac{1}{2})$ $(1\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
31(a).	A recent accounting graduate opened a new business and installed a computer system that costs ₹ 45,200. The computer system will be depreciated linearly over 3 years and will have a scrap value of ₹ 0. (i) What is the rate of depreciation ? (ii) Give a linear equation that describes the computer system's book value at the end of t^{th} year, where $0 \le t \le 3$. (iii) What will be the computer system's book value at the end of the first year and a half ?	
Sol.	(i) Annual amount of depreciation $= \frac{45200-0}{3} = ₹\ \frac{45200}{3}$ Rate of depreciation $= \frac{\frac{45200}{3}}{45200} \times 100 = 33.3\%$	**(1)**

	(ii) $\mathbf{v(t) = mt + C = -\frac{45200}{3}t + 45200}$ (iii) $\mathbf{v\left(1\frac{1}{2}\right) = -\frac{45200}{3} \times \frac{3}{2} + 45200 = ₹\,22600}$	**(1)** **(1)**
	OR	
31(b).	Find the effective rate which is equivalent to normal rate of 10% p.a. compounded : (i) semi-annually. (ii) quarterly. [Given $(1{\cdot}05)^2 = 1{\cdot}1025$, $(1{\cdot}025)^4 = 1{\cdot}1038$]	
Sol.	(i) $\mathbf{r_e = \left(1 + \frac{r}{200}\right)^2 - 1 = \left(1 + \frac{10}{200}\right)^2 - 1}$ $\mathbf{= (1.05)^2 - 1 = 1.1025 - 1 = 10.25\%}$ (ii) $\mathbf{r_e = \left(1 + \frac{r}{400}\right)^4 - 1 = \left(1 + \frac{10}{400}\right)^4 - 1}$ $\mathbf{= (1.025)^4 - 1 = 1.1038 - 1 = 10.38\%}$	**(1)** $(\frac{1}{2})$ **(1)** $(\frac{1}{2})$
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32.	A cistern has three pipes A, B and C. Pipes A and B are inlet pipes whereas C is an outlet pipe. Pipes A and B can fill the cistern separately in 3 hours and 4 hours respectively; while pipe C can empty the completely filled cistern in 1 hour. If the pipes A, B and C are opened in order at 5, 6 and 7 a.m. respectively, at what time will the cistern be empty ?	
Sol.	Let the cistern be emptied in n hours after 5 a.m. Clearly pipes A and B fill the cistern for n and $n-1$ hours respectively, while pipe C empties the tank for $n-2$ hours $\therefore \frac{n}{3} + \frac{n-1}{4} - \frac{n-2}{1} = 0$ Solving, we get $n = \frac{21}{5}$ i.e., 4 hours 12 minutes past 5 am i.e., at 9:12 am	$\mathbf{(1\frac{1}{2})}$ $\mathbf{(1\frac{1}{2})}$ **(1)** **(1)**

33(a). Find all the points of local maxima and local minima of the function :

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105.$$

Sol.

$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$

$f'(x) = -3x^3 - 24x^2 - 45x$ and $f''(x) = -9x^2 - 48x - 45$ **(1)**

$f'(x) = 0 \Rightarrow x = 0$ or $x^2 + 8x + 15 = 0$

i.e., $x = 0, -3, -5$ **(1)**

At $x = 0, f''(0) < 0 \Rightarrow 0$ is a point of local maxima **(1)**

At $x = -3, f''(-3) > 0 \Rightarrow -3$ is a point of local minima **(1)**

At $x = -5, f''(-5) < 0 \Rightarrow -5$ is a point of local maxima **(1)**

OR

33(b). Find the intervals in which the following function f is strictly increasing or strictly decreasing :

$$f(x) = 20 - 9x + 6x^2 - x^3.$$

Sol.

$f(x) = 20 - 9x + 6x^2 - x^3$

$f'(x) = -9 + 12x - 3x^2 = -3(x^2 - 4x + 3)$ **($\frac{1}{2}$)**

$= -3(x-1)(x-3)$ **(1)**

$f'(x) = 0 \Rightarrow x = 1,3$ **($\frac{1}{2}$)**

Now intervals are $(-\infty, 1), (1,3)$ and $(3, \infty)$ **($\frac{1}{2}$)**

Intervals	Sign of $f'(x)$
$(-\infty, 1)$	−ve
$(1,3)$	+ve
$(3, \infty)$	−ve

($1\frac{1}{2}$)

$\Rightarrow f(x)$ is strictly increasing in $(1,3)$ or $[1,3]$ **($\frac{1}{2}$)**

And $f(x)$ is strictly decreasing in $(-\infty, 1) \cup (3, \infty)$ or $(-\infty, 1] \cup [3, \infty)$ **($\frac{1}{2}$)**

34(a).	Let X denote the number of hours a Class 12 student studies during a randomly selected school day. The probability that X can take the values x_i, for an unknown constant 'k' : $P(X = k) = \begin{cases} 0{\cdot}1 & \text{if} \quad x_i = 0 \\ kx_i & \text{if} \quad x_i = 1 \text{ or } 2 \\ k(5 - x_i) & \text{if} \quad x_i = 3 \text{ or } 4 \end{cases}$ (i) Find the value of k. (ii) Determine the probability that the student studied for at least 2 hours. (iii) Determine the probability that the student studied for at most 2 hours.	
Sol.	(i) $\mathbf{0.1 + k + 2k + 2k + k = 1}$ $\Rightarrow \mathbf{0.1 + 6k = 1}$ $\Rightarrow \mathbf{k = \frac{3}{20}}$ (ii) $\mathbf{P(X \geq 2) = P(2) + P(3) + P(4)}$ $\mathbf{= 2k + 2k + k}$ $\mathbf{= 5k = \frac{3}{4}}$ (iii) $\mathbf{P(X \leq 2) = P(0) + P(1) + P(2)}$ $\mathbf{= 0.1 + k + 2k}$ $\mathbf{= \frac{1}{10} + \frac{9}{20} = \frac{11}{20}}$	**(1)** **(1)** **($\frac{1}{2}$)** **(1)** **($\frac{1}{2}$)** **(1)**
	OR	
34(b).	A river near a small town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation ? Also, calculate the probability of 3 or less overflows and floods in a 10-year interval. [Given $e^{-2} = 0{\cdot}13534$]	
Sol.	Here, mean expectation $= \boldsymbol{\lambda} = \mathbf{2}$ $\mathbf{P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}}$ $\mathbf{P(X \leq 3) = P(0) + P(1) + P(2) + P(3)}$ $\mathbf{= e^{-2}\left(1 + 2 + 2 + \frac{4}{3}\right)}$ $\mathbf{= 0.13534 \times \frac{19}{3} = 0.8571 \text{ or } 0.86}$	**(1)** **(1)** **(2)** **(1)**

35.	Amrita buys a car for which she makes a down payment of ₹ 2,50,000 and the balance is to be paid in 2 years by monthly instalments of ₹ 25,448 each. If the financer charges interest at the rate of 20% p.a, find the actual price of the car. [Given $\left(\frac{61}{60}\right)^{-24} = 0{\cdot}67253$]	
Sol.	$\mathbf{n = 2 \times 12 = 24,}$ $\mathbf{i = \frac{20}{1200} = \frac{1}{60}}$ EMI = $\frac{\mathbf{Pi}}{\mathbf{1-(1+i)^{-n}}}$ $\mathbf{25448 = \frac{P\times\frac{1}{60}}{1-(1+\frac{1}{60})^{-24}}}$ $\mathbf{P = 25448 \times 60 \left[1-\left(1+\frac{1}{60}\right)^{-24}\right]}$ $\mathbf{P = 25448 \times 60(1-0.67253)}$ $=$ ₹ $\mathbf{5,00,000}$ approx. Hence the actual price of the car is ₹ 7,50,000 approx.	$(\frac{1}{2})$ $(\frac{1}{2})$ **(1)** **(1)** $(\frac{1}{2})$ **(1)** $(\frac{1}{2})$
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36. On her birthday, Prema decides to donate some money to children of an orphanage home.

If there are 8 children less, everyone gets ₹ 10 more. However, if there are 16 children more, everyone gets ₹ 10 less.

Let the number of children in the orphanage home be x and the amount to be donated to each child be ₹ y.

Based on the above information, answer the following questions :

(i) Write the system of linear equations in x and y formed of the given situation. *1*

(ii) Write the system of linear equations, obtained in (i) above, in matrix form AX = B. *1*

(iii) (a) Find the inverse of matrix A. *2*

OR

(b) Determine the values of x and y. *2*

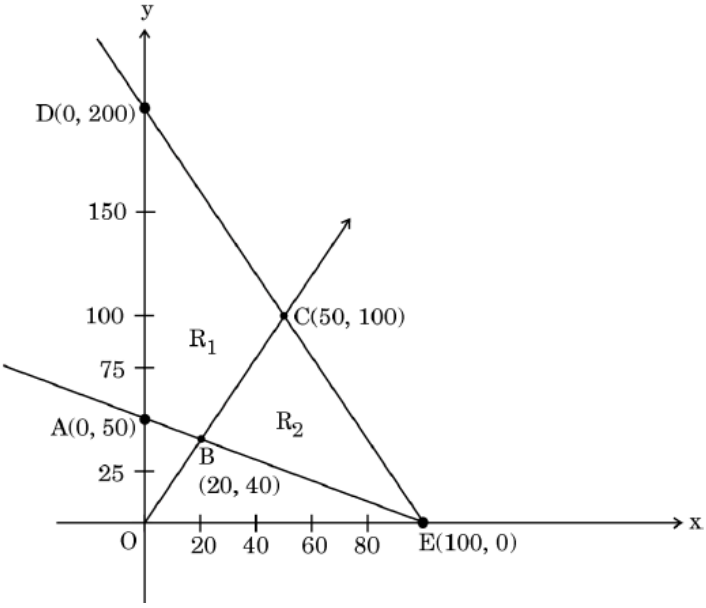
Sol.

(i) Let number of children be x and the amount donated to each child be ₹ y

$\therefore (x-8)(y+10) = xy$ and $(x+16)(y-10) = xy$ $\quad$ **$(\frac{1}{2})$**

i.e., $5x - 4y = 40$ and $5x - 8y = -80$ $\quad$ **$(\frac{1}{2})$**

(ii) $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$ $\quad$ **(1)**

	(iii) (a) $\lvert A\rvert = -40 + 20 = -20 \neq 0$ adj (A) $= \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$ $\Rightarrow A^{-1} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 8 & -4 \\ 5 & -5 \end{bmatrix}$ **OR** (iii) (b) $X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$ $= \frac{1}{20}\begin{bmatrix} 8 & -4 \\ 5 & -5 \end{bmatrix}\begin{bmatrix} 40 \\ -80 \end{bmatrix}$ $= \frac{1}{20}\begin{bmatrix} 640 \\ 600 \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$ $\Rightarrow x = 32, y = 30$	$(\frac{1}{2})$ **(1)** $(\frac{1}{2})$ $(\frac{1}{2})$ **(1)** $(\frac{1}{2})$
37.	**Case Study – 2** In number theory, it is often important to find factors of an integer N. The number N has two trivial factors, namely 1 and N. Any other factor, if exists, is called non-trivial factor of N. Naresh has plotted a graph of some constraints (linear inequations) with points A (0, 50), B (20, 40), C (50, 100), D (0, 200) and E (100, 0). This graph is constructed using three non-trivial constraints and two trivial constraints. One of the non-trivial constraints is x + 2y ≥ 100.  	

	Based on the above information, answer the following questions : (i) What are the two trivial constraints ? *1* (ii) (a) If R_1 is the feasible region, then what are the other two non-trivial constraints ? *2* **OR** (b) If R_2 is the feasible region, then what are the other two non-trivial constraints ? *2* (iii) If R_1 is the feasible region, then find the maximum value of the objective function $z = 5x + 2y$. *1*	
Sol.	(i) $x \geq 0, y \geq 0$	**(1)**
	(ii) (a) $2x - y \leq 0,$	**(1)**
	$2x + y \leq 200$	**(1)**
	OR	
	(ii) (b) $2x + y \leq 200,$	**(1)**
	$2x - y \geq 0$	**(1)**
	(iii) Corner points of R_1 are $A(0,50), B(20,40), C(50,100)$ and $D(0,200)$	
	$\mathbf{Z_A = 100;\ Z_B = 180; Z_C = 450;\ Z_D = 400}$	$\mathbf{(\frac{1}{2})}$
	So, Z is maximum at C and maximum value of $Z = 450$	$\mathbf{(\frac{1}{2})}$

38.

When observed over a long period of time, a time series data can predict trends that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

The table below shows the sale of an item in a district during 1996 – 2001 :

Year :	1996	1997	1998	1999	2000	2001
Sales (in lakh ₹) :	6·5	5·3	4·3	6·1	5·6	7·8

Based on the above information, answer the following questions :

(i) Determine the equation of the straight-line trend. *2*

(a) Tabulate the trend values of the years and also compute expected sales trend for the year 2002. *2*

OR

(b) Fit a straight-line trend by the method of least squares for the following data : *2*

Year :	2004	2005	2006	2007	2008	2009	2010
Profit (₹ '000)	114	130	126	144	138	156	164

Sol.

