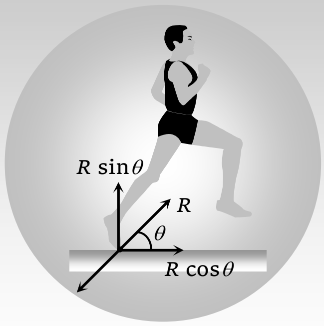
Chapter
4

Newton's Laws of Motion

Contents

Rocket propulsion is based upon the law of conservation of linear momentum. The fuel is burnt in the ignition chamber of the rocket engine. The gases so produced are ejected from an orifice in the rear of the rocket. A thrust acts on the rocket due to reaction of the movement of gases and thus the rocket starts moving in the forward direction.



Newton's Laws of Motion

4.1 Point Mass

(1) An object can be considered as a point object if during motion in a given time, it covers distance much greater than its own size.

(2) Object with zero dimension considered as a point mass.

(3) Point mass is a mathematical concept to simplify the problems.

4.2 Inertia

(1) Inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line by their own is called inertia.

(2) Inertia is not a physical quantity, it is only a property of the body which depends on mass of the body.

(3) Inertia has no units and no dimensions

(4) Two bodies of equal mass, one in motion and another is at rest, possess same inertia because it is a factor of mass only and does not depend upon the velocity.

4.3 Linear Momentum

(1) Linear momentum of a body is the quantity of motion contained in the body.

(2) It is measured in terms of the force required to stop the body in unit time.

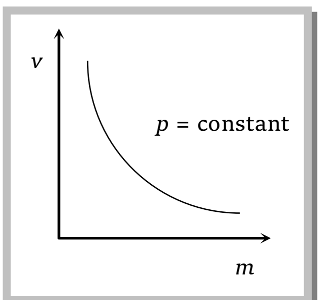
(3) It is measured as the product of the mass of the body and its velocity *i.e.*, Momentum = mass × velocity.

If a body of mass m is moving with velocity $\vec{v}$ then its linear momentum $\vec{p}$ is given by $\vec{p} = m\vec{v}$

(4) It is a vector quantity and it's direction is the same as the direction of velocity of the body.

(5) Units : *kg-m/sec* [S.I.], *g-cm/sec* [C.G.S.]

(6) Dimension : $[MLT^{-1}]$

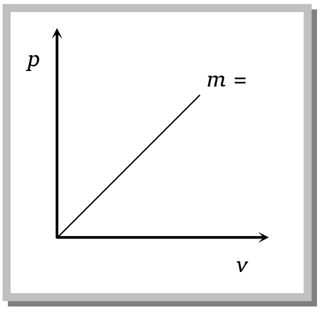


(7) If two objects of different masses have same momentum, the lighter body possesses greater velocity.

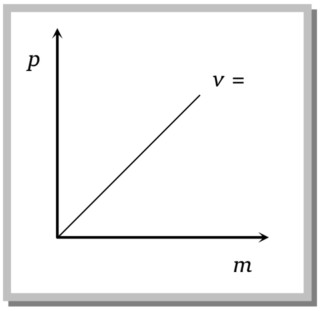
$$p = m_1 v_1 = m_2 v_2 = \text{constant}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1} \quad \textit{i.e.} \quad v \propto \frac{1}{m} \quad [\text{As } p \text{ is constant}]$$

(8) For a given body $p \propto v$



(9) For different bodies at same velocities $p \propto m$



4.4 Newton's First Law

A body continue to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

(1) If no net force acts on a body, then the velocity of the body cannot change *i.e.* the body cannot accelerate.

(2) Newton's first law defines inertia and is rightly called the law of inertia. Inertia are of three types :

Inertia of rest, Inertia of motion, Inertia of direction

(3) **Inertia of rest :** It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

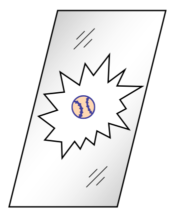
Example : (i) A person who is standing freely in bus, thrown backward, when bus starts suddenly.

When a bus suddenly starts, the force responsible for bringing bus in motion is also transmitted to lower part of body, so this part of the body comes in motion along with the bus. While the upper half of body (say above the waist) receives no force to overcome inertia of rest and so it stays in its original position. Thus there is a relative displacement between the two parts of the body and it appears as if the upper part of the body has been thrown backward.

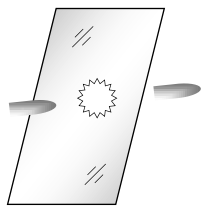
Note : ❑ If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk.

(ii) When a horse starts suddenly, the rider tends to fall backward on account of inertia of rest of upper part of the body as explained above.

(iii) A bullet fired on a window pane makes a clean hole through it while a stone breaks the whole window because the bullet has a speed much greater than the stone. So its time of contact with glass is small. So in case of bullet the motion is transmitted only to a small portion of the glass in that small time. Hence a clear hole is created in the glass window, while in case of ball, the time and the area of contact is large. During this time the motion is transmitted to the entire window, thus creating the cracks in the entire window.



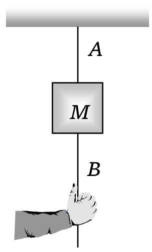
Cracks by the



Hole by the

(iv) In the arrangement shown in the figure :

(a) If the string B is pulled with a sudden jerk then it will experience tension while due to inertia of rest of mass M this force will not be transmitted to the string A and so the string B will break.



(b) If the string B is pulled steadily the force applied to it will be transmitted from string B to A through the mass M and as tension in A will be greater than in B by Mg (weight of mass M) the string A will break.

(v) If we place a coin on smooth piece of card board covering a glass and strike the card board piece suddenly with a finger. The cardboard slips away and the coin falls into the glass due to inertia of rest.

(vi) The dust particles in a durree falls off when it is beaten with a stick. This is because the beating sets the durree in motion whereas the dust particles tend to remain at rest and hence separate.

(4) **Inertia of motion :** It is the inability of a body to change itself its state of uniform motion *i.e.,* a body in uniform motion can neither accelerate nor retard by its own.

Example : (i) When a bus or train stops suddenly, a passenger sitting inside tends to fall forward. This is because the lower part of his body comes to rest with the bus or train but the upper part tends to continue its motion due to inertia of motion.

(ii) A person jumping out of a moving train may fall forward.

(iii) An athlete runs a certain distance before taking a long jump. This is because velocity acquired by running is added to velocity of the athlete at the time of jump. Hence he can jump over a longer distance.

(5) **Inertia of direction :** It is the inability of a body to change by itself direction of motion.

Example : (i) When a stone tied to one end of a string is whirled and the string breaks suddenly, the stone flies off along the tangent to the circle. This is because the pull in the string was forcing the stone to move in a circle. As soon as the string breaks, the pull vanishes. The stone in a bid to move along the straight line flies off tangentially.

(ii) The rotating wheel of any vehicle throw out mud, if any, tangentially, due to directional inertia.

(iii) When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

Sample problem based on Newton's first law

Problem **1.** When a bus suddenly takes a turn, the passengers are thrown outwards because of

[AFMC 1999; CPMT 2000, 2001]

(a) Inertia of motion (b) Acceleration of motion

(c) Speed of motion (d) Both (b) and (c)

Solution : (a)

Problem **2.** A person sitting in an open car moving at constant velocity throws a ball vertically up into air. The ball fall

[EAMCET (Med.) 1995]

(a) Outside the car (b) In the car ahead of the person

(c) In the car to the side of the person (d) Exactly in the hand which

Solution : (d) Because the horizontal component of velocity are same for both car and ball so they cover equal horizontal distances in given time interval.

4.5 Newton's Second Law

(1) The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.

(2) If a body of mass m, moves with velocity $\vec{v}$ then its linear momentum can be given by $\vec{p} = m\vec{v}$ and if force $\vec{F}$ is applied on a body, then

$$\vec{F} \propto \frac{d\vec{p}}{dt} \Rightarrow F = K\frac{d\vec{p}}{dt}$$

or $$\vec{F} = \frac{d\vec{p}}{dt}$$ (K = 1 in C.G.S. and S.I. units)

or $$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$ (As $a = \frac{d\vec{v}}{dt}$ = acceleration produced in the body)

$\therefore$ $$\vec{F} = m\vec{a}$$

Force = mass $\times$ acceleration

Sample problem based on Newton's second law

***Problem* 3.** A train is moving with velocity 20 m/sec. on this, dust is falling at the rate of 50 kg/min. The extra force required to move this train with constant velocity will be

(a) 16.66 N (b) 1000 N (c) 166.6 N (d)

Solution : (a) Force $F = v\frac{dm}{dt} = 20 \times \frac{50}{60} = 16.66\,N$

***Problem* 4.** A force of 10 Newton acts on a body of mass 20 kg for 10 seconds. Change in its momentum is **[MP PET 2002]**

(a) 5 $kg\ m/s$ (b) 100 $kg\ m/s$ (c) 200 $kg\ m/s$ (d)

Solution : (b) Change in momentum $=$ force $\times$ time $= 10 \times 10 = 100\ kg\,m/\text{sec}$

***Problem* 5.** A vehicle of 100 kg is moving with a velocity of 5 m/sec. To stop it in $\frac{1}{10}$ sec, the required force in opposite direction is

(a) 5000 N (b) 500 N (c) 50 N (d)

Solution : (a) $m = 100\ kg$ $u = 5\,m/s$, $v = 0$ $t = 0.1\ sec$

$$Force = \frac{mdv}{dt} = \frac{m(v-u)}{t} = \frac{100\,(0-5)}{0.1}$$

$$F = -5000\ N$$

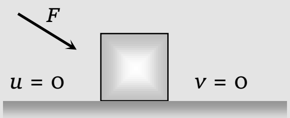
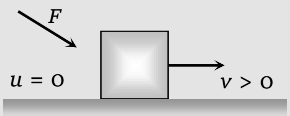
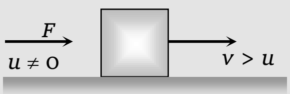
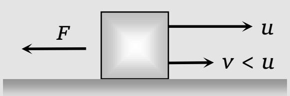
4.6 Force

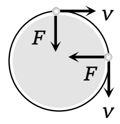
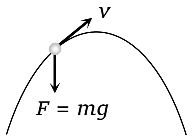
(1) Force is an external effect in the form of a push or pulls which

(i) Produces or tries to produce motion in a body at rest.

(ii) Stops or tries to stop a moving body.

(iii) Changes or tries to change the direction of motion of the body.

	Body remains at rest. Here force is trying to change the state of rest.
	Body starts moving. Here force changes the state of rest.
	In a small interval of time, force increases the magnitude of speed and direction of motion remains same.
	In a small interval of time, force decreases the magnitude of speed and direction of motion remains same.

	In uniform circular motion only direction of velocity changes, speed remains constant. Force is always perpendicular to velocity.
	In non-uniform circular motion, elliptical, parabolic or hyperbolic motion force acts at an angle to the direction of motion. In all these motions. Both magnitude and direction of velocity changes.

(2) Dimension : Force = mass × acceleration

$$[F]=[M][LT^{-2}]=[MLT^{-2}]$$

(3) Units : Absolute units : (i) *Newton* (S.I.) (ii) *Dyne* (C.G.S)

Gravitational units : (i) Kilogram-force (M.K.S.) (ii) Gram-force (C.G.S)

Newton : One Newton is that force which produces an acceleration of $1m/s^2$ in a body of mass 1 *Kilogram*. $\therefore 1$ *Newton* $=1kg\,m/s^2$

Dyne : One dyne is that force which produces an acceleration of $1cm/s^2$ in a body of mass 1 *gram*. $\therefore$ 1 *Dyne* $=1gm\,cm/\sec^2$

Relation between absolute units of force 1 *Newton* $=10^5$ *Dyne*

Kilogram-force : It is that force which produces an acceleration of $9.8m/s^2$ in a body of mass 1 *kg*. $\therefore$ 1 *kg-f* = 9.81 *Newton*

Gram-force : It is that force which produces an acceleration of $980\,cm/s^2$ in a body of mass 1*gm*. $\therefore$ 1 *gm-f* = 980 *Dyne*

Relation between gravitational units of force : 1 *kg-f* = 10^7 *gm-f*

(4) $\vec{F}=m\vec{a}$ formula is valid only if force is changing the state of rest or motion and the mass of the body is constant and finite.