(i)

Year (x_i)	Index Number (y)	$x = \frac{x_i - A}{0.5}$	x^2	xy
1996	6.5	−5	25	−32.5
1997	5.3	−3	9	−15.9
1998	4.3	−1	1	−4.3
1999	6.1	1	1	6.1
2000	5.6	3	9	16.8
2001	7.8	5	25	39
n = 6	$\sum y = 35.6$	$\sum x = 0$	$\sum x^2 = 70$	$\sum xy = 9.2$

(1) for correct table

$a = \frac{\sum y}{n} = \frac{35.6}{6}, b = \frac{\sum xy}{\sum x^2} = \frac{9.2}{70} = 0.13$ **$(\frac{1}{2})$**

∴ Equation of straight-line trend is given by

$y = a + bx = 5.9 + 0.13x$ **$(\frac{1}{2})$**

(ii) (a) Trend Values

1996	$5 \cdot 9 + (-5) \times 0 \cdot 13 = 5 \cdot 25$
1997	$5 \cdot 9 + (-3) \times 0 \cdot 13 = 5 \cdot 51$
1998	$5 \cdot 9 + (-1) \times 0 \cdot 13 = 5 \cdot 77$
1999	$5 \cdot 9 + (1) \times 0 \cdot 13 = 6.03$
2000	$5 \cdot 9 + (3) \times 0 \cdot 13 = 6.29$
2001	$5 \cdot 9 + (5) \times 0 \cdot 13 = 6.55$

(1) mark for correct trend values

Expected sales trend for 2002

$= 5.9 + 0.13\left(\frac{2002-1998.5}{0.5}\right)$ **$(\frac{1}{2})$**

= ₹ 6.81 lakhs **$(\frac{1}{2})$**

OR

(ii) (b)

Year (x_i)	Profit (y)	$x = x_i - A$	x^2	xy
2004	114	-3	9	-342
2005	130	-2	4	-260
2006	126	-1	1	-126
2007	144	0	0	0
2008	138	1	1	138
2009	156	2	4	312
2010	164	3	9	492
$n = 7$	$\sum y = 972$	$\sum x = 0$	$\sum x^2 = 28$	$\sum xy = 214$

(1) for correct table

$$a = \frac{\sum y}{n} = \frac{972}{7} = 138.86, b = \frac{\sum xy}{\sum x^2} = \frac{214}{28} = 7.64$$

$(\frac{1}{2})$

So, required equation of straight- line trend is

$$y = a + bx = 138.86 + 7.64x$$

$(\frac{1}{2})$

General Instructions :

Read the following instructions very carefully and strictly follow them :

- *(i) This question paper contains **38** questions. **All** questions are **compulsory**.*
- *(ii) This question paper is divided into **five** Sections – Section **A**, **B**, **C**, **D** and **E**.*
- *(iii) In Section – **A**, Questions Number **1** to **18** are Multiple Choice Questions (MCQs) and questions Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*
- *(iv) In Section – **B**, Questions Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*
- *(v) In Section – **C**, Questions Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*
- *(vi) In Section – **D**, Questions Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*
- *(vii) In Section – **E**, Questions Number **36** to **38** are case study based questions, carrying **4** marks each.*
- *(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **2** questions in Section – **C**, 2 questions in Section – **D** and **3** questions in Section – **E**.*
- *(ix) Use of calculators is **NOT** allowed.*

SECTION – A

This section comprises of Multiple Choice Questions (MCQs) of **1** mark each. Select the correct option (Question 1 to Question 18) :

1. $-41 \bmod 9$ is

(A) 5 (B) 4

(C) 3 (D) 0

2. If $a > b$ and $c < 0$, then which of the following is true ?

(A) $a + c < b + c$ (B) $a - c < b - c$

(C) $a\,c > b\,c$ (D) $a - c > b + c$

3. If A and B are symmetric matrices of the same order, then $(AB' - BA')$ is a

(A) symmetric matrix (B) null matrix

(C) diagonal matrix (D) skew symmetric matrix

4. The inverse of matrix $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$ is

(A) $\frac{1}{6}\begin{bmatrix} -4 & 2 \\ -1 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1/3 & 1/6 \\ 2/3 & -1/6 \end{bmatrix}$

(C) $\begin{bmatrix} 1/6 & 1/6 \\ -1/3 & 2/3 \end{bmatrix}$ (D) $\begin{bmatrix} -2/3 & 1/6 \\ -1/3 & -1/6 \end{bmatrix}$

5. If $\begin{vmatrix} 2x & 5 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix}$, then the value of x is

(A) $3/2$ (B) 6

(C) 3 (D) ± 3

6. The slope of the normal to the curve $y = \frac{x-3}{x-4}$ at $x = 6$ is

(A) 4 (B) $-\frac{1}{4}$

(C) -4 (D) $\frac{1}{4}$

7. The rate of change of population P(t) with respect to time (t), where α, β are the constant birth and death rates, respectively, is

(A) $\frac{dP}{dt} = (\alpha + \beta)P$ (B) $\frac{dP}{dt} = (\alpha - \beta)P$

(C) $\frac{dP}{dt} = \frac{\alpha + \beta}{P}$ (D) $\frac{dP}{dt} = \frac{\alpha - \beta}{P}$

8. A pair of dice is thrown two times. If X represents the number of doublets obtained, then the expectation of X is

(A) $\frac{1}{6}$ (B) 1

(C) $\frac{1}{3}$ (D) $\frac{11}{36}$

9. The mean of t-distribution is

(A) 0 (B) 1

(C) 2 (D) not defined

10. The variations which occur due to change in climate, festivals or weather conditions are known as

(A) secular variations (B) cyclic variations

(C) seasonal variations (D) irregular variations

11. In a LPP, the maximum value of $z = 3x + 4y$ subject to the constraints $x + y \leq 40$, $x + 2y \leq 60$, $x, y \geq 0$ is

(A) 120 (B) 140

(C) 150 (D) 130

12. The present value of a sequence of payments of ₹ 100 made at the end of every year and continuing forever, if the money is worth 5% compounded annually, is

(A) ₹ 2,000 (B) ₹ 20,000

(C) ₹ 5,000 (D) ₹ 12,000

13. The demand function of a monopolist is given by $p = 30 + 5x - 3x^2$, where x is the number of units demanded and p is the price per unit. The marginal revenue when 2 units are sold, is

(A) ₹ 28 (B) ₹ 23

(C) ₹ 1 (D) ₹ 14

14. If the cost function and revenue function of x items are respectively given as $C(x) = 100 + 0.015\,x^2$, $R(x) = 3x$, then the value of x for maximum profit is

(A) 50 (B) 100

(C) 150 (D) 200

15. If a random variable X has the probability distribution

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \text{ or } 2 \\ 0, & \text{otherwise,} \end{cases}$$

then the value of k is

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$

(C) $\frac{1}{6}$ (D) $\frac{1}{4}$

16. The test statistic t for testing the significance of differences between the means of two independent samples is given by

(A) $t = \frac{\overline{x} - \overline{y}}{\sqrt{s}}$ (B) $t = \frac{\overline{x} - \overline{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

(C) $t = \frac{\overline{x} - \overline{y}}{\frac{s}{\sqrt{n-1}}}$ (D) $t = \frac{\overline{x} + \overline{y}}{s\sqrt{\frac{1}{n_1} - \frac{1}{n_2}}}$

17. The effective rate of interest equivalent to a nominal rate of 4% compounded semi-annually, is

(A) 4.12% (B) 4.04%

(C) 4.08% (D) 4.14%

18. The CAGR of an investment, whose starting value is ₹ 5,000 and it grows to ₹ 25,000 in 4 years, is : [Given $(5)^{0.25} = 1.4953$]

(A) 49.53%	(B) 14.95%
(C) 495.3%	(D) 1.49%

Questions number **19** and **20** are Assertion – Reason based questions of **1** mark each. Two statements are given – one labelled Assertion (A) and other labelled Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below :

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not correct explanation for Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A)** : The area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -1$ and $x = 1$ is 2 square units.

Reason (R) : The area of the region bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is given by $\int_a^b f(x)\, dx$.

20. **Assertion (A)** : The differential equation representing the family of curves $y = mx$, m being an arbitrary constant, is $x\frac{dy}{dx} - y = 0$.

Reason (R) : For a family of curves, the differential equation is obtained by differentiating the equation of family of curves with respect to x and then eliminating the arbitrary constant, if any.

SECTION – B

Questions Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.

21. (a) The cost of Type I sugar is ₹ 25 per kg and Type II sugar is ₹ 35 per kg. If both Type I sugar and Type II sugar are mixed in the ratio 3:2, find the price per kg of the mixture.

OR

(b) Pipe A can fill a tank in 1 hour and Pipe B can fill it in 1½ hours. If both the pipes are opened in the empty tank, how much time will they take to fill the tank ?

22. A boat goes 3.5 km upstream and then returns. Total time taken is 1 hour and 12 minutes. If the speed of the current is 1 km/h, then find the speed of the boat in still water.

23. A runs $\frac{3}{2}$ times as fast as B. If A gives B a start of 40 m, how far must the winning post from the starting point be, so that A and B reach at the same time ?

24. Given $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 4 & 5 \\ 0 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -5 \\ -5 & 1 & -5 \\ 1 & -2 & 4 \end{bmatrix}$, find BA.

25. (a) If a fair coin is tossed 6 times, find the probability of getting atleast 4 heads.

OR

(b) Given that mean of a normal variate X is 9 and standard deviation is 3, then find :

(i) the z-score of the data point 15

(ii) the data point if its z-score is 4.

SECTION – C

Questions Number **26** to **31** are Short Answer (SA) type questions of **3** marks each.

26. Find the units digit in 7^{295}.

27. Two numbers are selected at random (without replacement) from first six positive integers. Let X denotes the smaller of the two numbers obtained. Calculate the mathematical expectation of X.

28. (a) If the mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively, then find $P(x = 1)$.

OR

(b) The mortality rate for a certain disease is 0.007. Using Poisson distribution, calculate the probability for 2 deaths in a group of 400 people. [Use $e^{-2.8} = 0.0608$]

29. (a) There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid. F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If F_1 costs ₹ 6 per kg and F_2 costs ₹ 5 per kg, how much of each type of fertilizer should be used so that the cost is minimum. Formulate a linear programming problem.

OR

(b) Solve the following linear programming problem graphically :

Maximise $z = 50x + 30y$

subject to $2x + y \leq 18$

$3x + 2y \leq 34$

$x, y \geq 0$.

30. A machinist is making engine parts with axle diameter of 0.7 cm. A random sample of 10 parts shows mean diameter 0.742 cm with a standard deviation of 0.04 cm. On the basis of this sample, find if you would say that the work is inferior. (Given t_9 (0.05) = 2.262)

31. Calculate EMI under Flat-Rate System for a loan of ₹ 5,00,000 with 7.5% annual interest rate for 5 years.

SECTION – D

Questions number **32** to **35** are Long Answer (LA) type questions of **5** marks each.

32. (a) If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3$

OR

(b) Using properties of determinants, prove that

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2\,abc\,(a+b+c)^3$$

33. If the supply function is $p = 4 - 5x + x^2$, then find the producer's surplus when price is 18.

34. (a) Compute the seasonal indices by 4-year moving averages from the given data of production of paper (in thousand tons) :

Year :	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Index number :	2450	1470	2150	1800	1210	1950	2300	2500	2480	2680

OR

(b) Fit a straight-line trend by method of least squares for the following data :

Year :	2011	2012	2013	2014	2015	2016
Production (in tons) :	210	225	275	220	240	235

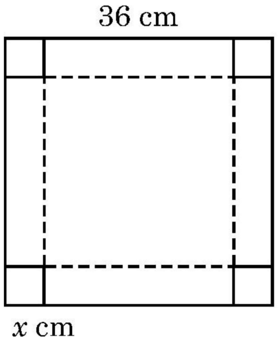
35. A machine costs ₹ 1,00,000 and its effective life is estimated to be 12 years. A sinking fund is created for replacing the machine by a new model at the end of its life time when its scrap realizes a sum of ₹ 5,000 only. Find what amount should be set aside at the end of each year, out of the profits for the sinking fund if it accumulates at 5% effective.

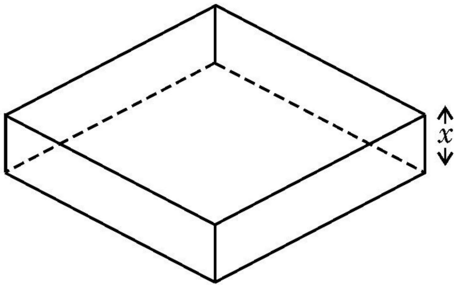
[Use $(1.05)^{12} = 1.7958$]

SECTION – E

Questions number **36** to **38** are case-study based questions of **4** marks each.

36. A man has an expensive square-shaped piece of golden board of side 36 cm. He wants to turn it into a box without top by cutting a square from each corner and folding the flaps. Let x cm be the side of square, which is cut from each corner.





Based on the above information, answer the following questions :

(i) Find the expression for the volume (V) of open box in terms of x.

(ii) Find $\frac{dV}{dx}$.

(iii) Find the value of x for which the volume (V) is maximum.

OR

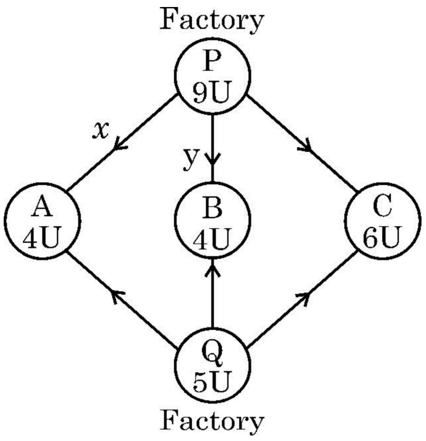
(iii) Find the maximum volume of the open box.

37. There are two factories located one at P and the other at Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 4, 4 and 6 units of the commodity while the production capacity of the factories at P and Q are 9 and 5 units respectively. The cost of transportation per unit is given as :

From / To	**Cost (in ₹)**		
	A	B	C
P	160	100	150
Q	100	120	100

Based on the above information, answer the following questions :

Let x units and y units of the commodity be transported from factory P to the depots at A and B respectively, then



(i) Find (in terms of x and y) how many units of commodity be transported from factory P to depot C.

(ii) Find how many units of commodity be transported from factory Q to A, B and C respectively.

(iii) Using (i) and (ii), find the total transportation cost z.

OR

(iii) Using (i) and (ii), find the constraint inequalities for minimum cost z.

38. Ramesh borrowed a home loan amount of ₹ 7,00,000 from a bank at an interest of 12% per annum for 30 years, to be paid in monthly installments.

Based on the above information, answer the following questions :

(i) Write the formula for calculating EMI by reducing balance method.

(ii) Write the values of P, i and n respectively.

(iii) Find the EMI. [Use $(1.01)^{-360} = 0.02781668$]

OR

(iii) If the loan is to be returned in 20 years, find EMI.

[Use $(1.01)^{-240} = 0.09180584$]

MARKING SCHEME

APPLIED MATHEMATICS (Subject Code–241)

(PAPER CODE: 465)

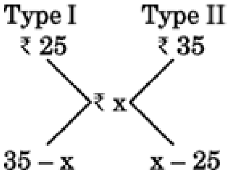
Section A

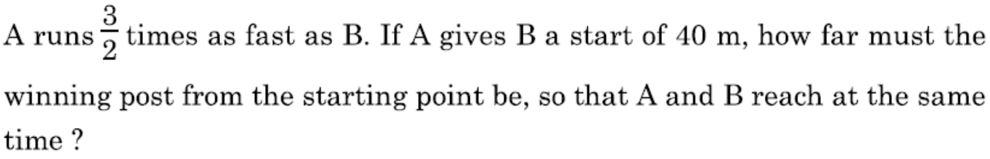
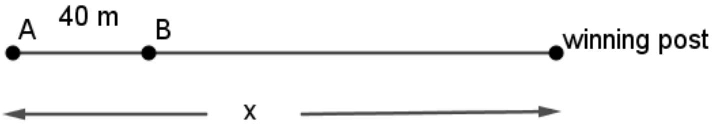
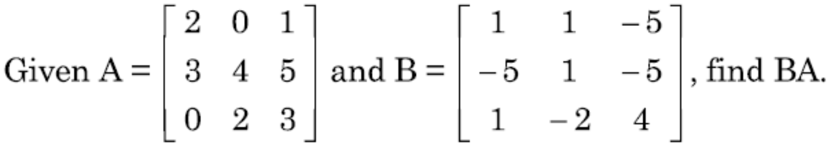
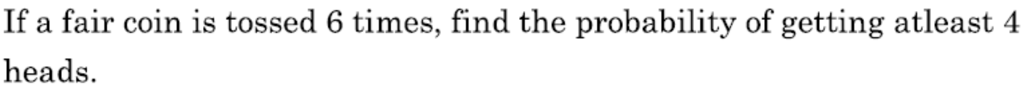
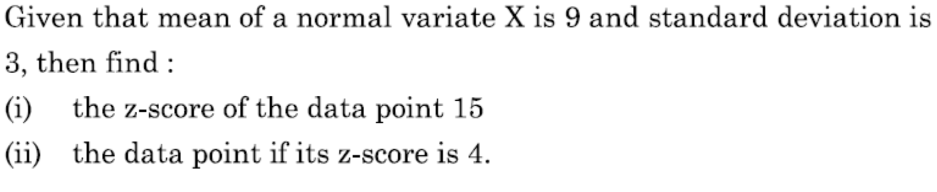
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	– 41 mod 9 is (A) 5 (B) 4 (C) 3 (D) 0	
Sol.	(B) 4	**1**
2.	If a > b and c < 0, then which of the following is true ? (A) a + c < b + c (B) a – c < b – c (C) a c > b c (D) a – c > b + c	
Sol.	(D) $a - c > b + c$	**1**
3.	If A and B are symmetric matrices of the same order, then $(AB' - BA')$ is a (A) symmetric matrix (B) null matrix (C) diagonal matrix (D) skew symmetric matrix	
Sol.	(D) skew symmetric matrix	**1**
4.	The inverse of matrix $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$ is (A) $\frac{1}{6}\begin{bmatrix} -4 & 2 \\ -1 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1/3 & 1/6 \\ 2/3 & -1/6 \end{bmatrix}$ (C) $\begin{bmatrix} 1/6 & 1/6 \\ -1/3 & 2/3 \end{bmatrix}$ (D) $\begin{bmatrix} -2/3 & 1/6 \\ -1/3 & -1/6 \end{bmatrix}$	
Sol.	(C) $\begin{bmatrix} 1/6 & 1/6 \\ -1/3 & 2/3 \end{bmatrix}$	**1**
5.	If $\begin{vmatrix} 2x & 5 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix}$, then the value of x is (A) $3/2$ (B) 6 (C) 3 (D) ± 3	
Sol.	(D) ± 3	**1**

6.	The slope of the normal to the curve y = $\frac{x-3}{x-4}$ at $x = 6$ is (A) 4 (B) $-\frac{1}{4}$ (C) -4 (D) $\frac{1}{4}$	
Sol.	(A) 4	**1**
7.	The rate of change of population P(t) with respect to time (t), where α, β are the constant birth and death rates, respectively, is (A) $\frac{dP}{dt} = (\alpha + \beta)P$ (B) $\frac{dP}{dt} = (\alpha - \beta)P$ (C) $\frac{dP}{dt} = \frac{\alpha + \beta}{P}$ (D) $\frac{dP}{dt} = \frac{\alpha - \beta}{P}$	
Sol.	(B) $\frac{dP}{dt} = (\alpha - \beta)P$	**1**
8.	A pair of dice is thrown two times. If X represents the number of doublets obtained, then the expectation of X is (A) $\frac{1}{6}$ (B) 1 (C) $\frac{1}{3}$ (D) $\frac{11}{36}$	
Sol.	(C) $\frac{1}{3}$	**1**
9.	The mean of t-distribution is (A) 0 (B) 1 (C) 2 (D) not defined	
Sol.	(A) 0	**1**
10.	The variations which occur due to change in climate, festivals or weather conditions are known as (A) secular variations (B) cyclic variations (C) seasonal variations (D) irregular variations	
Sol.	(C) seasonal variations	**1**
11.	In a LPP, the maximum value of z = $3x + 4y$ subject to the constraints $x + y \leq 40$, $x + 2y \leq 60$, $x, y \geq 0$ is (A) 120 (B) 140 (C) 150 (D) 130	
Sol.	(B) 140	**1**

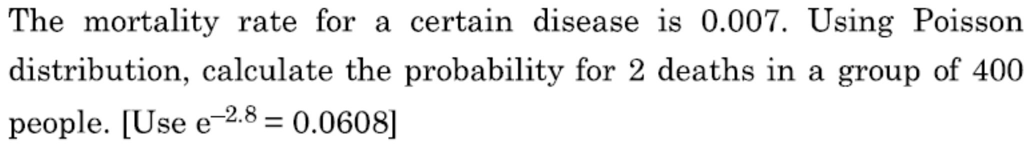
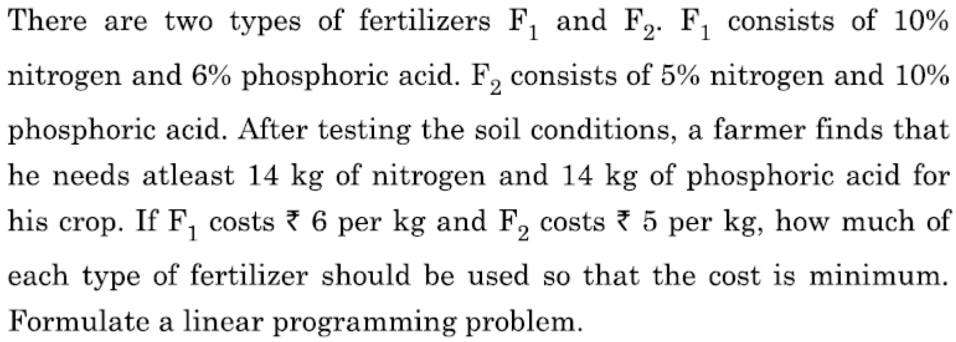
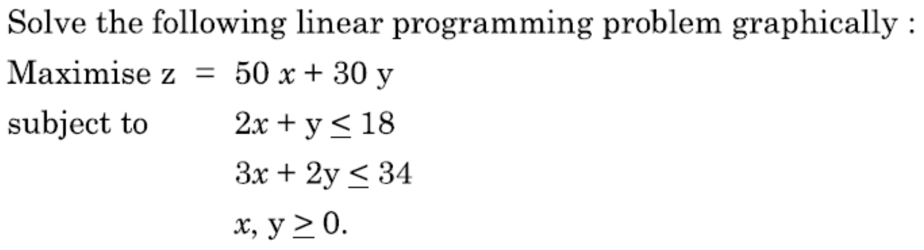
12.	The present value of a sequence of payments of ₹ 100 made at the end of every year and continuing forever, if the money is worth 5% compounded annually, is (A) ₹ 2,000 (B) ₹ 20,000 (C) ₹ 5,000 (D) ₹ 12,000	
Sol.	(A) ₹ 2,000	**1**
13.	The demand function of a monopolist is given by p = 30 + $5x - 3x^2$, where x is the number of units demanded and p is the price per unit. The marginal revenue when 2 units are sold, is (A) ₹ 28 (B) ₹ 23 (C) ₹ 1 (D) ₹ 14	
Sol.	(D) ₹ 14	**1**
14.	If the cost function and revenue function of x items are respectively given as $C(x) = 100 + 0.015\,x^2$, $R(x) = 3x$, then the value of x for maximum profit is (A) 50 (B) 100 (C) 150 (D) 200	
Sol.	(B) 100	**1**
15.	If a random variable X has the probability distribution $P(X=x)=\begin{cases} k, & \text{if } x=0 \\ 2k, & \text{if } x=1 \text{ or } 2 \\ 0, & \text{otherwise,} \end{cases}$ then the value of k is (A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$	
Sol.	(B) $\frac{1}{5}$	**1**
16.	The test statistic t for testing the significance of differences between the means of two independent samples is given by (A) $t=\frac{\overline{x}-\overline{y}}{\sqrt{s}}$ (B) $t=\frac{\overline{x}-\overline{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ (C) $t=\frac{\overline{x}-\overline{y}}{\frac{s}{\sqrt{n-1}}}$ (D) $t=\frac{\overline{x}+\overline{y}}{s\sqrt{\frac{1}{n_1}-\frac{1}{n_2}}}$	
Sol.	(B) $t=\frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$	**1**