(5) If *m* is not constant $\vec{F}=\frac{d}{dt}(m\vec{v})=m\frac{d\vec{v}}{dt}+\vec{v}\frac{dm}{dt}$

(6) If force and acceleration have three component along *x*, *y* and *z* axis, then

$$\vec{F}=F_x\hat{i}+F_y\hat{j}+F_z\hat{k} \text{ and } \vec{a}=a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$$

From above it is clear that $F_x=ma_x,\ F_y=ma_y, F_z=ma_z$

(7) No force is required to move a body uniformly along a straight line.

$\vec{F} = ma$ $\quad \therefore \vec{F} = 0 \quad$ (As $a = 0$)

(8) When force is written without direction then positive force means repulsive while negative force means attractive.

Example : *Positive force* – Force between two similar charges

Negative force – Force between two opposite charges

(9) Out of so many natural forces, for distance 10^{-15} *metre*, nuclear force is strongest while gravitational force weakest. $F_{\text{nuclear}} > F_{\text{electromagnetic}} > F_{\text{gravitational}}$

(10) Ratio of electric force and gravitational force between two electron $F_e / F_g = 10^{43}$
$\therefore F_e >> F_g$

(11) Constant force : If the direction and magnitude of a force is constant. It is said to be a constant force.

(12) Variable or dependent force :

(i) *Time dependent force* : In case of impulse or motion of a charged particle in an alternating electric field force is time dependent.

(ii) *Position dependent force* : Gravitational force between two bodies $\frac{Gm_1 m_2}{r^2}$

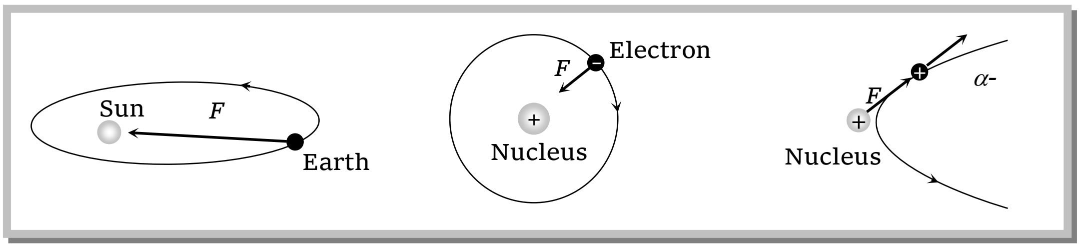
or Force between two charged particles $= \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$.

(iii) *Velocity dependent force* : Viscous force $(6\pi\eta rv)$

Force on charged particle in a magnetic field $(qvB \sin\theta)$

(13) Central force : If a position dependent force is always directed towards or away from a fixed point it is said to be central otherwise non-central.

Example : Motion of earth around the sun. Motion of electron in an atom. Scattering of α-particles from a nucleus.



(14) Conservative or non conservative force : If under the action of a force the work done in a round trip is zero or the work is path independent, the force is said to be conservative otherwise non conservative.

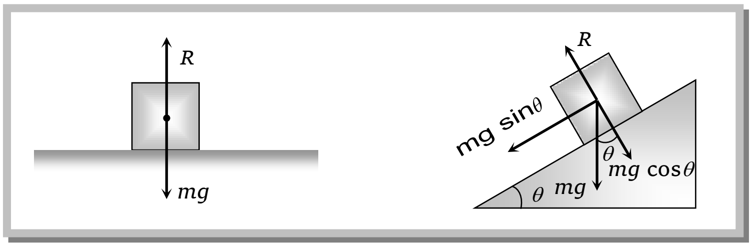
Example : Conservative force : Gravitational force, electric force, elastic force.

Non conservative force : Frictional force, viscous force.

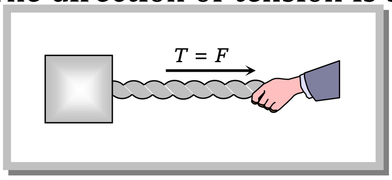
(15) Common forces in mechanics :

(i) *Weight* : Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

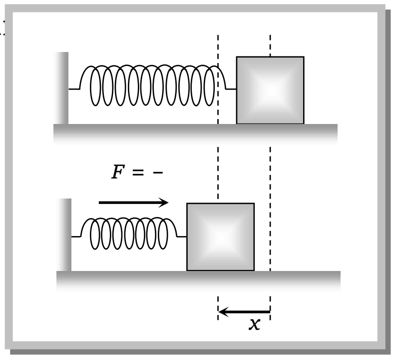
(ii) *Reaction or Normal force :* When a body is placed on a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. Then force is called 'Normal force' or 'Reaction'.



(iii) *Tension* : The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is so as to pull the body.



(iv) *Spring force :* Every spring resists any attempt to change its length. This resistive force increases with change in length. Spring force is given by $F = -Kx$; where x is the change in length and K is the spring constant (u



4.7 Equilibrium of Concurrent Force

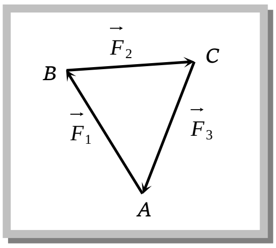
(1) If all the forces working on a body are acting on the same point, then they are said to be concurrent.

(2) A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.

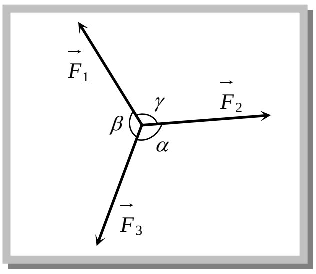
(3) The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.

(4) Mathematically for equilibrium $\sum F_{\text{net}} = 0$ or $\sum F_x = 0$; $\sum F_y = 0$; , $\sum F_z = 0$

(5) Three concurrent forces will be in equilibrium, if they can be represented completely by three sides of a triangle taken in order.



(6) Lami's Theorem : For concurrent forces $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$



Sample problem based on force and equilibrium

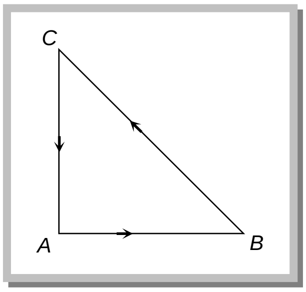
***Problem* 6.** Three forces starts acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle *ABC* (as shown). The particle will now move with velocity

(a) $\vec{v}$ remaining unchanged

(b) Less than $\vec{v}$

(c) Greater than $\vec{v}$

(d) $\vec{v}$ in the direction of the largest force *BC*



Solution : (a) Given three forces are in equilibrium *i.e.* net force will be zero. It means the particle will move with same velocity.

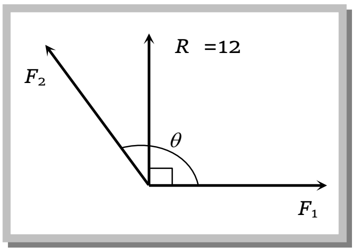
***Problem* 7.** Two forces are such that the sum of their magnitudes is 18 *N* and their resultant is perpendicular to the smaller force and magnitude of resultant is 12. Then the magnitudes of the forces are **[AIEEE 2002]**

(a) 12 N, 6 N (b) 13 N, 5N (c) 10 N, 8 N (d)

Solution : (b) Let two forces are F_1 and $F_2 (F_1 < F_2)$.

According to problem: $F_1 + F_2 = 18$ .....(i)

Angle between F_1 and resultant (R) is 90°



$\therefore \quad \tan 90 = \dfrac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} = \infty$

$\Rightarrow F_1 + F_2 \cos\theta = 0$

$\Rightarrow \cos\theta = -\dfrac{F_1}{F_2}$ .....(ii)

and $R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\theta$

$144 = F_1^2 + F_2^2 + 2F_1F_2 \cos\theta$ .....(iii)

by solving (i), (ii) and (iii) we get $F_1 = 5N$ and $F_2 = 13\,N$

***Problem* 8.** The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is

(a) 60^o (b) 120^o (c) 150^o (d) 90^o

Solution: (b) Let forces are F and 2F and angle between them is θ and resultant makes an angle α with the force F.

$\tan\alpha = \dfrac{2F\sin\theta}{F + 2F\cos\theta} = \tan 90 = \infty$

$\Rightarrow F + 2F\cos\theta = 0 \qquad \therefore \cos\theta = -1/2$ or $\theta = 120°$

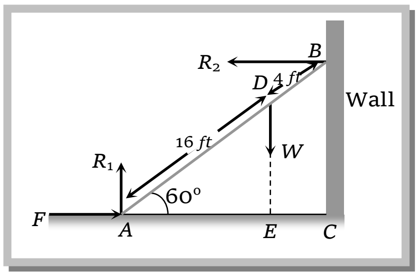
***Problem* 9.** A weightless ladder, 20 *ft* long rests against a frictionless wall at an angle of 60° with the horizontal. A 150 pound man is 4 *ft* from the top of the ladder. A horizontal force is needed to prevent it from slipping. Choose the correct magnitude from the following

(a) 175 *lb* (b) 100 *lb* (c) 70 *lb* (d) 150 *lb*

Solution: (c) Since the system is in equilibrium therefore $\sum F_x = 0$ and $\sum F_y = 0 \;\therefore\; F = R_2$ and $W = R_1$

Now by taking the moment of forces about point B.

$F.(BC) + W.(EC) = R_1(AC)$ [from the figure EC= 4 cos

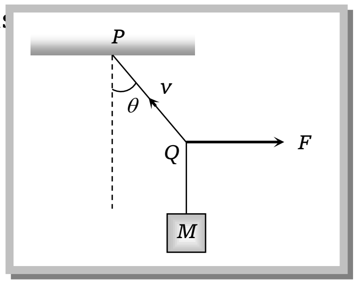


$F.(20 \sin 60) + W(4\cos 60) = R_1(20\cos 60)$

$10\sqrt{3}F + 2W = 10\,R_1$ [As $R_1 = W$]

$\therefore F = \dfrac{8W}{10\sqrt{3}} = \dfrac{8 \times 150}{10\sqrt{3}} = 70\,lb$

Problem **10.** A mass M is suspended by a rope from a rigid support at P as shown in the figure. Another rope is tied at the end Q, and it is pulled horizontally with a force F. If the rope PQ makes angle θ with the vertical then the tension in the string PQ is



(a) $F\sin\theta$

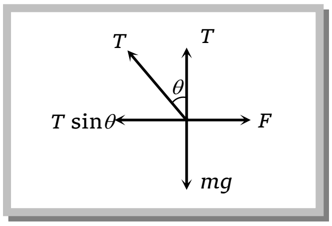
(b) $F/\sin\theta$

(c) $F\cos\theta$

(d) $F/\cos\theta$

Solution: (b) From the figure

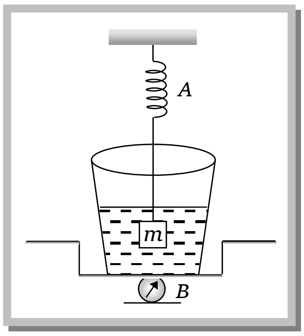
For horizontal equilibrium



$T\sin\theta = F$

$\therefore\ T = \dfrac{F}{\sin\theta}$

Problem **11.** A spring balance A shows a reading of 2 *kg*, when an aluminium block is suspended from it. Another balance B shows a reading of 5 *kg*, when a beaker full of liquid is placed in its pan. The two balances are arranged such that the *Al* – block is completely immersed inside the liquid as shown in the figure. Then **[IIT-JEE 1985]**



(a) The reading of the balance A will be more than 2 *kg*

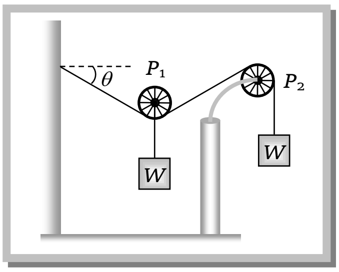
(b) The reading of the balance B will be less than 5 *kg*

(c) The reading of the balance A will be less than 2 *kg*. and that of B will be more than 5 *kg*

(d) The reading of balance A will be 2 *kg*. and that of B will be 5 *kg*.