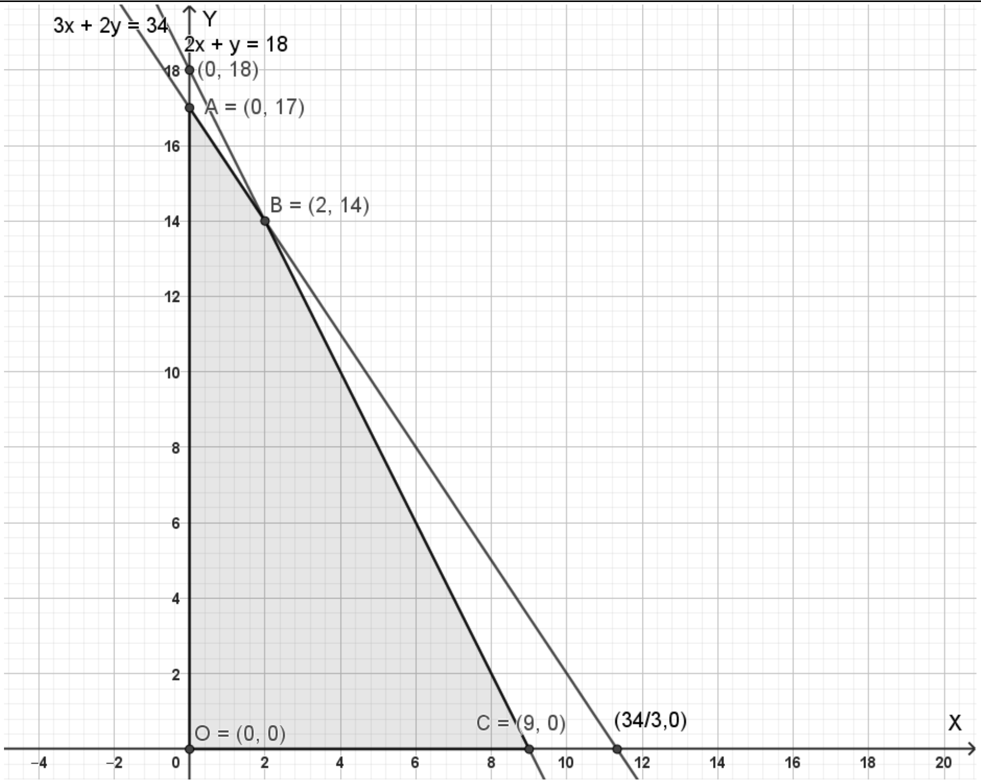
17.	The effective rate of interest equivalent to a nominal rate of 4% compounded semi-annually, is (A) 4.12% (B) 4.04% (C) 4.08% (D) 4.14%	
Sol.	(B) 4.04 %	**1**
18.	The CAGR of an investment, whose starting value is ₹ 5,000 and it grows to ₹ 25,000 in 4 years, is : [Given $(5)^{0.25} = 1.4953$] (A) 49.53% (B) 14.95% (C) 495.3% (D) 1.49%	
Sol.	(A) 49.53 %	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	**Assertion (A)** : The area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -1$ and $x = 1$ is 2 square units. **Reason (R)** : The area of the region bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is given by $\int_a^b f(x)\,dx$.	
Sol.	**(A)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
20.	**Assertion (A)** : The differential equation representing the family of curves $y = mx$, m being an arbitrary constant, is $x\frac{dy}{dx} - y = 0$. **Reason (R)** : For a family of curves, the differential equation is obtained by differentiating the equation of family of curves with respect to x and then eliminating the arbitrary constant, if any.	
Sol.	**(A)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**

	SECTION B This section comprises very short answer (VSA) type questions of 2 marks each.	
21(a).	The cost of Type I sugar is ₹ 25 per kg and Type II sugar is ₹ 35 per kg. If both Type I sugar and Type II sugar are mixed in the ratio 3:2, find the price per kg of the mixture.	
Sol.	Let the price per kg of mixture be ₹ **x**  ∴ $\frac{\mathbf{35-x}}{\mathbf{x-25}} = \frac{\mathbf{3}}{\mathbf{2}}$ $\Rightarrow 70 - 2x = 3x - 75$ Solving, we get $x = 29$ ∴ Price per kg of the mixture is ₹ 29	**1** **1**
	OR	
21(b).	Pipe A can fill a tank in 1 hour and Pipe B can fill it in 1½ hours. If both the pipes are opened in the empty tank, how much time will they take to fill the tank ?	
Sol.	Let the required time taken be n minutes ∴ $\frac{1}{n} = \frac{1}{60} + \frac{1}{90}$ $\Rightarrow \frac{1}{n} = \frac{3+2}{180} = \frac{5}{180} = \frac{1}{36}$ ∴ $n = 36$ minutes or $\frac{3}{5}$ hours	**1** **1**
22.	A boat goes 3.5 km upstream and then returns. Total time taken is 1 hour and 12 minutes. If the speed of the current is 1 km/h, then find the speed of the boat in still water.	
Sol.	Let the speed of the boat be x km/h ∴ Speed of the boat upstream $= (x-1)$ km/h and speed of the boat downstream $= (x+1)$ km/h ∴ $\frac{3.5}{x-1} + \frac{3.5}{x+1} = 1 + \frac{12}{60} = \frac{6}{5}$ $\Rightarrow\ 3.5\,(2x)5 = 6\,(x^2-1)$ $\Rightarrow 6x^2 - 35x - 6 = 0$ Solving, we get $x = 6$ km/h (rejecting the -ve value)	½ ½ ½ ½

23.	 A runs $\frac{3}{2}$ times as fast as B. If A gives B a start of 40 m, how far must the winning post from the starting point be, so that A and B reach at the same time ?	
Sol.	 Let the winning post be x metres away from the starting point. $\therefore \frac{x}{3/2} = \frac{x-40}{1}$ $\Rightarrow \frac{x}{2} = \frac{3}{2} \times 40 = 60 \Rightarrow x = 120$ metres	 **1** **1**
24.	 Given A = $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 4 & 5 \\ 0 & 2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & -5 \\ -5 & 1 & -5 \\ 1 & -2 & 4 \end{bmatrix}$, find BA.	
Sol.	BA = $\begin{bmatrix} 1 & 1 & -5 \\ -5 & 1 & -5 \\ 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 3 & 4 & 5 \\ 0 & 2 & 3 \end{bmatrix}$ Obtaining at least 4 correct entries $= \begin{bmatrix} 5 & -6 & -9 \\ -7 & -6 & -15 \\ -4 & 0 & 3 \end{bmatrix}$	 **1** **1**
25 (a).	 If a fair coin is tossed 6 times, find the probability of getting atleast 4 heads.	
Sol.	Here, $n = 6, p = \frac{1}{2}, q = \frac{1}{2}$ P (at least 4 heads in 6 throws) = $P(X \geq 4)$ $= 6_{C_4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 + 6_{C_5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 + 6_{C_6} \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0$ $= 22 \left(\frac{1}{2}\right)^6 = \frac{22}{64}\ or \frac{11}{32}$	½ **1** ½
	OR	
25 (b).	 Given that mean of a normal variate X is 9 and standard deviation is 3, then find : (i) the z-score of the data point 15 (ii) the data point if its z-score is 4.	
Sol.	(i) $Z = \frac{X-\mu}{\sigma} = \frac{15-9}{3} = 2$ (ii) $4 = \frac{X-9}{3} \Rightarrow X = 21$	**1** **1**

<table>
<tr><td></td><td>SECTION C
This section comprises short answer (SA) type questions of 3 marks each.</td><td></td></tr>
<tr><td>26.</td><td>Find the units digit in 7^{295}.</td><td></td></tr>
<tr><td>Sol.</td><td>$7^2 = 49 \equiv -1(mod\ 10)$
$7^{295} = (7^2)^{147} \times 7$
Now, $(7^2)^{147} \equiv (-1)^{147}(mod\ 10) \equiv -1(mod\ 10)$
$\Longrightarrow\ 7^{295} = (7^2)^{147} \times 7 \equiv -7(mod\ 10) \equiv 3(mod\ 10)$
∴ Units digit is 3.</td><td>
1
1
½
½</td></tr>
<tr><td>27.</td><td>Two numbers are selected at random (without replacement) from first six positive integers. Let X denotes the smaller of the two numbers obtained. Calculate the mathematical expectation of X.</td><td></td></tr>
<tr><td>Sol.</td><td>Numbers on the dice are 1, 2, 3, 4, 5, 6
∴ X can take the values 1, 2, 3, 4, 5
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>P(X)</td><td>$\frac{5}{15}$</td><td>$\frac{4}{15}$</td><td>$\frac{3}{15}$</td><td>$\frac{2}{15}$</td><td>$\frac{1}{15}$</td></tr>
<tr><td>X P(X)</td><td>$\frac{5}{15}$</td><td>$\frac{8}{15}$</td><td>$\frac{9}{15}$</td><td>$\frac{8}{15}$</td><td>$\frac{5}{15}$</td></tr>
</table>
$E(X) = \sum X\ P(X) = \frac{35}{15} = \frac{7}{3}$</td><td>
½
1
1
½</td></tr>
<tr><td>28 (a).</td><td>If the mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively, then find $P(x = 1)$.</td><td></td></tr>
<tr><td>Sol.</td><td>Mean $= np = \frac{4}{3}$, variance $= npq = \frac{8}{9}$
$\Longrightarrow q = \frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$
$\therefore p = 1 - \frac{2}{3} = \frac{1}{3}$
$n \times \frac{1}{3} = \frac{4}{3} \Longrightarrow n = 4$
$P(x = 1) = 4_{C_1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3 = 4 \times \frac{1}{3} \times \frac{8}{27}$
$= \frac{32}{81}$</td><td>½
½
½
½
½
½</td></tr>
<tr><td></td><td>OR</td><td></td></tr>
</table>

28 (b).	 The mortality rate for a certain disease is 0.007. Using Poisson distribution, calculate the probability for 2 deaths in a group of 400 people. [Use $e^{-2.8} = 0.0608$]	
Sol.	Given, $p = 0.007, n = 400$ $\therefore\ \lambda = np = 400 \times 0.007 = 2.8$ Now, $P(\mathrm{X} = 2) = \frac{(2.8)^2 e^{-2.8}}{2!}$ $= \frac{7.84}{2} \times 0.0608 = 0.2383$	½ **1** **1** ½
29 (a).	 There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid. F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If F_1 costs ₹ 6 per kg and F_2 costs ₹ 5 per kg, how much of each type of fertilizer should be used so that the cost is minimum. Formulate a linear programming problem.	
Sol.	Let x kg of nitrogen and y kg of phosphoric acid is used for minimum cost. $\therefore$ the objective function is Minimize $Z = 6x + 5y$ Subject to the constraints $10\% \times x + 5\% \times y \geq 14$ or $2x + y \geq 280$ and $6\% \times x + 10\% \times y \geq 14$ or $3x + 5y \geq 700$ $x, y \geq 0$ **Note: *** Marks should be awarded for the formation of equations $2x + y = 280$ and $3x + 5y = 700$ instead of inequations in Hindi medium only.	 ½ **1** **1** ½
	OR	
29 (b).	 Solve the following linear programming problem graphically : Maximise z = $50x + 30y$ subject to $2x + y \leq 18$ $3x + 2y \leq 34$ $x, y \geq 0$.	

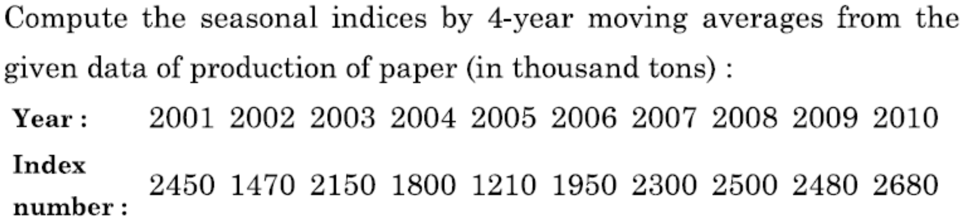
<table>
<tr>
<td>Sol.</td>
<td>


<table>
<tr><th>Corner Points</th><th>Value of Z</th></tr>
<tr><td>O (0,0)</td><td>0</td></tr>
<tr><td>A (0, 17)</td><td>510</td></tr>
<tr><td>B (2,14)</td><td>520</td></tr>
<tr><td>C (9, 0)</td><td>450</td></tr>
</table>
∴ Maximum Z = 520 at B (2, 14)
</td>
<td>1½ for correct graph
1 for correct table
½</td>
</tr>
<tr>
<td>30.</td>
<td>A machinist is making engine parts with axle diameter of 0.7 cm. A random sample of 10 parts shows mean diameter 0.742 cm with a standard deviation of 0.04 cm. On the basis of this sample, find if you would say that the work is inferior. (Given t_9 (0.05) = 2.262)</td>
<td></td>
</tr>
<tr>
<td>Sol.</td>
<td>
$\bar{x} = 0.742,\ \mu = 0.7$
$n = 10, s = 0.04$
H_0: Null hypothesis : If there is no significant difference between $\bar{x}$ and μ
H_1: Alternate hypothesis : If there is a significant difference between $\bar{x}$ and μ
$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \frac{0.742-0.7}{\frac{0.04}{\sqrt{9}}} = 3.15$
Df = 9 and $t_9(0.05) = 2.262$
Since $|t| = 3.15 > 2.262$
∴ Null hypothesis is rejected
</td>
<td>½
1½
½</td>
</tr>
</table>

	Hence the work is inferior **Note:*** $t=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ should also be accepted.	½
31.	Calculate EMI under Flat-Rate System for a loan of ₹ 5,00,000 with 7.5% annual interest rate for 5 years.	
Sol.	$P=₹\,5,00,000$ Interest $=\frac{PRT}{100}$ $=\frac{500000\times7.5\times5}{100}=₹\,1,87,500$ $n=5$ years $=60$ months $\therefore$ EMI $=\frac{P+I}{n}$ $=\frac{500000+187500}{60}$ $=₹\,11{,}458.33$	 **1** ½ **1** ½
	SECTION D This section comprises of Long Answer (LA) type questions of 5 marks each.	
32 (a).	If A $=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations : $2x-3y+5z=11,\ 3x+2y-4z=-5,\ x+y-2z=-3$	
Sol.	$\lvert A\rvert=2(-4+4)+3(-6+4)+5(3-2)=-1\neq0$ $\therefore A^{-1}$ exists Now, $\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}11\\-5\\-3\end{bmatrix}$ or $\quad AX=B\Longrightarrow X=A^{-1}B$ $\text{adj(A)}=\begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}$ $A^{-1}=\frac{1}{\lvert A\rvert}\text{adj(A)}=\frac{1}{-1}\begin{bmatrix}0&-1&2\\2&-9&23\\1&-5&13\end{bmatrix}=\begin{bmatrix}&0&1&-2\\-&2&9&-23\\&-1&5&-13\end{bmatrix}$ $\therefore\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}&0&1&-2\\-&2&9&-23\\&-1&5&-13\end{bmatrix}\begin{bmatrix}11\\-5\\-3\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$ So, $x=1, y=2, z=3$	½ **1** **1½** ½ **1** ½

	OR	
32 (b).	Using properties of determinants, prove that $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2\text{ abc }(a+b+c)^3$	
Sol.	LHS = $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$ Applying, $C_1 \rightarrow C_1 - C_3$ $C_2 \rightarrow C_2 - C_3$ $\Delta = \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (c+a)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$ $= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$ Applying, $R_1 \rightarrow R_3 - R_1 - R_2$ $= 2(a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -b & -a & ab \end{vmatrix}$ Expanding, we get $\Delta = 2abc(a+b+c)^3$	**1+1** **1** **2**
33.	If the supply function is $p = 4 - 5x + x^2$, then find the producer's surplus when price is 18.	
Sol.	$p = 4 - 5x + x^2$ $p_0 = 18,$ we have $18 = 4 - 5x + x^2$ or $x^2 - 5x - 14 = 0$ $\Longrightarrow (x-7)(x+2) = 0$ $\therefore x = 7, x = -2$ is rejected $p_0 x_0 = 18 \times 7 = 126$ $\text{PS} = p_0 x_0 - \int_0^7 (x^2 - 5x + 4)\, dx$ $= 126 - \left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x\right]_0^7$ $= 126 - \frac{119}{6}$ $= \frac{637}{6}$ or 106.17 approx.	**1½** **½** **1** **1** **1**

34(a).



Compute the seasonal indices by 4-year moving averages from the given data of production of paper (in thousand tons) :

Year :	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Index number :	2450	1470	2150	1800	1210	1950	2300	2500	2480	2680

Sol.

Year	Index Number	4 yearly moving total	4 yearly moving Average	Centered Total	Centered moving average
2001	2450				
2002	1470				
		7870	1967.5		
2003	2150			3625	1812.5
		6630	1657.5		
2004	1800			3435	1717.5
		7110	1777.5		
2005	1210			3592.5	1796.25
		7260	1815		
2006	1950			3805	1902.5
		7960	1990		
2007	2300			4297.5	2148.75
		9230	2307.5		
2008	2500			4797.5	2398.75
		9960	2490		
2009	2480				
2010	2680				

4 yearly moving total – 1 mark

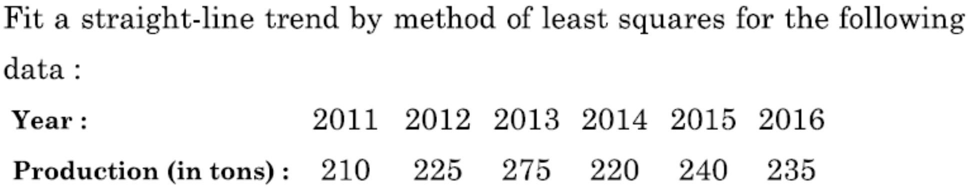
4 yearly moving average – 1½ marks

centered total - 1 mark

centered moving average - 1½ marks

OR

34(b).



Fit a straight-line trend by method of least squares for the following data :

Year :	2011	2012	2013	2014	2015	2016
Production (in tons) :	210	225	275	220	240	235

Sol.

Year (x_i)	Index Number (Y)	$X = \frac{x_i - A}{0.5} = \frac{x_i - 2013.5}{0.5}$	X^2	XY	$Y_t = a + bx$
2011	210	-5	25	-1050	$234.17 + (-5)1.64 = 225.97$
2012	225	-3	9	-675	229.25
2013	275	-1	1	-275	232.53
2014	220	1	1	220	235.81
2015	240	3	9	720	239.09
2016	235	5	25	1175	242.37
$n = 6$	1405	$\sum X = 0$	$\sum X^2 = 70$	$\sum XY = 115$	

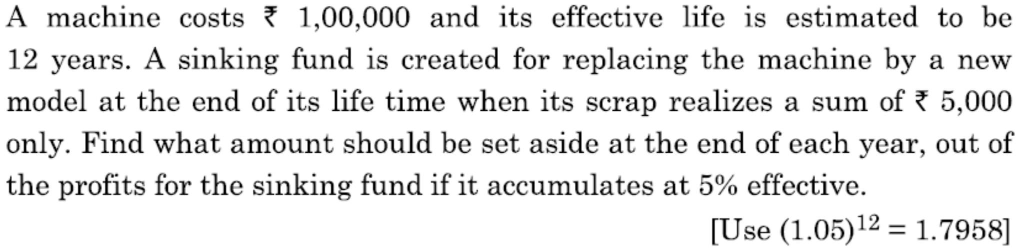
2½ for the correct table

$a = \frac{\sum Y}{n} = \frac{1405}{6} = 234.17$ (approx.) **1**

$b = \frac{\sum XY}{\sum X^2} = \frac{115}{70} = 1.64$ (approx.) **1**

Required line is $Y = a + bx = 234.17 + 1.64\,x$ ½

35.



A machine costs ₹ 1,00,000 and its effective life is estimated to be 12 years. A sinking fund is created for replacing the machine by a new model at the end of its life time when its scrap realizes a sum of ₹ 5,000 only. Find what amount should be set aside at the end of each year, out of the profits for the sinking fund if it accumulates at 5% effective.

[Use $(1.05)^{12} = 1.7958$]

Sol.

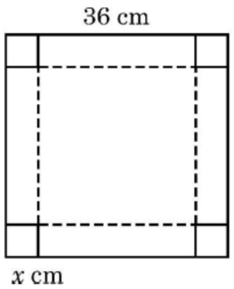
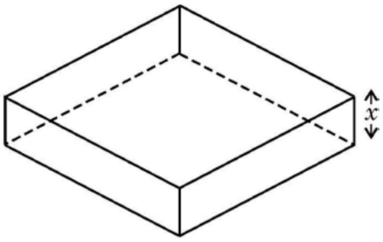
Amount needed after 12 years = ₹ 1,00,000 – ₹ 5,000 = ₹ 95,000 **1**

The payments into sinking fund consist of 12 annual payments at the rate of 5% per year.

$A = R\left[\frac{(1+i)^n - 1}{i}\right]$

$95000 = R\left[\frac{(1.05)^{12} - 1}{0.05}\right]$ **2**

$\Rightarrow R = \frac{95000 \times 0.05}{0.7958} =$ ₹ 5968.84 (or $\frac{4750}{0.8} =$ ₹ 5937.50 using approximations) **2**

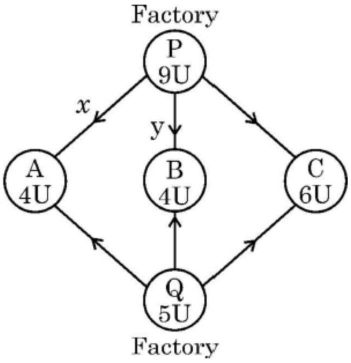
	SECTION E This section comprises of 3 case-study based questions of 4 marks each.	
36.	A man has an expensive square-shaped piece of golden board of side 36 cm. He wants to turn it into a box without top by cutting a square from each corner and folding the flaps. Let x cm be the side of square, which is cut from each corner.   Based on the above information, answer the following questions : (i) Find the expression for the volume (V) of open box in terms of x. (ii) Find $\frac{dV}{dx}$. (iii) Find the value of x for which the volume (V) is maximum. **OR** (iii) Find the maximum volume of the open box.	
Sol.	(i) $V = x\,(36 - 2x)^2$ (ii) $\frac{dV}{dx} = (36 - 2x)^2 + 2x(36 - 2x)(-2)$ $= (36 - 2x)(36 - 2x - 4x)$ $= (36 - 2x)(36 - 6x)$ $= 12\,(18 - x)(6 - x)$ (iii) $\frac{dV}{dx} = 0 \Rightarrow x = 18$ or $x = 6$ Rejecting $x = 18$, we have $x = 6$ and $\frac{d^2V}{dx^2} = 12(18 - x)(-1) + 12\,(-1)(6 - x)$ $\Rightarrow \frac{d^2V}{dx^2} < 0$ at $x = 6$ $\therefore$ volume is maximum for $x = 6$ **OR** (iii) $\frac{dV}{dx} = 0 \Rightarrow x = 18$ or $x = 6$ Rejecting $x = 18$, we have $x = 6$ and $\frac{d^2V}{dx^2} = 12(18 - x)(-1) + 12\,(-1)(6 - x)$ $\Rightarrow \frac{d^2V}{dx^2} < 0$ at $x = 6$ $\therefore$ Max V $= 6\,(36 - 12)^2 = 6\,(24)^2 = 3456\,cm^3$	**1** **1** **1** ½ ½ **1** **1**

37. There are two factories located one at P and the other at Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 4, 4 and 6 units of the commodity while the production capacity of the factories at P and Q are 9 and 5 units respectively. The cost of transportation per unit is given as :

From / To	**Cost (in ₹)**		
	A	B	C
P	160	100	150
Q	100	120	100

Based on the above information, answer the following questions :

Let x units and y units of the commodity be transported from factory P to the depots at A and B respectively, then



(i) Find (in terms of x and y) how many units of commodity be transported from factory P to depot C.

(ii) Find how many units of commodity be transported from factory Q to A, B and C respectively.

(iii) Using (i) and (ii), find the total transportation cost z.

OR

(iii) Using (i) and (ii), find the constraint inequalities for minimum cost z.

Sol.

(i) P to C $= 9 - (x + y)$ **1**

(ii) Q to A $= (4 - x)$

Q to B $= (4 - y)$ $\Big\}$ ½

Q to C $= 6 - [9 - x - y] = (x + y - 3)$ ½

(iii) $Z = 160\,x + 100\,y + 150\,(9 - x - y) + 100\,(4 - x) + 120(4 - y) + 100(x + y - 3)$ **1**

$= 10x - 70y + 1930$ **1**

OR

	(iii) $x + y \leq 9$ $x + y \geq 3$ $\left.\begin{array}{l} x \leq 4, y \leq 4 \\ x, y \geq 0 \end{array}\right\}$	½ ½ 1
38.	Ramesh borrowed a home loan amount of ₹ 7,00,000 from a bank at an interest of 12% per annum for 30 years, to be paid in monthly installments. Based on the above information, answer the following questions : (i) Write the formula for calculating EMI by reducing balance method. (ii) Write the values of P, i and n respectively. (iii) Find the EMI. [Use $(1.01)^{-360} = 0.02781668$] **OR** (iii) If the loan is to be returned in 20 years, find EMI. [Use $(1.01)^{-240} = 0.09180584$]	
Sol.	(i) $E = \frac{P\,i}{1-(1+i)^{-n}}$ or $\frac{P\,i\,(1+i)^{n}}{(1+i)^{n}-1}$ (ii) $P = ₹\,7,00,0000, i = \frac{12}{1200} = 0.01, n = 12 \times 30 = 360$ months (iii) $E = ₹\,\frac{7,00,000 \times 0.01}{1-(1.01)^{-360}}$ $= \frac{7000}{0.97218332} = ₹\,7200.29$ (or $\frac{7000}{0.97} = ₹\,7216.49$ using approximations) **OR** (iii) $E = ₹\,\frac{7,00,000 \times 0.01}{1-(1.01)^{-240}}$ $= \frac{7000}{0.90819416} = ₹\,7707.60$ (or $\frac{7000}{0.91} = ₹\,7692.31$ using approximations)	1 1 1 1 1 1

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are Multiple Choice Questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are Very Short Answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are Short Answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are Long Answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 4 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

(ix) *Use of calculator is* ***not*** *allowed.*

SECTION A

This section comprise 20 Multiple Choice Questions (MCQs) of 1 mark each.