Solution: (c) Due to buoyant force on the aluminium block the reading of spring balance A will be less than 2 kg but it increase the reading of balance B.

Problem **12.** In the following diagram, pulley P_1 is movable and pulley P_2 is fixed. The value of angle θ will be



(a) 60°

(b) 30°

(c) 45°

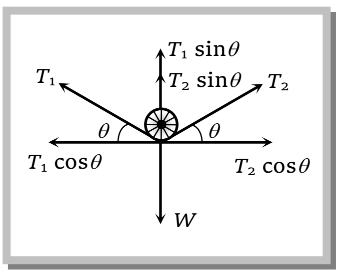
(d) 15°

Solution: (b) Free body diagram of pulley P_1 is shown in the figure

For horizontal equilibrium $T_1\cos\theta = T_2\cos\theta\ \therefore\ T_1 = T_2$

and $T_1 = T_2 = W$

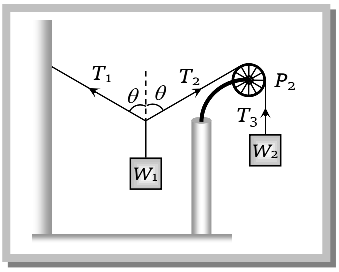
For vertical equilibrium



$T_1 \sin\theta + T_2 \sin\theta = W \Rightarrow W\sin\theta + W\sin\theta = W$

$\therefore \sin\theta = \frac{1}{2}$ or $\theta = 30°$

***Problem* 13.** In the following figure, the pulley is massless and frictionless. The relation between T_1, T_2 and T_3 will be



(a) $T_1 = T_2 \neq T_3$

(b) $T_1 \neq T_2 = T_3$

(c) $T_1 \neq T_2 \neq T_3$

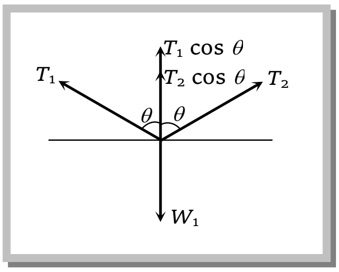
(d) $T_1 = T_2 = T_3$

Solution : (d) Since through a single string whole system is attached so $W_2 = T_3 = T_2 = T_1$

***Problem* 14.** In the above problem (13), the relation between W_1 and W_2 will be

(a) $W_2 = \frac{W_1}{2\cos\theta}$ (b) $2W_1 \cos\theta$ (c) $W_2 = W_1$ (d) $W_2 = \frac{2\cos\theta}{W_1}$

Solution : (a) For vertical equilibrium

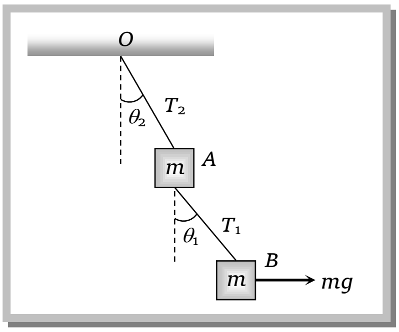


$T_1 \cos\theta + T_2 \cos\theta = W_1$ $[\text{As} T_1 = T_2 = W_2]$

$2W_2 \cos\theta = W_1$

$\therefore W_2 = \frac{W_1}{2\cos\theta}$.

***Problem* 15.** In the following figure the masses of the blocks *A* and *B* are same and each equal to *m*. The tensions in the strings *OA* and *AB* are T_2 and T_1 respectively. The system is in equilibrium with a constant horizontal force mg on *B*. The T_1 is



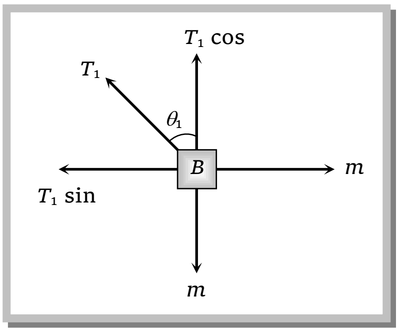
(a) *mg*

(b) $\sqrt{2}\, mg$

(c) $\sqrt{3}\, mg$

(d) $\sqrt{5}\, mg$

Solution : (b) From the free body diagram of block *B*



$T_1 \cos\theta_1 = mg$ ......(i)

$T_1 \sin\theta_1 = -mg$ .....(ii)

by squaring and adding $T_1^2\left(\sin^2\theta_1 + \cos^2\theta_1\right) = 2(mg)^2$

$\therefore T_1 = \sqrt{2}mg$

Problem **16.** In the above problem (15), the angle θ_1 is

(a) 30° (b) 45° (c) 60° (d) $\tan^{-1}\left(\frac{1}{2}\right)$

Solution : (b) From the solution (15) by dividing equation(ii) by equation (i)

$$\frac{T_1 \sin\theta_1}{T_1 \cos\theta_1} = \frac{mg}{mg}$$

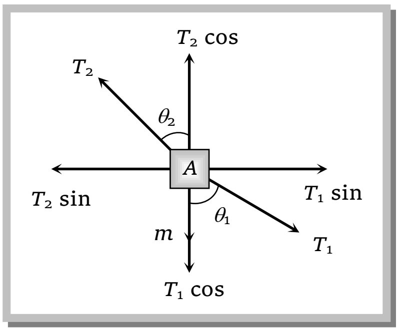
$$\therefore \tan\theta_1 = 1 \text{ or } \theta_1 = 45°$$

Problem **17.** In the above problem (15) the tension T_2 will be

(a) mg (b) $\sqrt{2}\,mg$ (c) $\sqrt{3}\,mg$ (d) $\sqrt{5}\,mg$

Solution : (d) From the free body diagram of block *A*

For vertical equilibrium $T_2 \cos\theta_2 = mg + T_1 \cos\theta_1$



$$T_2 \cos\theta_2 = mg + \sqrt{2}mg \cos 45°$$

$$T_2 \cos\theta_2 = 2mg \quad .....(i)$$

For horizontal equilibrium $T_2 \sin\theta_2 = T_1 \sin\theta_1 = \sqrt{2}mg \sin 45°$

$$T_2 \sin\theta_2 = mg \quad .....(ii)$$

by squaring and adding (i) and (ii) equilibrium

$$T_2^2 = 5(mg)^2 \text{ or } T_2 = \sqrt{5}mg$$

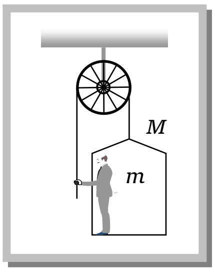
Problem **18,** In the above problem (15) the angle θ_2 will be

(a) 30° (b) 45° (c) 60° (d) $\tan^{-1}\left(\frac{1}{2}\right)$

Solution : (d) From the solution (17) by dividing equation(ii) by equation (i)

$$\frac{\sin\theta_2}{\cos\theta_2} = \frac{mg}{2mg} \Rightarrow \tan\theta_2 = \frac{1}{2} \qquad \therefore \theta_2 = \tan^{-1}\left[\frac{1}{2}\right]$$

Problem **19.** A man of mass *m* stands on a crate of mass *M*. He pulls on a light rope passing over a smooth light pulley. The other end of the rope is attached to the crate. For the system to be in equilibrium, the force exerted by the men on the rope will be



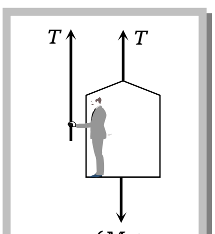
(a) $(M + m)g$

(b) $\frac{1}{2}(M+m)g$

(c) Mg

(d) mg

Solution : (b) From the free body diagram of man and crate system:



For vertical equilibrium

$2T = (M+m)g$

$\therefore T = \frac{(M+m)g}{2}$

***Problem* 20.** Two forces, with equal magnitude *F*, act on a body and the magnitude of the resultant force is $\frac{F}{3}$. The angle between the two forces is

(a) $\cos^{-1}\left(-\frac{17}{18}\right)$ (b) $\cos^{-1}\left(-\frac{1}{3}\right)$ (c) $\cos^{-1}\left(\frac{2}{3}\right)$ (d) $\cos^{-1}\left(\frac{8}{9}\right)$

Solution : (a) Resultant of two vectors A and B, which are working at an angle θ, can be given by

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ [As $A = B = F$ and $R = \frac{F}{3}$]

$\left(\frac{F}{3}\right)^2 = F^2 + F^2 + 2F^2\cos\theta$

$\frac{F^2}{9} = 2F^2 + 2F^2\cos\theta \Rightarrow \frac{-17}{9}F^2 = 2F^2\cos\theta \Rightarrow \cos\theta = \left(\frac{-17}{18}\right)$ or $\theta = \cos^{-1}\left(\frac{-17}{18}\right)$

***Problem* 21.** A cricket ball of mass 150 *gm* is moving with a velocity of 12 *m*/*s* and is hit by a bat so that the ball is turned back with a velocity of 20 *m*/*s*. The force of blow acts for 0.01*s* on the ball. The average force exerted by the bat on the ball is

(a) 480 *N* (b) 600 *N* (c) 500 *N* (d) 400 *N*

Solution : (a) $v_1 = -12\, m/s$ and $v_2 = +20\, m/s$ [because direction is reversed]

$m = 150\, gm = 0.15\, kg$, $t = 0.01$ sec

Force exerted by the bat on the ball $F = \frac{m[v_2 - v_1]}{t} = \frac{0.15[20 - (-12)]}{0.01}$ = 480 *Newton*

4.8 Newton's Third Law

To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

(1) When a body exerts a force on any other body, the second body also exerts an equal and opposite force on the first.

(2) Forces in nature always occurs in pairs. A single isolated force is not possible.

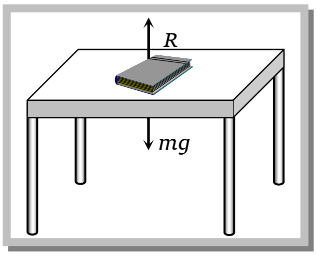
(3) Any agent, applying a force also experiences a force of equal magnitude but in opposite direction. The force applied by the agent is called '*Action*' and the counter force experienced by it is called '*Reaction*'.

(4) Action and reaction never act on the same body. If it were so the total force on a body would have always been zero *i.e.* the body will always remain in equilibrium.

(5) If $\vec{F}_{AB}$ = force exerted on body A by body B (Action) and $\vec{F}_{BA}$ = force exerted on body B by body A (Reaction)

Then according to Newton's third law of motion $\vec{F}_{AB} = -\vec{F}_{BA}$

(6) Example : (i) A book lying on a table exerts a force on the table which is equal to the weight of the book. This is the force of action.