1. The smallest positive integer (mod 11) to which 282 is congruent, is :

(A) 3 (B) 7

(C) 9 (D) 17

2. A man can row 6 km/h in still water. It takes him twice as long to row up as to row down the river. Then, the speed of the stream is :

(A) 2 km/h (B) 4 km/h

(C) 6 km/h (D) 8 km/h

3. If $\frac{|x+1|}{x+1} > 0, x \in R$, then

(A) $x \in [-1, \infty)$

(B) $x \in (-1, \infty)$

(C) $x \in (-\infty, -1)$

(D) $x \in (-\infty, -1]$

4. If $P = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $P = Q^2$, then x equals :

(A) ± 1 (B) -1

(C) 1 (D) 2

5. If A is an invertible matrix, then which of the following is ***not*** true ?

(A) $|A^{-1}| = |A|^{-1}$ (B) $(A^2)^{-1} = (A^{-1})^2$

(C) $(A')^{-1} = (A^{-1})'$ (D) $|A| \neq 0$

6. The system of linear equations

$2x + ky = 7$

$3x + 2y = 7$

will be consistent, if :

(A) $k = \frac{4}{3}$ (B) $k \neq \frac{4}{3}$

(C) $k \neq \frac{3}{4}$ (D) $k = \frac{3}{4}$

7. If $y = x^y$, then $\frac{dy}{dx}$ is :

(A) $x^y (\log x + 1)$ (B) $\frac{y^2}{x(1 + y \log x)}$

(C) $x^y (\log x - 1)$ (D) $\frac{y^2}{x(1 - y \log x)}$

8. The function $f(x) = a^x$ is increasing on R, if :

(A) $a > 0$

(B) $a > 1$

(C) $a < 0$

(D) $0 < a < 1$

9. A function $f : R \to R$ is defined as $f(x) = x^3 + 1$. The function f has :

(A) no maximum value

(B) no minimum value

(C) both maximum and minimum values

(D) neither maximum nor minimum value

10. The relation between "Marginal Cost (MC)" and "Average Cost (AC)" of producing 'x' units of a product is :

(A) $\frac{d}{dx}(AC) = x\,(MC - AC)$

(B) $\frac{d}{dx}(AC) = x\,(AC - MC)$

(C) $\frac{d}{dx}(AC) = \frac{1}{x}\,(MC - AC)$

(D) $\frac{d}{dx}(AC) = \frac{1}{x}\,(AC - MC)$

11. For a random variable X, $E(X) = 3$ and $E(X^2) = 11$. The variance of X is :

(A)	8	(B)	5
(C)	2	(D)	1

12. If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then the value of the parameter p is :

(A) $\frac{5}{6}$ (B) $\frac{1}{6}$

(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

13. If the variance of a Poisson distribution is 2, then P(X = 2) is :

(A) $4e^2$ (B) $2e^2$

(C) $\frac{2}{e^2}$ (D) $\frac{4}{e^2}$

14. Normal distribution is symmetric about :

(A) Variance (B) Co-variance

(C) Mean (D) Standard deviation

15. Using the flat rate method, the EMI to repay a loan of ₹ 20,000 in $2\frac{1}{2}$ years at an interest rate of 8% per annum is :

(A) ₹ 100 (B) ₹ 700

(C) ₹ 800 (D) ₹ 1,000

16. The graph of the inequality $3x + 2y > 6$ is the :

(A) entire XOY plane

(B) whole XOY plane excluding the points on the line $3x + 2y = 6$

(C) half plane that contains the origin

(D) half plane that neither contains the origin nor the points on the line $3x + 2y = 6$

17. The straight line trend is represented by the equation :

(A) $y = a + bx$

(B) $y = a - bx$

(C) $y = na + b \sum x$

(D) $y = na - b \sum x$

18. If for the purpose of t-test of significance, a random sample of size (n) 34 is drawn from a normal population, then the degree of freedom (N) is :

(A) 32

(B) 33

(C) 35

(D) 36

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* Solution set of inequality $|3x - 2| \le \frac{1}{2}, x \in R$ is $\left[\frac{1}{2}, \frac{5}{6}\right]$.

Reason (R) : $|x - a| \le r \Leftrightarrow x \le a - r$ or $x \ge a + r$.

20. *Assertion (A) :* Matrix $A = \begin{bmatrix} 0 & -6 & 7 \\ 6 & 5 & -1 \\ -7 & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

Reason (R) : A matrix A is skew-symmetric if $A' = -A$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Two pipes P and Q together can fill a tank in 10 minutes. If pipe P takes 15 minutes less than Q to fill the tank alone, then find the time taken by pipe Q to fill the tank alone.

OR

(b) In a 200 m race, A beats B by 35 m or 7 seconds. Find the time taken by A to complete the race.

22. Find the probability distribution of a number of successes in two tosses of a die, where a success is defined as getting a number greater than 4.

23. Suppose that a 95% confidence interval states that population mean is greater than 100 and less than 300. How would you interpret this statement ?

24. Fill in the blanks :

(a) t-distribution curve is symmetrical about the line __________.

(b) The variable t of t-distribution lies between __________.

(c) The mean of the t-distribution is __________.

(d) The variance of the t-distribution is __________.

25. (a) Find the present value of a perpetuity of ₹ 4,200 payable at the beginning of each year, if money is worth 5% compounded annually.

OR

(b) Find the present value of a perpetuity of ₹ 5,000 payable at the end of each year, if money is worth 5% compounded annually.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Prove that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

OR

(b) Prove that $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$

27. Find the inverse (if it exists) of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

28. (a) Prove that the function $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on the interval $(-1, 1)$.

OR

(b) Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

29. (a) A person invested ₹ 5,000 in a fund for 5 years. The value of the investment was ₹ 4,800 at the end of the second year, ₹ 6,000 at the end of the third year, ₹ 6,700 at the end of the fourth year and on maturity, the final investment sold at ₹ 8,000. Find the CAGR.

[Use $(1 \cdot 6)^{\frac{1}{5}} = 1 \cdot 098$]

OR

(b) The annual depreciation of an asset is ₹ 50,000 and its scrap value after useful life of 10 years is ₹ 60,000. Find the original cost of the asset, using linear depreciation method.

30. (a) Find : $\int \frac{dx}{(x+1)^2 (x^2+1)}$

OR

(b) Solve the differential equation : $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

31. Minimise Z = 5x + 10y, subject to the constraints

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x, y \geq 0$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. Solve for x : $1 \leq |x - 2| \leq 3$

33. (a) A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when the length of the rectangular area is twice its breadth.

OR

(b) Solve the differential equation : $x \frac{dy}{dx} + 2y = x^2 \log x$

34. (a) Fit a straight line trend by the method of least squares to the following data and find the trend values.

Year	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹)	65	68	70	72	75	67	73

OR

(b) Find the trend values by taking 4-yearly moving averages for the following data.

Year	2015	2016	2017	2018	2019	2020	2021	2022
Sales (in thousand ₹)	108	112	110	120	140	120	100	135

35. A machine costs a company ₹ 52,000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of ₹ 2,500 only. The price of the new model is estimated to be 25% more than the price of the present one. Find what amount should be set aside at the end of each year out of the profits for the sinking fund, if it accumulates at 3·5% per annum compound. [Given $(1{\cdot}035)^{25} = 2{\cdot}3632$]

SECTION E

This section comprises 3 case-study-based questions of 4 marks each.

Case Study – 1

36. According to an educational board survey, it was observed that class XII students apply at least one to four weeks ahead of college application deadlines. Let X represent the week when an average student applies ahead of a college's application deadline and the probability of the student to get admission in the college P(X = x) is given as follows :

$$P(X=x)=\begin{cases}\dfrac{kx}{6}, \text{ when } x = 0, 1 \text{ or } 2\\[2mm] \dfrac{(1-k)\,x}{6}, \text{ when } x = 3\\[2mm] \dfrac{kx}{2}, \text{ when } x = 4\\[2mm] 0, \text{ when } x > 4\end{cases}$$

where k is a real number.

Based on the above information, answer the following questions :

(i) Determine the value of k. *1*

(ii) What is the probability that Mahesh will get admission in the college, given that he applied at least 3 weeks ahead of application deadline ? *1*

(iii) (a) Calculate the mathematical expectation of number of weeks taken by a student to apply ahead of a college's application deadline. *2*

OR

(iii) (b) To promote early admissions, the college is offering scholarships to the students for applying ahead of deadline as follows :

₹ 50,000 for applying 4 weeks ahead

₹ 20,000 for applying 3 weeks ahead

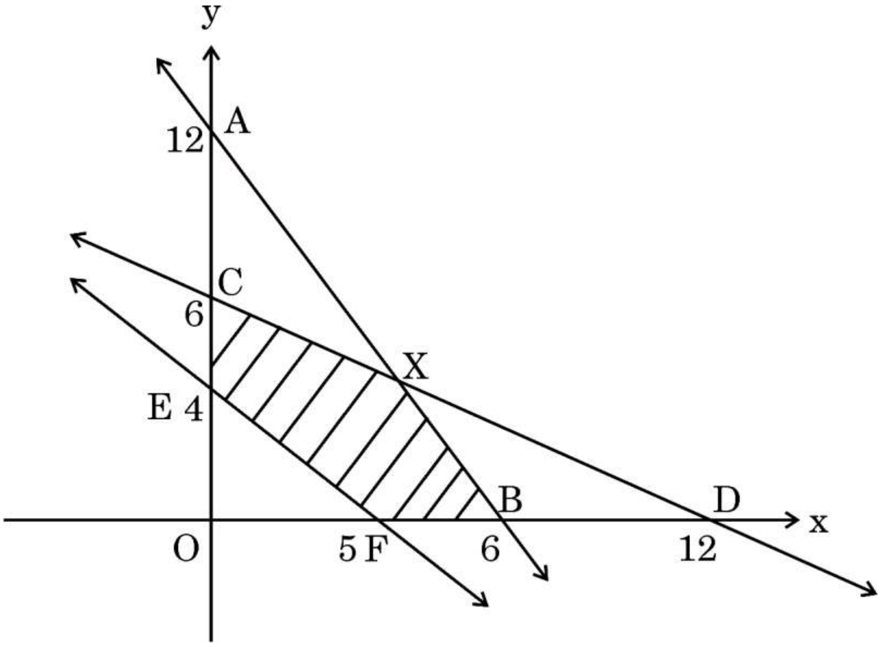
₹ 12,000 for applying 2 weeks ahead

and ₹ 9,600 for applying 1 week ahead

Determine the expected scholarship offered by the college. *2*

Case Study – 2

37. The feasible region for an LPP is shown in the graph given below :



Based on the above information, answer the following questions :

(i) Determine the equation of CD. *1*

(ii) Determine the equation EF. *1*

(iii) (a) Determine all the constraints for the LPP. *2*

OR

(iii) (b) Find the maximum value of the objective function $Z = 600x + 400y$. *2*

Case Study – 3

38. A man took a home loan of ₹ 40,00,000 from a bank at the interest of 6·75% per annum compounded monthly which is to be amortized by equal payments at the end of each month for 10 years.

Based on the above information, answer the following questions :

(i) Find the monthly instalment. *1*

[Use $(1{\cdot}005625)^{-120} = 0{\cdot}510120$]

(ii) Find the principal outstanding at the beginning of 61st month. *1*

[Use $(1{\cdot}005625)^{60} = 1{\cdot}400115$]

(iii) (a) Find the interest amount paid in the 61st instalment. *2*

OR

(iii) (b) Find the principal amount paid in the 61st instalment. *2*

MARKING SCHEME
APPLIED MATHEMATICS (Subject Code–241)
(PAPER CODE: 465/S)

Section A

Q. No.	**EXPECTED OUTCOMES/VALUE POINTS**	**Marks**
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.**	
1.	The smallest positive integer (mod 11) to which 282 is congruent, is : (A) 3 (B) 7 (C) 9 (D) 17	
Sol.	(B) 7	**1**
2.	A man can row 6 km/h in still water. It takes him twice as long to row up as to row down the river. Then, the speed of the stream is : (A) 2 km/h (B) 4 km/h (C) 6 km/h (D) 8 km/h	
Sol.	(A) 2 km/h	**1**
3.	If $\frac{\lvert x+1 \rvert}{x+1} > 0, x \in R$, then (A) $x \in [-1, \infty)$ (B) $x \in (-1, \infty)$ (C) $x \in (-\infty, -1)$ (D) $x \in (-\infty, -1]$	
Sol.	(B) $x \in (-1, \infty)$	**1**
4.	If $P = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, Q = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $P = Q^2$, then x equals : (A) ± 1 (B) -1 (C) 1 (D) 2	
Sol.	(C) 1	**1**

5.	If A is an invertible matrix, then which of the following is ***not*** true ? (A) $\lvert A^{-1}\rvert = \lvert A\rvert^{-1}$ (B) $(A^2)^{-1} = (A^{-1})^2$ (C) $(A')^{-1} = (A^{-1})'$ (D) $\lvert A\rvert \neq 0$	
Sol.	(B) $(A^2)^{-1} = (A^{-1})^2$	**1**
6.	The system of linear equations $2x + ky = 7$ $3x + 2y = 7$ will be consistent, if : (A) $k = \frac{4}{3}$ (B) $k \neq \frac{4}{3}$ (C) $k \neq \frac{3}{4}$ (D) $k = \frac{3}{4}$	
Sol.	(B) $k \neq \frac{4}{3}$	**1**
7.	If $y = x^y$, then $\frac{dy}{dx}$ is : (A) $x^y (\log x + 1)$ (B) $\frac{y^2}{x(1 + y \log x)}$ (C) $x^y (\log x - 1)$ (D) $\frac{y^2}{x(1 - y \log x)}$	
Sol.	(D) $\frac{y^2}{x\,(1-y\log x)}$	**1**
8.	The function $f(x) = a^x$ is increasing on R, if : (A) $a > 0$ (B) $a > 1$ (C) $a < 0$ (D) $0 < a < 1$	
Sol.	(B) $a > 1$	**1**

9.	A function f : R → R is defined as $f(x) = x^3 + 1$. The function f has : (A) no maximum value (B) no minimum value (C) both maximum and minimum values (D) neither maximum nor minimum value	
Sol.	(D) neither maximum nor minimum value	**1**
10.	The relation between “Marginal Cost (MC)” and “Average Cost (AC)” of producing ‘x’ units of a product is : (A) $\frac{d}{dx}(AC) = x\,(MC - AC)$ (B) $\frac{d}{dx}(AC) = x\,(AC - MC)$ (C) $\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC)$ (D) $\frac{d}{dx}(AC) = \frac{1}{x}(AC - MC)$	
Sol.	(C) $\frac{d}{dx}(AC) = \frac{1}{x}(MC - AC)$	**1**
11.	For a random variable X, $E(X) = 3$ and $E(X^2) = 11$. The variance of X is : (A) 8 (B) 5 (C) 2 (D) 1	
Sol.	(C) 2	**1**
12.	If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then the value of the parameter p is : (A) $\frac{5}{6}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$	
Sol.	(D) $\frac{2}{3}$	**1**

13.	If the variance of a Poisson distribution is 2, then $P(X = 2)$ is : (A) $4e^2$ (B) $2e^2$ (C) $\frac{2}{e^2}$ (D) $\frac{4}{e^2}$	
Sol.	(C) $\frac{2}{e^2}$	**1**
14.	Normal distribution is symmetric about : (A) Variance (B) Co-variance (C) Mean (D) Standard deviation	
Sol.	(C) Mean	**1**
15.	Using the flat rate method, the EMI to repay a loan of ₹ 20,000 in $2\frac{1}{2}$ years at an interest rate of 8% per annum is : (A) ₹ 100 (B) ₹ 700 (C) ₹ 800 (D) ₹ 1,000	
Sol.	(C) ₹ 800	**1**
16.	The graph of the inequality $3x + 2y > 6$ is the : (A) entire XOY plane (B) whole XOY plane excluding the points on the line $3x + 2y = 6$ (C) half plane that contains the origin (D) half plane that neither contains the origin nor the points on the line $3x + 2y = 6$	
Sol.	(D) half plane that neither contains the origin nor the points on the line $3x + 2y = 6$	**1**
17.	The straight line trend is represented by the equation : (A) $y = a + bx$ (B) $y = a - bx$ (C) $y = na + b \sum x$ (D) $y = na - b \sum x$	
Sol.	(A) $y = a + bx$	**1**

18.	If for the purpose of t-test of significance, a random sample of size (n) 34 is drawn from a normal population, then the degree of freedom (N) is : (A) 32 (B) 33 (C) 35 (D) 36	
Sol.	(B) 33	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* Solution set of inequality $\lvert 3x-2 \rvert \le \frac{1}{2}, x \in R$ is $\left[\frac{1}{2}, \frac{5}{6}\right]$. *Reason (R) :* $\lvert x-a \rvert \le r \Leftrightarrow x \le a-r$ or $x \ge a+r$.	
Sol.	(C) Assertion (A) is true, but Reason (R) is false.	**1**
20.	*Assertion (A) :* Matrix $A = \begin{bmatrix} 0 & -6 & 7 \\ 6 & 5 & -1 \\ -7 & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix. *Reason (R) :* A matrix A is skew-symmetric if $A' = -A$.	
Sol.	(D) Assertion (A) is false, but Reason (R) is true.	**1**
	SECTION B This section comprises very short answer (VSA) type questions of 2 marks each.	
21(a).	Two pipes P and Q together can fill a tank in 10 minutes. If pipe P takes 15 minutes less than Q to fill the tank alone, then find the time taken by pipe Q to fill the tank alone.	
Sol.	Let pipe Q fills the tank in x minutes, then P will fill the tank in $x-15$ minutes.	

<table>
<tr><td></td><td>$\frac{1}{x}+\frac{1}{x-15}=\frac{1}{10}$
$\Rightarrow\ x^2-35x+150=0$
$\Rightarrow\ x=30$
($x = 5$ rejected)
∴ Q can fill the tank alone in 30 minutes.</td><td>1
½
½</td></tr>
<tr><td></td><td>OR</td><td></td></tr>
<tr><td>21(b).</td><td>In a 200 m race, A beats B by 35 m or 7 seconds. Find the time taken by A to complete the race.</td><td></td></tr>
<tr><td>Sol.</td><td>B covers a distance of 35 metres in 7 seconds.
So, speed of B is 5 m/s
So, B completes the race of 200 m in 40 seconds.
Hence, A covers the race is (40 – 7) = 33 seconds.</td><td>½
1
½</td></tr>
<tr><td>22.</td><td>Find the probability distribution of a number of successes in two tosses of a die, where a success is defined as getting a number greater than 4.</td><td></td></tr>
<tr><td>Sol.</td><td>X can take the values 0, 1 and 2
$p=\frac{2}{6}=\frac{1}{3}, q=\frac{2}{3}$
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{4}{9}$</td><td>$\frac{4}{9}$</td><td>$\frac{1}{9}$</td></tr></table></td><td>½
1½</td></tr>
<tr><td>23.</td><td>Suppose that a 95% confidence interval states that population mean is greater than 100 and less than 300. How would you interpret this statement ?</td><td></td></tr>
<tr><td>Sol.</td><td>95% are confident that true population mean lies in the interval (100, 300).</td><td>2</td></tr>
</table>

24.	Fill in the blanks : (a) t-distribution curve is symmetrical about the line __________. (b) The variable t of t-distribution lies between __________. (c) The mean of the t-distribution is __________. (d) The variance of the t-distribution is __________.	
Sol.	(a) $t = 0$ (b) $-\infty$ to $+\infty$ (c) 0 (d) greater than 1	½ ½ ½ ½
25 (a).	Find the present value of a perpetuity of ₹ 4,200 payable at the beginning of each year, if money is worth 5% compounded annually.	
Sol.	Here, R = ₹4,200 and $i = \frac{5}{100} = 0.05$ $P = R + \frac{R}{i}$ $P = 4200 + \frac{4200}{0.05}$ $= 88{,}200$ Thus, present value of perpetuity is ₹88,200.	1 1
	OR	
25 (b).	Find the present value of a perpetuity of ₹ 5,000 payable at the end of each year, if money is worth 5% compounded annually.	
Sol.	$P = \frac{R}{i}$ $P = \frac{5000}{0.05}$ $= 1{,}00{,}000$ Thus, present value of perpetuity is ₹1,00,000.	1 1

	SECTION C This section comprises short answer (SA) type questions of 3 marks each.	
26 (a).	Prove that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$	
Sol.	$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = \begin{vmatrix} x & x & x \\ 5x & 4x & 2x \\ 10x & 8x & 3x \end{vmatrix} + \begin{vmatrix} y & x & x \\ 4y & 4x & 2x \\ 8y & 8x & 3x \end{vmatrix}$ $= x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + x^2y \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 2 \\ 8 & 8 & 3 \end{vmatrix}$ $= x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + x^2y \times 0$ $= x^3 \begin{vmatrix} 1 & 0 & 0 \\ 5 & -1 & -3 \\ 10 & -2 & -7 \end{vmatrix}$ $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ $= x^3\,[7 - 6] = x^3$	½ 1 ½ 1
	OR	
26 (b).	Prove that $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$	
Sol.	$\Delta = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = \begin{vmatrix} 0 & z & y \\ -2x & z+x & x \\ -2x & x & x+y \end{vmatrix}$ $C_1 \to C_1 - C_2 - C_3$ $= -2x \begin{vmatrix} 0 & z & y \\ 1 & x+z & x \\ 1 & x & x+y \end{vmatrix}$ $= -2x \begin{vmatrix} 0 & z & y \\ 0 & z & -y \\ 1 & x & x+y \end{vmatrix}$ $R_2 \to R_2 - R_3$ $= -2x(-2zy) = 4xyz$	1 1 ½ ½

27.	Find the inverse (if it exists) of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.	
Sol.	$\lvert A \rvert = 1 \neq 0$ $\therefore$ A is invertible and so A^{-1} exists. adj $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$	**1** **1½** **½**
28 (a).	Prove that the function $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on the interval $(-1, 1)$.	
Sol.	$f'(x) = 2x - 1$ $f'(x) = 0 \Rightarrow x = \frac{1}{2}$ $\Rightarrow f'(x) > 0$ when $x > \frac{1}{2}$ and $f(x) < 0$ when $x < \frac{1}{2}$ Since $f'(x)$ changes its sign in the interval $(-1,1)$, hence f(x) is neither strictly increasing nor strictly decreasing on $(-1, 1)$.	**½** **½** **½ + ½** **1**
	OR	
28 (b).	Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.	
Sol.	Let $P = y^x$, $Q = x^y$, $R = x^x$ Then, $\log P = x \log y \quad \Rightarrow \frac{dP}{dx} = y^x \left[\log y + \frac{x}{y}\frac{dy}{dx}\right]$ $\log Q = y \log x \quad \Rightarrow \frac{dQ}{dx} = x^y \left[\frac{y}{x} + \log x \frac{dy}{dx}\right]$ $\log R = x \log x \quad \Rightarrow \frac{dR}{dx} = x^x [1 + \log x]$ $\frac{dP}{dx} + \frac{dQ}{dx} + \frac{dR}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{y^x \log y + yx^{y-1} + x^x (1 + \log x)}{x[y^{x-1} + x^{y-1} \log x]}$	**½** **½** **½** **½** **1**

29 (a).	A person invested ₹ 5,000 in a fund for 5 years. The value of the investment was ₹ 4,800 at the end of the second year, ₹ 6,000 at the end of the third year, ₹ 6,700 at the end of the fourth year and on maturity, the final investment sold at ₹ 8,000. Find the CAGR. [Use $(1\cdot6)^{\frac{1}{5}} = 1\cdot098$]	
Sol.	$i = \left(\frac{8000}{5000}\right)^{\frac{1}{5}} - 1$ $= (1.6)^{\frac{1}{5}} - 1$ $= 0.098$ $\therefore$ CAGR = 9.8 %	**1** **1** **½** **½**
	OR	
29 (b).	The annual depreciation of an asset is ₹ 50,000 and its scrap value after useful life of 10 years is ₹ 60,000. Find the original cost of the asset, using linear depreciation method.	
Sol.	Let C denotes the original cost of the Asset. Then annual depreciation (D) is given by $D = \frac{C-S}{n}$ $\Rightarrow \quad 50000 = \frac{C-60000}{10}$ $\Rightarrow \quad C = 5{,}60{,}000$ So, the original cost of the asset is ₹ 5,60,000	 **2** **1**
30 (a).	Find : $\int \frac{dx}{(x+1)^2\,(x^2+1)}$	
Sol.	Let $\frac{1}{(x+1)^2\,(x^2+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1}$ Getting the values, $A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{-1}{2}$ and $D = 0$ Thus $I = \int \frac{1}{2(x+1)}\,dx + \frac{1}{2}\int \frac{1}{(x+1)^2}\,dx - \frac{1}{2}\int \frac{x\,dx}{x^2+1}$ $= \frac{1}{2}\log(x+1) - \frac{1}{2(x+1)} - \frac{1}{4}\log\,\lvert x^2+1 \rvert + C$	**½** **1** **1½**

OR

30 (b).

Solve the differential equation : $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

Sol.

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

½

$$\Rightarrow \frac{dy}{e^{-y}} = (e^x + x^2)\,dx$$

1

$$\int e^y dy = \int (e^x + x^2)\,dx$$

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + C$$

1½

31.