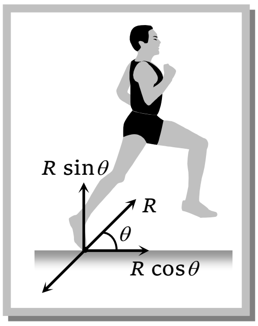
The table supports the book, by exerting an equal force on the book. This is the force of reaction.

As the system is at rest, net force on it is zero. Therefore force of action and reaction must be equal and opposite.

(ii) Swimming is possible due to third law of motion.

(iii) When a gun is fired, the bullet moves forward (action). The gun recoils backward (reaction)

(iv) Rebounding of rubber ball takes place due to third law of motion.



(v) While walking a person presses the ground in the backward direction (action) by his feet. The ground pushes the person in forward direction with an equal force (reaction). The component of reaction in horizontal direction makes the person move forward.

(vi) It is difficult to walk on sand or ice.

(vii) Driving a nail into a wooden block without holding the block is difficult.

Sample problem based on Newton's third law

***Problem* 22.** You are on a frictionless horizontal plane. How can you get off if no horizontal force is exerted by pushing against the surface

(a) By jumping
(b) By splitting or sneezing
(c) By rolling your body on the surface
(d) By running on the plane

Solution : (b) By doing so we can get push in backward direction in accordance with *Newton's* third law of motion.

4.9 Frame of Reference

(1) A frame in which an observer is situated and makes his observations is known as his 'Frame of reference'.

(2) The reference frame is associated with a co-ordinate system and a clock to measure the position and time of events happening in space. We can describe all the physical quantities like position, velocity, acceleration etc. of an object in this coordinate system.

(3) Frame of reference are of two types : (i) Inertial frame of reference (ii) Non-inertial frame of reference.

(i) **Inertial frame of reference :**

(a) A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.

(b) In inertial frame of reference Newton's laws of motion holds good.

(c) Inertial frame of reference are also called unaccelerated frame of reference or Newtonian or Galilean frame of reference.

(d) Ideally no inertial frame exist in universe. For practical purpose a frame of reference may be considered as inertial if it's acceleration is negligible with respect to the acceleration of the object to be observed.

(e) To measure the acceleration of a falling apple, earth can be considered as an inertial frame.

(f) To observe the motion of planets, earth can not be considered as an inertial frame but for this purpose the sun may be assumed to be an inertial frame.

Example : The lift at rest, lift moving (up or down) with constant velocity, car moving with constant velocity on a straight road.

(ii) **Non inertial frame of reference :**

(a) Accelerated frame of references are called non-inertial frame of reference.

(b) Newton's laws of motion are not applicable in non-inertial frame of reference.

Example : Car moving in uniform circular motion, lift which is moving upward or downward with some acceleration, plane which is taking off.

4.10 Impulse

(1) When a large force works on a body for very small time interval, it is called impulsive force.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. In such case we measure the total effect of force.

(2) Impulse of a force is a measure of total effect of force.

(3) $\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt$.

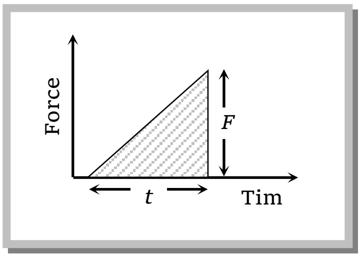
(4) Impulse is a vector quantity and its direction is same as that of force.

(5) Dimension : [MLT^{-1}]

(6) Units : *Newton-second* or $Kg\text{-}m\text{-}s^{-1}$ (S.I.) and *Dyne-second* or $gm\text{-}cm\text{-}s^{-1}$ (C.G.S.)

(7) Force-time graph : Impulse is equal to the area under *F-t* curve.

If we plot a graph between force and time, the area under the curve and time axis gives the value of impulse.



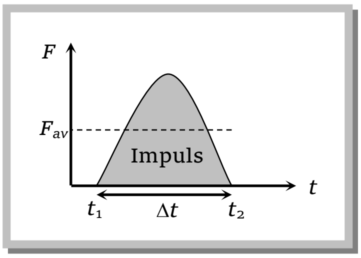
$I =$ Area between curve and time axis

$= \frac{1}{2} \times$ Base $\times$ Height

$= \frac{1}{2} F\, t$

(8) If F_{av} is the average magnitude of the force then

$$I = \int_{t_1}^{t_2} F\, dt = F_{av} \int_{t_1}^{t_2} dt = F_{av} \Delta t$$



(9) From Newton's second law $\vec{F} = \frac{d\vec{p}}{dt}$

or $\int_{t_1}^{t_2} \vec{F}\, dt = \int_{p_1}^{p_2} d\vec{p} \Rightarrow \vec{I} = \vec{p}_2 - \vec{p}_1 = \overrightarrow{\Delta p}$

i.e. The impulse of a force is equal to the change in momentum.

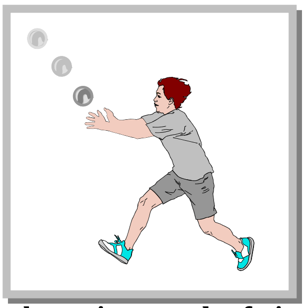
This statement is known as *Impulse momentum theorem*.

(10) *Examples* : Hitting, kicking, catching, jumping, diving, collision *etc.*

In all these cases an impulse acts. $I = \int F\, dt = F_{av} . \Delta t = \Delta p =$ constant

So if time of contact Δt is increased, average force is decreased (or diluted) and vice-versa.

(i) In hitting or kicking a ball we decrease the time of contact so that large force acts on the ball producing greater acceleration.



(ii) In catching a ball a player by drawing his hands backwards increases the time of contact and so, lesser force acts on his hands and his hands are saved from getting hurt.

(iii) In jumping on sand (or water) the time of contact is increased due to yielding of sand or water so force is decreased and we are not injured. However if we jump on cemented floor the motion stops in a very short interval of time resulting in a large force due to which we are seriously injured.

(iv) An athlete is advised to come to stop slowly after finishing a fast race. So that time of stop increases and hence force experienced by him decreases.

(v) China wares are wrapped in straw or paper before packing.

Sample problem based on Impulse

***Problem* 23.** A ball of mass 150*g* moving with an acceleration $20 m/s^2$ is hit by a force, which acts on it for 0.1 *sec*. The impulsive force is

(a) 0.5 *N-s* (b) 0.1 *N-s* (c) 0.3 *N-s* (d) 1.2 *N-s*

Solution : (c) Impulsive force $= \text{force} \times \text{time}$ $= m a \times t$ $= 0.15 \times 20 \times 0.1 =$ 0.3 *N-s*

***Problem* 24.** A force of 50 *dynes* is acted on a body of mass 5 *g* which is at rest for an interval of 3 *seconds*, then impulse is

[AFMC 1998]

(a) $0.15 \times 10^{-3} N\text{-}s$ (b) $0.98 \times 10^{-3} N\text{-}s$ (c) $1.5 \times 10^{-3} N\text{-}s$ (d) $2.5 \times 10^{-3} N\text{-}s$

Solution : (c) $\text{Impulse} = \text{force} \times \text{time}$ $= 50 \times 10^{-5} \times 3$ $= 1.5 \times 10^{-3} N\text{-s}$

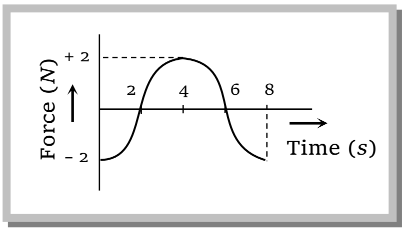
***Problem* 25.** The force-time (*F* – *t*) curve of a particle executing linear motion is as shown in the figure. The momentum acquired by the particle in time interval from zero to 8 *second* will be

(a) – 2 *N-s*

(b) + 4 *N-s*

(c) 6 *N-s*

(d) Zero



Solution : (d) Momentum acquired by the particle is numerically equal to the area enclosed between the *F-t* curve and time Axis. For the given diagram area in a upper half is positive and in lower half is negative (and equal to the upper half). So net area is zero. Hence the momentum acquired by the particle will be zero.

4.11 Law of Conservation of Linear Momentum

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

(1) According to this law for a system of particles $\vec{F} = \frac{d\vec{p}}{dt}$

In the absence of external force $\vec{F} = 0$ then $\vec{p} =$ constant

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ... =$ constant.

or $m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + ... =$ constant

This equation shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

(2) Law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.

(3) Conservation of linear momentum is equivalent to Newton's third law of motion.

For a system of two particles in absence of external force by law of conservation of linear momentum.

$$\vec{p}_1 + \vec{p}_2 = \text{constant.}$$

$\therefore$ $$m_1\vec{v}_1 + m_2\vec{v}_2 = \text{constant.}$$

Differentiating above with respect to time

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0 \quad \Rightarrow \quad m_1\vec{a}_1 + m_2\vec{a}_2 = 0 \quad \Rightarrow \quad \vec{F}_1 + \vec{F}_2 = 0$$

$\therefore$ $$\vec{F}_2 = -\vec{F}_1$$

i.e. for every action there is equal and opposite reaction which is Newton's third law of motion.

(4) Practical applications of the law of conservation of linear momentum

(i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.

(ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

(iii) **Recoiling of a gun :** For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffe

Let m_G = mass of gun, m_B = mass of bullet,

v_G = velocity of gun, v_B = velocity of bullet

Initial momentum of system = 0

Final momentum of system $= m_G\vec{v}_G + m_B\vec{v}_B$

By the law of conservation linear momentum

$$m_G\vec{v}_G + m_B\vec{v}_B = 0$$

So recoil velocity $$\vec{v}_G = -\frac{m_B}{m_G}\vec{v}_B$$

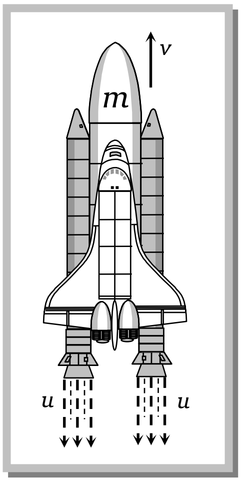
(a) Here negative sign indicates that the velocity of recoil $\vec{v}_G$ is opposite to the velocity of the bullet.

(b) $v_G \propto \frac{1}{m_G}$ *i.e.* higher the mass of gun, lesser the velocity of recoil of gun.

(c) While firing the gun must be held tightly to the shoulder, this would save hurting the shoulder because in this condition the body of the shooter and the gun behave as one body. Total mass become large and recoil velocity becomes too small.

$$v_G \propto \frac{1}{m_G + m_{\text{man}}}$$

(iv) **Rocket propulsion :** The initial momentum of the rocket on its launching pad is zero. When it is fired from the launching pad, the exhaust gases rush downward at a high speed and to conserve momentum, the rocket moves upwards.



Let $m_0 =$ initial mass of rocket,

m = mass of rocket at any instant 't' (instantaneous mass)

$m_r =$ residual mass of empty container of the rocket

u = velocity of exhaust gases,

v = velocity of rocket at any instant 't' (instantaneous velocity)

$\frac{dm}{dt} =$ rate of change of mass of rocket = rate of fuel consumption

= rate of ejection of the fuel.

(a) Thrust on the rocket : $F = -u\frac{dm}{dt} - mg$

Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

$$F = -u\frac{dm}{dt} \text{ (if effect of gravity is neglected)}$$

(b) Acceleration of the rocket : $a = \frac{u}{m}\frac{dm}{dt} - g$

and if effect of gravity is neglected $a = \frac{u}{m}\frac{dm}{dt}$

(c) Instantaneous velocity of the rocket : $v = u\log_e\left(\frac{m_0}{m}\right) - gt$

and if effect of gravity is neglected $v = u\log_e\left(\frac{m_0}{m}\right) = 2.303\,u\log_{10}\left(\frac{m_0}{m}\right)$

(d) Burnt out speed of the rocket : $v_b = v_{\max} = u\log_e\left(\frac{m_0}{m_r}\right)$

The speed attained by the rocket when the complete fuel gets burnt is called burnt out speed of the rocket. It is the maximum speed acquired by the rocket.

Sample Problem based on conservation of momentum

***Problem* 26.** A wagon weighing 1000 *kg* is moving with a velocity 50 *km*/*h* on smooth horizontal rails. A mass of 250 *kg* is dropped into it. The velocity with which it moves now is

(a) 12.5 *km/hour* (b) 20 *km/hour* (c) 40 *km/hour* (d) 50 *km/hour*

Solution : (c) Initially the wagon of mass 1000 *kg* is moving with velocity of 50 *km*/*h*

So its momentum $= 1000 \times 50 \frac{kg \times km}{h}$

When a mass 250*kg* is dropped into it. New mass of the system $= 1000 + 250 = 1250 kg$

Let *v* is the velocity of the system.

By the conservation of linear momentum : Initial momentum = Final momentum $\Rightarrow$ $1000 \times 50 = 1250 \times v$

$\therefore \quad v = \frac{50,000}{1250} = 40 km/h.$

***Problem* 27.** The kinetic energy of two masses m_1 and m_2 are equal their ratio of linear momentum will be **[RPET 1988]**

(a) m_1/m_2 (b) m_2/m_1 (c) $\sqrt{m_1/m_2}$ (d) $\sqrt{m_2/m_1}$

Solution : (c) Relation between linear momentum (*P*), man (*m*) and kinetic energy (*E*)

$P = \sqrt{2mE} \Rightarrow P \propto \sqrt{m}$ [as *E* is constant] $\therefore \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$

***Problem* 28.** Which of the following has the maximum momentum

(a) A 100 *kg* vehicle moving at 0.02 ms^{-1} (b) A 4 *g* weight moving at 10000 cms^{-1}

(c) A 200 *g* weight moving with kinetic energy 10^{-6} *J* (d) A 20 *g* weight after falling 1 *kilometre*

Solution : (d) Momentum of body for given options are :

(a) $P = mv = 100 \times 0.02 = 2 kgm/\sec$

(b) $P = mv = 4 \times 10^{-3} \times 100 = 0.4 kgm/\sec$

(c) $P = \sqrt{2mE} = \sqrt{2 \times 0.2 \times 10^{-6}} = 6.3 \times 10^{-4} kgm/\sec$

(d) $P = m\sqrt{2gh} = 20 \times 10^{-3} \times \sqrt{2 \times 10 \times 10^3} = 2.82 kgm/\sec$

So for option (d) momentum is maximum.

***Problem* 29.** A rocket with a lift-off mass 3.5×10^4 *kg* is blasted upwards with an initial acceleration of $10 m/s^2$. Then the initial thrust of the blast is

(a) $1.75 \times 10^5 N$ (b) $3.5 \times 10^5 N$ (c) $7.0 \times 10^5 N$ (d) $14.0 \times 10^5 N$

Solution : (c) Initial thrust on the rocket $F = m(g + a) = 3.5 \times 10^4 (10 + 10) = 7.0 \times 10^5 N$

***Problem* 30.** In a rocket of mass 1000 *kg* fuel is consumed at a rate of 40 *kg/s*. The velocity of the gases ejected from the rocket is $5\times10^4 m/s$. The thrust on the rocket is

(a) $2\times10^3 N$ (b) $5\times10^4 N$ (c) $2\times10^6 N$ (d) $2\times10^9 N$

Solution : (c) Thrust on the rocket $F=\frac{udm}{dt} = 5\times10^4(40) = 2\times10^6 N$

***Problem* 31.** If the force on a rocket moving with a velocity of 300 *m/s* is 210 *N*, then the rate of combustion of the fuel is

[CBSE PMT 1999]

(a) 0.7 *kg/s* (b) 1.4 *kg/s* (c) 0.07 *kg/s* (d) 10.7 *kg/s*

Solution : (a) Force on the rocket $=\frac{udm}{dt}$ $\therefore$ Rate of combustion of fuel $\left(\frac{dm}{dt}\right)=\frac{F}{u}=\frac{210}{300}=0.7 kg/s.$

***Problem* 32.** A rocket has a mass of 100 *kg*. 90% of this is fuel. It ejects fuel vapours at the rate of 1 *kg/sec* with a velocity of 500 *m/sec* relative to the rocket. It is supposed that the rocket is outside the gravitational field. The initial upthrust on the rocket when it just starts moving upwards is **[NCERT 1978]**

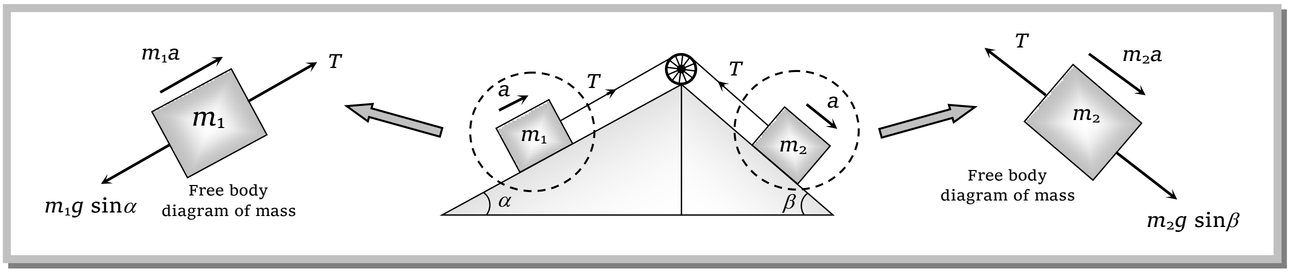
(a) Zero (b) 500 *N* (c) 1000 *N* (d) 2000 *N*

Solution : (b) Up thrust force $F=u\left(\frac{dm}{dt}\right)=500\times1=500 N$

4.12 Free Body Diagram

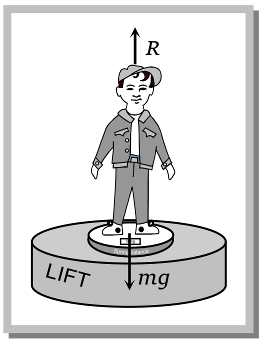
In this diagram the object of interest is isolated from its surroundings and the interactions between the object and the surroundings are represented in terms of forces.

Example :

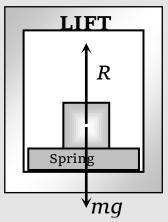
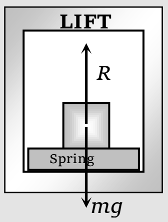
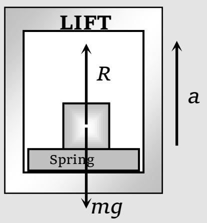
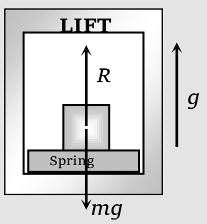
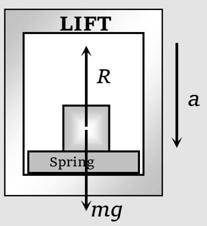
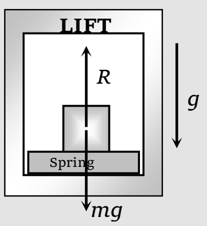


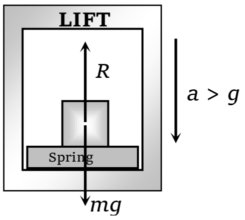
4.13 Apparent Weight of a Body in a Lift

When a body of mass *m* is placed on a weighing machine which is placed in a lift, then actual weight of the body is *mg*.



This acts on a weighing machine which offers a reaction *R* given by the reading of weighing machine. This reaction exerted by the surface of contact on the body is the *apparent weight* of the body.

Condition	Figure	Velocity	Acceleration	Reaction	Conclusion
Lift is at rest		$v = 0$	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift moving upward or downward with constant velocity		v = constant	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift accelerating upward at the rate of 'a'		v = variable	$a < g$	$R - mg = ma$ $\therefore R = m(g + a)$	Apparent weight > Actual weight
Lift accelerating upward at the rate of 'g'		v = variable	$a = g$	$R - mg = mg$ $R = 2mg$	Apparent weight = 2 Actual weight
Lift accelerating downward at the rate of 'a'		v = variable	$a < g$	$mg - R = ma$ $\therefore R = m(g - a)$	Apparent weight < Actual weight
Lift accelerating downward at the rate of 'g'		v = variable	$a = g$	$mg - R = mg$ $R = 0$	Apparent weight = Zero (weightlessness)

Lift accelerating downward at the rate of a(>g)		v = variable	$a > g$	$mg - R = ma$ $R = mg - ma$ $R = -ve$	Apparent weight negative means the body will rise from the floor of the lift and stick to the ceiling of the lift.

Sample problems based on lift

Problem **33.** A man weighs $80kg$. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of $5m/s^2$. What would be the reading on the scale. $(g = 10\,m/s^2)$

(a) 400 *N* (b) 800 *N* (c) 1200 *N* (d) Zero

Solution : (c) Reading of weighing scale $= m(g+a) = 80\,(10+5) = 1200\ N$

Problem **34.** A body of mass 2 *kg* is hung on a spring balance mounted vertically in a lift. If the lift descends with an acceleration equal to the acceleration due to gravity '*g*', the reading on the spring balance will be

(a) 2 *kg* (b) $(4 \times g)$ *kg* (c) $(2 \times g)$ *kg* (d) Zero

Solution : (d) $R = m(g-a) = (g-g) = 0$ [because the lift is moving downward with *a* = *g*]

Problem **35.** In the above problem, if the lift moves up with a constant velocity of 2 *m/sec*, the reading on the balance will be

(a) 2 *kg* (b) 4 *kg* (c) Zero (d) 1 *kg*

Solution : (a) $R = mg = 2g$ *Newton* or $2kg$ [because the lift is moving with the zero acceleration]

Problem **36.** If the lift in problem, moves up with an acceleration equal to the acceleration due to gravity, the reading on the spring balance will be

(a) 2 *kg* (b) $(2 \times g)$ *kg* (c) $(4 \times g)$ *kg* (d) 4 *kg*

Solution : (d) $R = m(g+a) = m(g+g)$ [because the lift is moving upward with *a* =*g*]

$= 2mg\ \ R = 2 \times 2g\ N = 4g\ N$ or $4\,kg$

Problem **37.** A man is standing on a weighing machine placed in a lift, when stationary, his weight is recorded as 40 *kg*. If the lift is accelerated upwards with an acceleration of $2\,m/s^2$, then the weight recorded in the machine will be $(g = 10\,m/s^2)$

(a) 32 *kg* (b) 40 *kg* (c) 42 *kg* (d) 48 *kg*

Solution : (d) $R = m\,(g+a) = 40(10+2) = 480\ N$ or $48kg$

Problem **38.** An elevator weighing 6000 *kg* is pulled upward by a cable with an acceleration of $5ms^{-2}$. Taking *g* to be $10ms^{-2}$, then the tension in the cable is

(a) 6000 *N* (b) 9000 *N* (c) 60000 *N* (d) 90000 *N*

Solution : (d) $T = m\,(g+a) = 6000\ (10+5)\ \ T = 90{,}000\ N$

Problem **39.** The ratio of the weight of a man in a stationary lift and when it is moving downward with uniform acceleration '*a*' is 3 : 2. The value of '*a*' is (*g*- Acceleration due to gravity on the earth) **[MP PET 1997]**

(a) $\frac{3}{2}g$ (b) $\frac{g}{3}$ (c) $\frac{2}{3}g$ (d) *g*

Solution : (b) $\frac{\text{weight of a man in stationary lift}}{\text{weight of a man in downward moving lift}} = \frac{mg}{m(g-a)} = \frac{3}{2}$

$\therefore \frac{g}{g-a} = \frac{3}{2} \quad \Rightarrow \quad 2g = 3g - 3a \quad \text{or} \quad a = \frac{g}{3}$

***Problem* 40.** A 60 *kg* man stands on a spring scale in the lift. At some instant he finds, scale reading has changed from 60 *kg* to 50 *kg* for a while and then comes back to the original mark. What should we conclude

(a) The lift was in constant motion upwards

(b) The lift was in constant motion downwards

(c) The lift while in constant motion upwards, is stopped suddenly

(d) The lift while in constant motion downwards, is suddenly stopped

Solution : (c) For retarding motion of a lift $R = m(g+a)$ for downward motion

$R = m(g-a)$ for upward motion

Since the weight of the body decrease for a while and then comes back to original value it means the lift was moving upward and stops suddenly.