Minimise Z = 5x + 10y, subject to the constraints

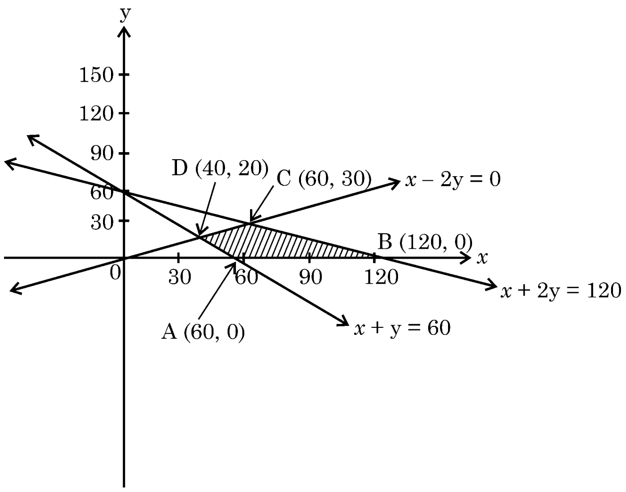
$$x + 2y \le 120$$

$$x + y \ge 60$$

$$x - 2y \ge 0$$

$$x, y \ge 0$$

Sol.



1½ for correct graph

Corner Points	Z = 5x+10y
A (60, 0)	300
B (120, 0)	600
C (60, 30)	600
D (40, 20)	400

1 for correct table

Z is minimum at A(60, 0)

½

	SECTION D This section comprises of Long Answer (LA) type questions of 5 marks each.	
32.	Solve for x : $1 \le \lvert x-2 \rvert \le 3$	
Sol.	$\lvert x-2 \rvert \ge 1$	
	$\Rightarrow x-2 \ge 1$ or $x-2 \le -1$	
	$\Rightarrow x \ge 3$ or $x \le 1$	
	$\Rightarrow x \in (-\infty, 1] \cup [3, \infty)$ --- (i)	**2**
	$\lvert x-2 \rvert \le 3$	
	$\Rightarrow -3 \le x-2 \le 3 \Rightarrow -1 \le x \le 5$	
	$\Rightarrow x \in [-1,5]$ --- (ii)	**2**
	From (i) and (ii)	
	$x \in [-1,1] \cup [3,5]$	**1**
33 (a).	A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when the length of the rectangular area is twice its breadth.	
Sol.	Let the length and breadth of rectangle be x and y units respectively.	
	Let L be the length of fencing required, then	
	$L = x + 2y$	**1**
	If 'a' denotes the area of the rectangular region. Then	
	$a = xy$	**½**
	$\Rightarrow L = x + \frac{2a}{x}$	**½**
	$\Rightarrow \frac{dL}{dx} = 1 - \frac{2a}{x^2}$	**1**
	$\frac{dL}{dx} = 0 \Rightarrow x = \sqrt{2a}$	**1**
	$\left(\frac{d^2L}{dx^2}\right)_{x=\sqrt{2a}} = \frac{2}{\sqrt{2a}} > 0$	**½**
	Thus L is least when $x = \sqrt{2a}$	
	Length of fence $= x = \sqrt{2a}$	

Breadth of fence $= y = \frac{a}{x} = \frac{a}{\sqrt{2a}} = \frac{1}{2}\sqrt{2a} = \frac{1}{2}x$ ½

Hence, length $= 2 \times$ breadth

OR

33 (b). Solve the differential equation : $x\frac{dy}{dx} + 2y = x^2 \log x$

Sol. $\frac{dy}{dx} = \frac{x^2 \log x - 2y}{x}$

Full marks may be awarded to the student who has attempted the question till finding correct value of $\frac{dy}{dx}$ as Linear differential equations is not explicitly mentioned in the curriculum.

34(a). Fit a straight line trend by the method of least squares to the following data and find the trend values.

Year	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹)	65	68	70	72	75	67	73

Sol.

Year	Index Number (Y)	$X = x_i - A$ $= x_i - 2014$	X^2	XY	$Y_t = a + bX$
2010	65	-4	16	-260	69.75 $+(-4)0.75$ $= 66.75$
2012	68	-2	4	-136	68.25
2013	70	-1	1	-70	69
2014	72	0	0	0	69.75
2015	75	1	1	75	70.5
2016	67	2	4	134	71.25
2019	73	5	25	365	73.5
$n = 7$	$\sum Y = 490$	$\sum X = 1$	$\sum X^2 = 51$	$\sum XY = 108$	

2½ for the correct table

Equation of trend line is $Y = a + bX$

$\sum Y = na + b\sum X$ and $\sum XY = a\sum X + b\sum X^2$

$\Rightarrow 490 = 7a + b$ ½

and $108 = a + 51b$ ½

Solving the two equations, we get $a = 69.75$ and $b = 0.75$ (approx.) **1**

Required line is $Y = a + bX = 69.75 + 0.75\,X$ ½

OR

34(b). Find the trend values by taking 4-yearly moving averages for the following data.

Year	2015	2016	2017	2018	2019	2020	2021	2022
Sales (in thousand ₹)	108	112	110	120	140	120	100	135

Sol.

Year	Sales	4 yearly moving total	4 yearly moving Average	Centered moving average
2015	108			
2016	112			
		450	112.5	
2017	110			116.5
		482	120.5	
2018	120			121.5
		490	122.5	
2019	140			121.25
		480	120	
2020	120			121.875
		495	123.75	
2021	100			
2022	135			

4 yearly moving total: **1½ marks**

4 yearly moving average: **1½ marks**

centered moving average: **2 marks**

35. A machine costs a company ₹ 52,000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of ₹ 2,500 only. The price of the new model is estimated to be 25% more than the price of the present one. Find what amount should be set aside at the end of each year out of the profits for the sinking fund, if it accumulates at 3·5% per annum compound. [Given $(1{\cdot}035)^{25} = 2{\cdot}3632$]

Sol.	Let R be the amount set aside each year.	
	Price of new model = 52000 + 25% of 52000 = ₹ 65000	**1**
	Scrap Value = ₹ 2500	
	Net amount = ₹ 62500	**1**
	$R = \frac{Si}{(1+i)^{n}-1}$	
	$= \frac{62500 \times 0.035}{(1+0.035)^{25}-1} = \frac{62500 \times 0.035}{1.3632}$	**2**
	= ₹ 1604.68	**1**
	SECTION E This section comprises of 3 case-study based questions of 4 marks each.	
36.	According to an educational board survey, it was observed that class XII students apply at least one to four weeks ahead of college application deadlines. Let X represent the week when an average student applies ahead of a college's application deadline and the probability of the student to get admission in the college P(X = x) is given as follows :	
	$P(X=x)= \begin{cases} \frac{kx}{6}, \text{ when } x = 0, 1 \text{ or } 2 \\ \frac{(1-k)x}{6}, \text{ when } x = 3 \\ \frac{kx}{2}, \text{ when } x = 4 \\ 0, \text{ when } x > 4 \end{cases}$	
	where k is a real number.	
	Based on the above information, answer the following questions :	
	(i) Determine the value of k.	
	(ii) What is the probability that Mahesh will get admission in the college, given that he applied at least 3 weeks ahead of application deadline ?	
	(iii) (a) Calculate the mathematical expectation of number of weeks taken by a student to apply ahead of a college's application deadline.	
	OR	
	(iii) (b) To promote early admissions, the college is offering scholarships to the students for applying ahead of deadline as follows : ₹ 50,000 for applying 4 weeks ahead ₹ 20,000 for applying 3 weeks ahead ₹ 12,000 for applying 2 weeks ahead and ₹ 9,600 for applying 1 week ahead Determine the expected scholarship offered by the college.	

Sol.

(i) $\frac{k}{6}+\frac{2k}{6}+\frac{3(1-k)}{6}+\frac{4k}{2}=1$ ½

$\Longrightarrow 3+12k=6$

$\Longrightarrow k=\frac{3}{12}=\frac{1}{4}$ ½

(ii) Required Probability $= P(X=3)+P(X=4)$

$=\frac{3}{8}+\frac{1}{2}$ ½

$=\frac{7}{8}$ ½

(iii) (a)

X	0	1	2	3	4
P(X)	0	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$

1

$E(X)=1\times\frac{1}{24}+2\times\frac{1}{12}+3\times\frac{3}{8}+4\times\frac{1}{2}=\frac{10}{3}$ weeks or $3\frac{1}{3}$ weeks **1**

OR

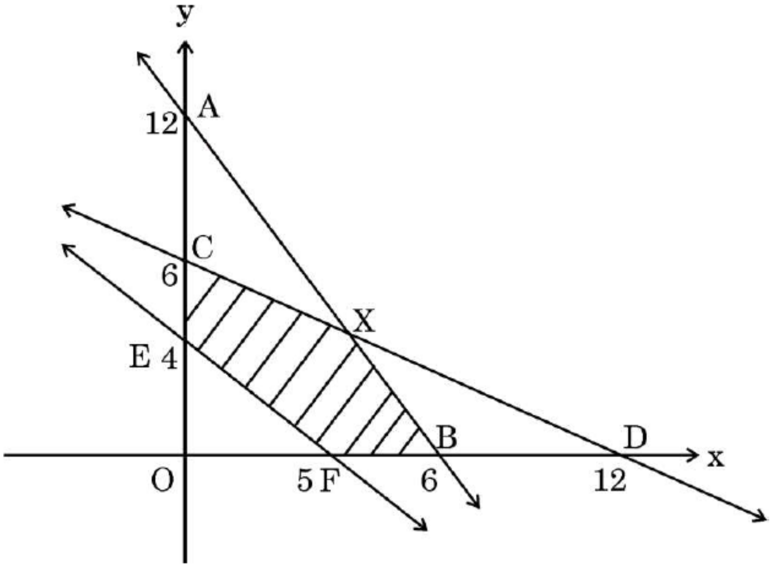
(iii) (b)

X	9600	12000	20000	50000
$P(X)$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$

1

$E(X)=\ 9600\times\frac{1}{24}+12000\times\frac{1}{12}+20000\times\frac{3}{8}+50000\times\frac{1}{2}=₹\ 33900$ **1**

37. The feasible region for an LPP is shown in the graph given below :



Based on the above information, answer the following questions :

(i) Determine the equation of CD.

(ii) Determine the equation EF.

(iii) (a) Determine all the constraints for the LPP.

OR

(iii) (b) Find the maximum value of the objective function Z = 600x + 400y.

Sol.

(i) $x + 2y = 12$ **1**

(ii) $4x + 5y = 20$ **1**

(iii) (a)

$x + 2y \leq 12, \quad 4x + 5y \geq 20,$ **½**

$2x + y \leq 12,$ **1**

$x \geq 0, y \geq 0$ **½**

OR

(iii) (b)

Corner Points	Value of Z
B (6,0)	3600
X (4,4)	4000
C (0,6)	2400
E (0,4)	1600
F (5,0)	3000

1½

Maximum value of Z = 4000 **½**

38. A man took a home loan of ₹ 40,00,000 from a bank at the interest of 6·75% per annum compounded monthly which is to be amortized by equal payments at the end of each month for 10 years.

Based on the above information, answer the following questions :

(i) Find the monthly instalment.

[Use $(1{\cdot}005625)^{-120} = 0{\cdot}510120$]

(ii) Find the principal outstanding at the beginning of 61st month.

[Use $(1{\cdot}005625)^{60} = 1{\cdot}400115$]

(iii) (a) Find the interest amount paid in the 61st instalment.

OR

(iii) (b) Find the principal amount paid in the 61st instalment.

Sol.

$P = ₹\ 40{,}00{,}000,\ i = \frac{6.75}{1200} = 0.005625,\ n = 120$ months

(i) EMI

$$= \frac{Pi}{1-(1+i)^{-n}}$$

$$= \frac{4000000 \times 0.005625}{1-(1.005625)^{-120}}$$ ½

$$= \frac{22500}{1-0.510120} = \frac{22500}{0.48988}$$

$$\approx ₹\ 45{,}930$$ ½

(ii) Principal outstanding at beginning of 61st instalment

$$= \frac{E[(1+i)^{120-61+1}-1]}{i(1+i)^{120-61+1}}$$

$$= \frac{45930\ [(1.005625)^{60}-1]}{0.005625\ (1.005625)^{60}}$$ ½

$$= \frac{45930 \times (1.400115-1)}{0.005625 \times 1.400115}$$

$$= \frac{45930 \times 0.400115}{0.005625 \times 1.400115}$$

$$\approx ₹\ 23{,}33{,}431$$ ½

(iii) (a) Interest paid in 61st instalment $= 2333431 \times 0.005625$ **1**

$$\approx ₹\ 13{,}126$$ **1**

	OR	
	(iii) (b) Interest paid in 61st instalment $= 2333431 \times 0.005625$	
	$\approx$ ₹ 13,126	**1**
	Principal amount paid in 61st instalment = EMI – Interest	
	= 45,930 – 13,126	½
	= ₹ 32,804	½