Note : ❑ Generally we use $R = m(g+a)$ for upward motion

$R = m(g-a)$ for downward motion

here *a*= acceleration, but for the given problem *a*= retardation

***Problem* 41.** A bird is sitting in a large closed cage which is placed on a spring balance. It records a weight placed on a spring balance. It records a weight of 25 *N*. The bird (mass = 0.5*kg*) flies upward in the cage with an acceleration of $2m/s^2$. The spring balance will now record a weight of **[MP PMT 1999]**

(a) 24 *N* (b) 25 *N* (c) 26 *N* (d) 27 *N*

Solution : (b) Since the cage is closed and we can treat bird cage and air as a closed (Isolated) system. In this condition the force applied by the bird on the cage is an internal force due to this reading of spring balance will not change.

***Problem* 42.** A bird is sitting in a wire cage hanging from the spring balance. Let the reading of the spring balance be W_1. If the bird flies about inside the cage, the reading of the spring balance is W_2. Which of the following is true

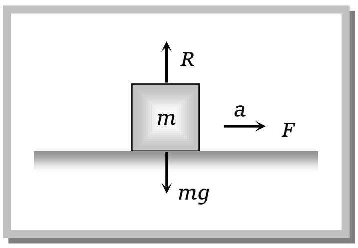
(a) $W_1 = W_2$ (b) $W_1 > W_2$

(c) $W_1 < W_2$ (d) Nothing definite can be predicted

Solution : (b) In this problem the cage is wire-cage the momentum of the system will not be conserved and due to this the weight of the system will be lesser when the bird is flying as compared to the weight of the same system when bird is resting is $W_2 < W_1$.

4.14 Acceleration of Block on Horizontal Smooth Surface

(1) **When a pull is horizontal**



$R = mg$

and $F = ma$

$\therefore$ $a = F/m$

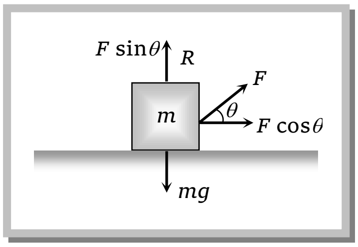
(2) **When a pull is acting at an angle (θ) to the horizontal (upward)**

$$R + F \sin\theta = mg$$

$\Rightarrow$ $$R = mg - F\sin\theta$$

and $$F\cos\theta = ma$$

$\therefore$ $$a = \frac{F\cos\theta}{m}$$

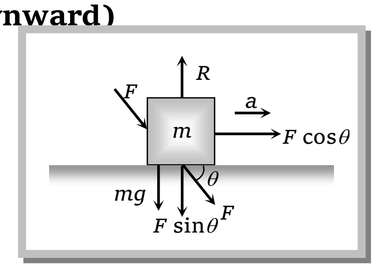


(3) **When a push is acting at an angle (θ) to the horizontal (downward)**

$$R = mg + F\sin\theta$$

and $$F\cos\theta = ma$$

$$a = \frac{F\cos\theta}{m}$$



4.15 Acceleration of Block on Smooth Inclined Plane

(1) **When inclined plane is at rest**

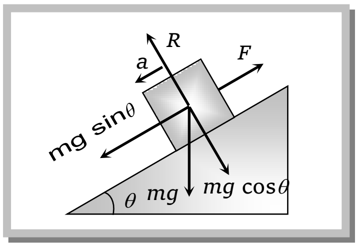
Normal reaction $R = mg\cos\theta$

Force along a inclined plane

$$F = mg\sin\theta$$

$$ma = mg\ \sin\theta$$

$\therefore$ $$a = g\sin\theta$$



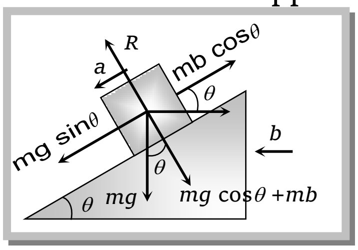
(2) **When a inclined plane given a horizontal acceleration 'b'**

Since the body lies in an accelerating frame, an inertial force (mb) acts on it in the opposite direction.

Normal reaction $R = mg\cos\theta + mb\sin\theta$

and $$ma = mg\ \sin\theta - mb\cos\theta$$

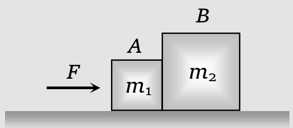
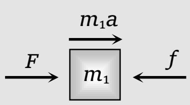
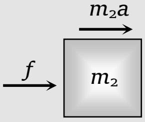
$\therefore$ $$a = g\sin\theta - b\cos\theta$$

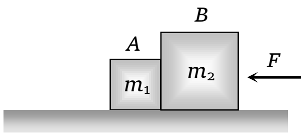
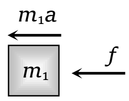
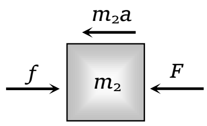
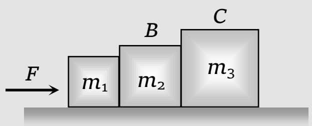
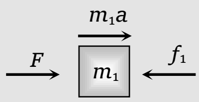
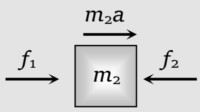
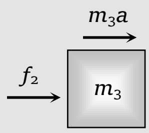
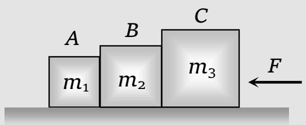
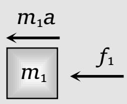
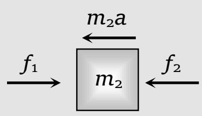
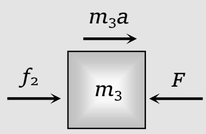


Note : ❑ The condition for the body to be at rest relative to the inclined plane : $a = g\sin\theta - b\cos\theta = 0$

$\therefore$ $$b = g\tan\theta$$

4.16 Motion of Blocks in Contact

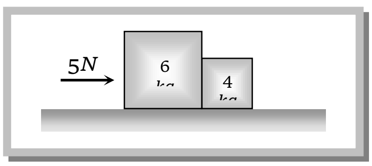
Condition	Free body diagram	Equation	Force and acceleration
	 	$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$

		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$
		$f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_2 - f_1 = m_2 a$	$f_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - f_2 = m_3 a$	$f_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

Sample problems based on motion of blocks in contact

***Problem* 43.** Two blocks of mass 4 *kg* and 6 *kg* are placed in contact with each other on a frictionless horizontal surface. If we apply a push of 5 *N* on the heavier mass, the force on the lighter mass will be

(a) 5 *N*



(b) 4 N

(c) 2 N

(d) None of the above

Solution : (c) Let $m_1 = 6kg, m_2 = 4kg$ and $F = 5N$ (given)

Force on the lighter mass = $\frac{m_2 \times F}{m_1 + m_2} = \frac{4 \times 5}{6+4} = 2N$

Problem **44.** In the above problem, if a push of 5 N is applied on the lighter mass, the force exerted by the lighter mass on the heavier mass will be

(a) 5 N (b) 4 N (c) 2 N (d) None of the above

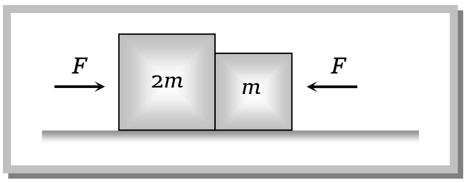
Solution : (d) Force on the heavier mass $= \frac{m_1 F}{m_1 + m_2} = \frac{6 \times 5}{6+4} = 3N$

Problem **45.** In the above problem, the acceleration of the lighter mass will be

(a) $0.5\, ms^{-2}$ (b) $\frac{5}{4}\, ms^{-2}$ (c) $\frac{5}{6}\, ms^{-2}$ (d) None of the above

Solution : (a) Acceleration $= \frac{\text{Net force on the system}}{\text{Total mass of the system}} = \frac{5}{10} = 0.5\, m/s^2$

Problem **46.** Two blocks are in contact on a frictionless table one has a mass m and the other 2 m as shown in figure. Force F is applied on mass 2m then system moves towards right. Now the same force F is applied on m. The ratio of force of contact between the two blocks will be in the two cases respectively.



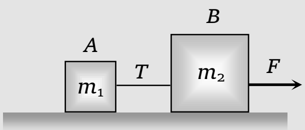
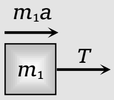
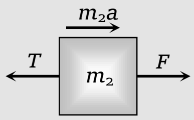
(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

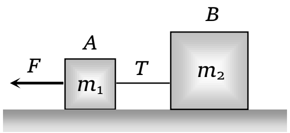
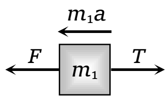
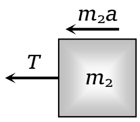
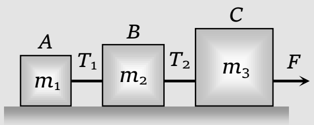
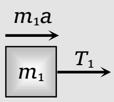
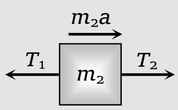
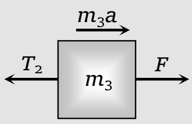
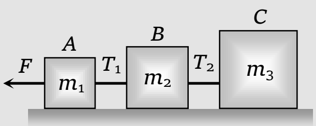
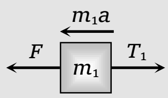
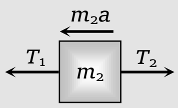
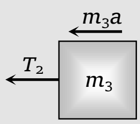
Solution : (b) When the force is applied on mass 2m contact force $f_1 = \frac{m}{m+2m} g = \frac{g}{3}$

When the force is applied on mass m contact force $f_2 = \frac{2m}{m+2m} g = \frac{2}{3} g$

Ratio of contact forces $\frac{f_1}{f_2} = \frac{1}{2}$

4.17 Motion of Blocks Connected by Mass Less String

Condition	**Free body diagram**	**Equation**	**Tension and acceleration**
	 	$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$

		$F - T = m_2 a$	$T = \dfrac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \dfrac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \dfrac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \dfrac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$
		$F - T_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$
		$T_1 - T_2 = m_2 a$	$T_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$T_2 = m_3 a$	$T_2 = \dfrac{m_3 F}{m_1 + m_2 + m_3}$

Sample problems based on motion of blocks connected by mass less string

***Problem* 47.** A monkey of mass 20 *kg* is holding a vertical rope. The rope will not break when a mass of 25 *kg* is suspended from it but will break if the mass exceeds 25 *kg*. What is the maximum acceleration with which the monkey can climb up along the rope $(g = 10\, m/s^2)$

(a) $10\,m/s^2$ (b) $25\,m/s^2$ (c) $2.5m/s^2$ (d) $5m/s^2$

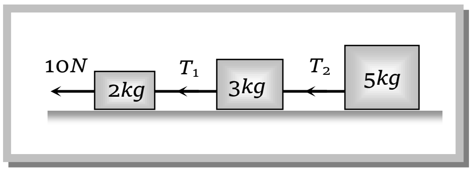
Solution : (c) Maximum tension that string can bear (T_{max}) = $25 \times g\ N = 250\ N$

Tension in rope when the monkey climb up $T = m(g+a)$

For limiting condition $T = T_{max} \Rightarrow m(g+a) = 250 \Rightarrow 20\,(10+a) = 250 \quad \therefore\ a = 2.5\,m/s^2$

Problem **48.** Three blocks of masses 2 *kg*, 3 *kg* and 5 *kg* are connected to each other with light string and are then placed on a frictionless surface as shown in the figure. The system is pulled by a force $F = 10N$, then tension $T_1 =$

[Orissa JEE 2002]

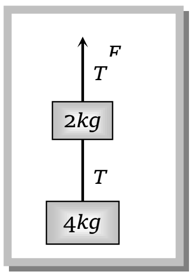


(a) 1*N* (b) 5 *N* (c) 8 *N* (d) 10 *N*

Solution : (c) By comparing the above problem with general expression. $T_1 = \frac{(m_2+m_3)F}{m_1+m_2+m_3}$

$= \frac{(3+5)10}{2+3+5} = 8\ Newton$

Problem **49.** Two blocks are connected by a string as shown in the diagram. The upper block is hung by another string. A force *F* applied on the upper string produces an acceleration of $2m/s^2$ in the upward direction in both the blocks. If *T* and T' be the tensions in the two parts of the string, then **[AMU (Engg.) 2000]**



(a) $T = 70.8N$ and $T' = 47.2N$

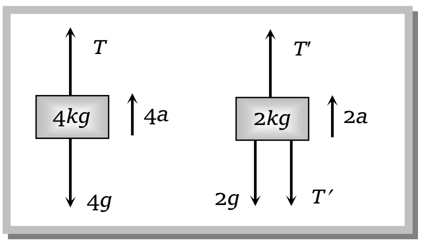
(b) $T = 58.8N$ and $T' = 47.2N$

(c) $T = 70.8N$ and $T' = 58.8N$

(d) $T = 70.8N$ and $T' = 0$

Solution : (a) From F.B.D. of mass 4 *kg* $\quad 4a = T' - 4g$ .....(i)

From F.B.D. of mass 2 *kg* $\quad 2a = T - T' - 2g$ .....(ii)



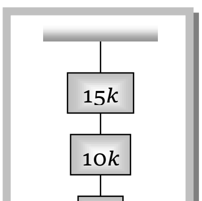
For total system upward force

$F = T = (2+4)(g+a) = 6\,(18+2)N$ = 70.8 N

by substituting the value of T in equation (i) and (ii)

and solving we get $T' = 47.2N$

Problem **50.** Three masses of 15 *kg*. 10 *kg* and 5 *kg* are suspended vertically as shown in the fig. If the string attached to the support breaks and the system falls freely, what will be the tension



in the string between 10 *kg* and 5 *kg* masses. Take $g = 10\, ms^{-2}$. It is assumed that the string remains tight during the motion

(a) 300 *N* (b) 250 *N* (c) 50 *N* (d) Zero

Solution : (d) In the condition of free fall, tension becomes zero.

***Problem* 51.** A sphere is accelerated upwards with the help of a cord whose breaking strength is five times its weight. The maximum acceleration with which the sphere can move up without cord breaking is

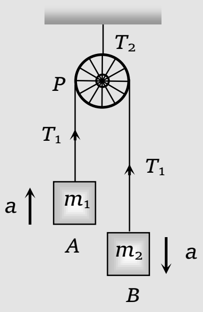
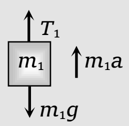
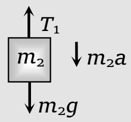
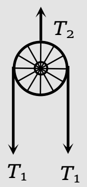
(a) $4g$ (b) $3g$ (c) $2g$ (d) g

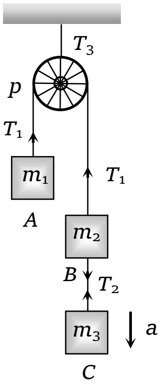
Solution : (a) Tension in the cord = $m(g + a)$ and breaking strength = 5 mg

For critical condition $m(g+a) = 5mg \Rightarrow a = 4g$

This is the maximum acceleration with which the sphere can move up with cord breaking.

4.18 Motion of Connected Block Over a Pulley

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1 a = T_1 - m_1 g$	$T_1 = \frac{2m_1 m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T_1$	$T_2 = \frac{4m_1 m_2}{m_1 + m_2} g$
		$T_2 = 2T_1$	$a = \left[\frac{m_2 - m_1}{m_1 + m_2}\right] g$

<table>
<tr><td rowspan="4">
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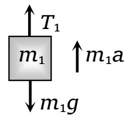
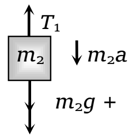
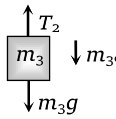
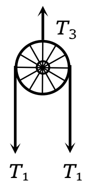
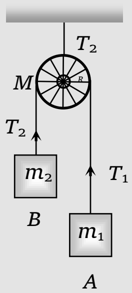
<td>
</td>
<td>$m_1a = T_1 - m_1g$</td>
<td>$T_1 = \frac{2m_1[m_2 + m_3]}{m_1 + m_2 + m_3}g$</td></tr>
<tr><td>
</td>
<td>$m_2a = m_2g + T_2 - T_1$</td>
<td>$T_2 = \frac{2m_1m_3}{m_1 + m_2 + m_3}g$</td></tr>
<tr><td>
</td>
<td>$m_3a = m_3g - T_2$</td>
<td>$T_3 = \frac{4m_1[m_2 + m_3]}{m_1 + m_2 + m_3}g$</td></tr>
<tr><td>
</td>
<td>$T_3 = 2T_1$</td>
<td>$a = \frac{[(m_2 + m_3) - m_1]g}{m_1 + m_2 + m_3}$</td></tr>
</table>

<table>
<tr><th>Condition</th><th>Free body diagram</th><th>Equation</th><th>Tension and acceleration</th></tr>
<tr><td rowspan="3">When pulley have a finite mass M and radius R then tension in two segments of string are different

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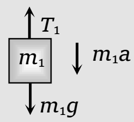
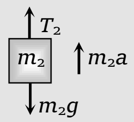
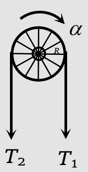
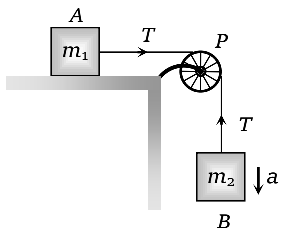
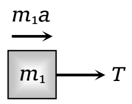
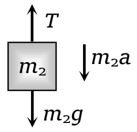
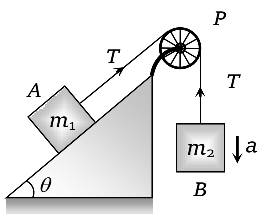
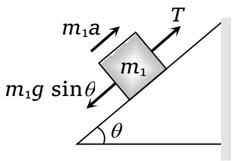
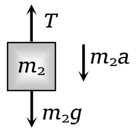
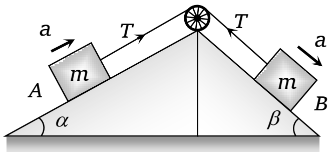
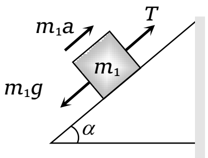
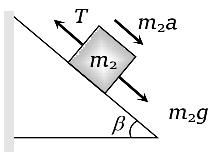
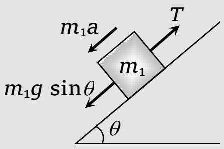
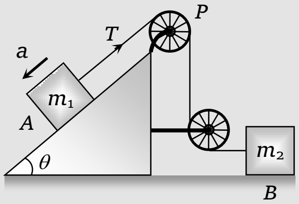
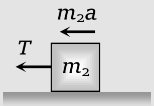
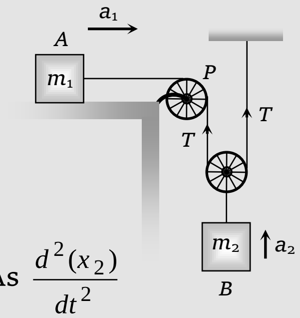
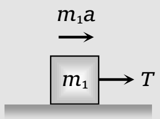
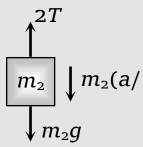
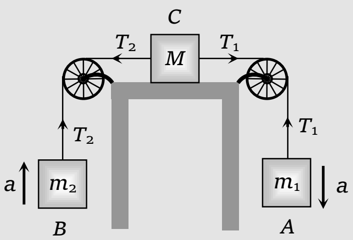
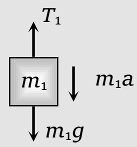
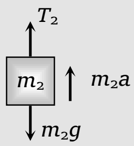
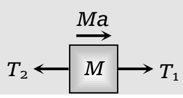
<td>
</td>
<td>$m_1a = m_1g - T_1$</td>
<td rowspan="3">$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{M}{2}}$
$T_1 = \frac{m_1\left[2m_2 + \frac{M}{2}\right]}{m_1 + m_2 + \frac{M}{2}}g$
$T_2 = \frac{m_2\left[2m_1 + \frac{M}{2}\right]}{m_1 + m_2 + \frac{M}{2}}g$</td></tr>
<tr><td>
</td>
<td>$m_2a = T_2 - m_2g$</td></tr>
<tr><td>
</td>
<td>Torque $= (T_1 - T_2)R = I\alpha$
$(T_1 - T_2)R = I\frac{a}{R}$
$(T_1 - T_2)R = \frac{1}{2}MR^2\frac{a}{R}$
$T_1 - T_2 = \frac{Ma}{2}$</td></tr>
</table>

		$T = m_1 a$	$a = \frac{m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2}{m_1 + m_2} g$
		$m_1 a = T - m_1 g \sin\theta$	$a = \left[\frac{m_2 - m_1 \sin\theta}{m_1 + m_2}\right] g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g$
		$T - m_1 g \sin\alpha = m_1 a$	$a = \frac{(m_2 \sin\beta - m_1 \sin\alpha)}{m_1 + m_2} g$
		$m_2 a = m_2 g \sin\beta - T$	$T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1 + m_2} g$

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1 g \sin\theta - T = m_1 a$	$a = \frac{m_1 g \sin\theta}{m_1 + m_2}$
			

		$T = m_2 a$	$T = \frac{2m_1 m_2}{4m_1 + m_2} g$
 As $\frac{d^2(x_2)}{dt^2} = \frac{1}{2}\frac{d^2(x_1)}{dt^2}$ $\therefore a_2 = \frac{a_1}{2}$ a_1 = acceleration of block *A* a_2 = acceleration of block *B*		$T = m_1 a$	$a_1 = a = \frac{2m_2 g}{4m_1 + m_2}$ $a_2 = \frac{m_2 g}{4m_1 + m_2}$ $T = \frac{2m_1 m_2 g}{4m_1 + m_2}$
		$m_2 \frac{a}{2} = m_2 g - 2T$	
		$m_1 a = m_1 g - T_1$	$a = \frac{(m_1 - m_2)}{[m_1 + m_2 + M]} g$
		$m_2 a = T_2 - m_2 g$	$T_1 = \frac{m_1(2m_2 + M)}{[m_1 + m_2 + M]} g$
		$T_1 - T_2 = Ma$	$T_2 = \frac{m_2(2m_2 + M)}{[m_1 + m_2 + M]} g$

Sample problems based on motion of blocks over pulley

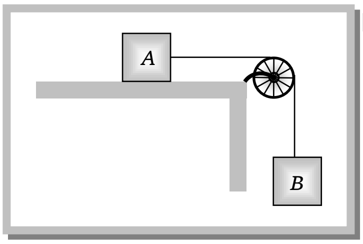
***Problem* 52.** A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$ then the ratio of the masses is

(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3

Solution : (b) $a=\left(\dfrac{m_2-m_1}{m_1+m_2}\right)g = \dfrac{g}{8}$; by solving $\dfrac{m_2}{m_1}=9/7$

Problem **53.** A block *A* of mass 7 *kg* is placed on a frictionless table. A thread tied to it passes over a frictionless pulley and carries a body *B* of mass 3 *kg* at the other end. The acceleration of the system is (given g = 10 ms^{-2})

[Kerala (Engg.) 2000]



(a) $100\,ms^{-2}$

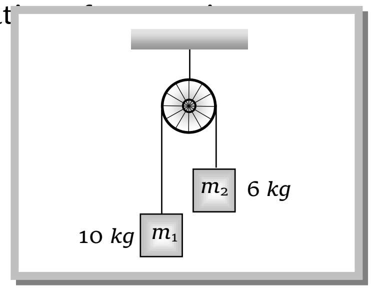
(b) $3ms^{-2}$

(c) $10ms^{-2}$

(d) $30ms^{-2}$

Solution : (b) $a=\left(\dfrac{m_2}{m_1+m_2}\right)g = \left(\dfrac{3}{7+3}\right)10 = 3m/s^2$

Problem **54.** Two masses m_1 and m_2 are attached to a string which passes over a frictionless smooth pulley. When $m_1=10\,kg,\ m_2=6\,kg$, the accelerat



(a) 20 m/s^2

(b) $5m/s^2$

(c) 2.5 m/s^2

(d) $10\,m/s^2$

Solution : (c) $a=\dfrac{m_1-m_2}{m_1+m_2}g = \left(\dfrac{10-6}{10+6}\right)10 = 2.5\,m/s^2$

Problem **55.** Two weights W_1 and W_2 are suspended from the ends of a light string passing over a smooth fixed pulley. If the pulley is pulled up with an acceleration g, the tension in the string will be

(a) $\dfrac{4W_1W_2}{W_1+W_2}$ (b) $\dfrac{2W_1W_2}{W_1+W_2}$ (c) $\dfrac{W_1W_2}{W_1+W_2}$ (d) $\dfrac{W_1W_2}{2(W_1+W_2)}$

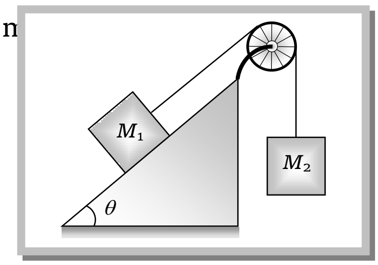
Solution : (a) When the system is at rest tension in string $T=\dfrac{2m_1m_2}{(m_1+m_2)}g$

If the system moves upward with acceleration g then $T=\dfrac{2m_1m_2}{m_1+m_2}(g+g) = \dfrac{4m_1m_2}{m_1+m_2}g$ or

$T=\dfrac{4w_1w_2}{w_1+w_2}$

Problem **56.** Two masses M_1 and M_2 are attached to the ends of a string which passes over a pulley attached to the top of an inclined plane. The angle of inclination of the plane in θ. Take g = 10 ms^{-2}.

If M_1 = 10 *kg*, M_2 = 5 *kg*, θ = 30°, what is the acceleration of n



(a) $10ms^{-2}$

(b) $5ms^{-2}$

(c) $\frac{2}{3} ms^{-2}$

(d) Zero

Solution : (d) Acceleration $= \frac{m_2 - m_1 \sin\theta}{m_1 + m_2} g = \frac{5 - 10 \,.\sin 30}{5 + 10} g = \frac{5-5}{5+10} g = 0$

Problem **57.** In the above problem, what is the tension in the string

(a) 100 N (b) 50 N (c) 25 N (d) Zero

Solution : (b) $T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g = \frac{10 \times 5(1 + \sin 30).10}{10 + 5} = 50\,N$

Problem **58.** In the above problem, given that $M_2 = 2M_1$ and M_2 moves vertically downwards with acceleration a. If the position of the masses are reversed the acceleration of M_2 down the inclined plane will be

(a) 2 a (b) a (c) $a/2$ (d) None of the above

Solution : (d) If $m_2 = 2m_1$, then m_2 moves vertically downward with acceleration

$$a = \frac{m_2 - m_1 \sin\theta}{m_1 + m_2} g = \frac{2m_1 - m_1 \sin 30}{m_1 + 2m_1} g = g/2$$

If the position of masses are reversed then m_2 moves downward with acceleration

$$a' = \frac{m_2 \sin\theta - m_1}{m_1 + m_2} g = \frac{2m_1 \sin 30 - m_1}{m_1 + 2m_1} . g = 0 \qquad [\text{As } m_2 = 2m_1]$$

i.e. the m_2 will not move.

Problem **59.** In the above problem, given that $M_2 = 2M_1$ and the tension in the string is T. If the positions of the masses are reversed, the tension in the string will be

(a) 4 T (b) 1 T (c) T (d) $T/2$

Solution : (c) Tension in the string $T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g$

If the position of the masses are reversed then there will be no effect on tension.

Problem **60.** In the above problem, given that $M_1 = M_2$ and $\theta = 30^o$. What will be the acceleration of the system

(a) $10 ms^{-2}$ (b) $5 ms^{-2}$ (c) $2.5 ms^{-2}$ (d) Zero

Solution : (c) $a = \frac{m_2 - m_1 \sin\theta}{m_1 + m_2} g = \frac{1 - \sin 30}{2} g = \frac{g}{4} = 2.5\, m/s^2$ [As $m_1 = m_2$]

Problem **61.** In the above problem, given that $M_1 = M_2 = 5\,kg$ and $\theta = 30^o$. What is tension in the string

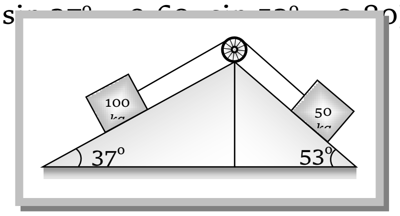
(a) 37.5 N (b) 25 N (c) 12.5 N (d) Zero

Solution : (a) $T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g = \frac{5 \times 5 (1 + \sin 30)}{5 + 5} \times 10 = 37.5\,N$

Problem **62.** Two blocks are attached to the two ends of a string passing over a smooth pulley as shown in the figure. The acceleration of the block will be (in m/s^2) (s[illegible]0)

(a) 0.33

(b) 0.133

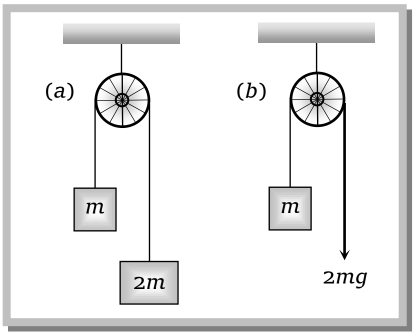


(c) 1

(d) 0.066

Solution : (b) $a = \frac{m_2 \sin\beta - m_1 \sin\alpha}{m_1 + m_2} g = \frac{50 \sin 53° - 100 \sin 37°}{100 + 50} g = -0.133 \, m/s^2$

Problem 63. The two pulley arrangements shown in the figure are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In (b). m is lifted up by pulling the other end of the rope with a constant downward force of $2mg$. The ratio of accelerations in two cases will be



(a) 1 : 1

(b) 1 : 2

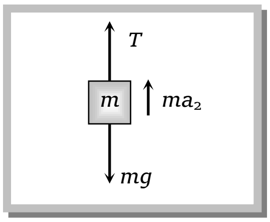
(c) 1 : 3

(d) 1 : 4

Solution : (c) For first case $a_1 = \frac{m_2 - m_1}{m_1 + m_2} g = \frac{2m - m}{m + 2m} = \frac{g}{3}$ .....(i)

For second case

from free body diagram of m



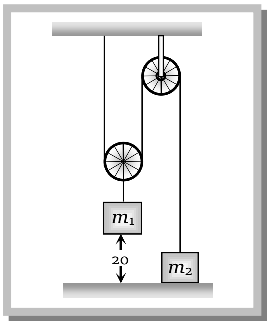
$ma_2 = T - mg$

$ma_2 = 2mg - mg$ [As $T = 2mg$]

$a_2 = g$ ....(ii)

From (i) and (ii) $\frac{a_1}{a_2} = \frac{g/3}{g} = 1/3$

Problem 64. In the adjoining figure $m_1 = 4m_2$. The pulleys are smooth and light. At time $t = 0$, the system is at rest. If the system is released and if the acceleration of mass m_1 is a, then the acceleration of m_2 will be



(a) g

(b) a

(c) $\frac{a}{2}$

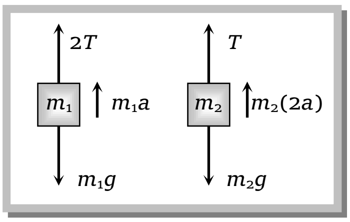
(d) $2a$

Solution : (d) Since the mass m_2 travels double distance in comparison to mass m_1 therefore its acceleration will be double *i.e.* $2a$

Problem 65. In the above problem (64), the value of a will be

(a) g (b) $\frac{g}{2}$ (c) $\frac{g}{4}$ (d) $\frac{g}{8}$

Solution : (c) By drawing the FBD of m_1 and m_2



$m_1 a = m_1 g - 2T$ .....(i)

$m_2(2a) = T - m_2 g$ .....(ii)

by solving these equation $a = g/4$

***Problem* 66.** In the above problem, the tension T in the string will be

(a) $m_2 g$ (b) $\frac{m_2 g}{2}$ (c) $\frac{2}{3} m_2 g$ (d) $\frac{3}{2} m_2 g$

Solution : (d) From the solution (65) by solving equation

$$T = \frac{3}{2} m_2 g$$

***Problem* 67.** In the above problem, the time taken by m_1 in coming to rest position will be

(a) 0.2 *s* (b) 0.4 *s* (c) 0.6 *s* (d) 0.8 *s*

Solution : (b) Time taken by mass m_2 to cover the distance 20 *cm*

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \times 0.2}{g/4}} = \sqrt{\frac{2 \times 0.2}{2.5}} = 0.4 \text{ sec}$$

***Problem* 68.** In the above problem, the distance covered by m_2 in 0.4 *s* will be

(a) 40 *cm* (b) 20 *cm* (c) 10 *cm* (d) 80 *cm*

Solution : (a) Since the m_2 mass cover double distance therefore $S = 2 \times 20 = 40$ *cm*

***Problem* 69.** In the above problem, the velocity acquired by m_2 in 0.4 second will be

(a) 100 *cm/s* (b) 200 *cm/s* (c) 300 *cm/s* (d) 400 *cm/s*

Solution : (b) Velocity acquired by mass m_2 in 0.4 sec

From $v = u + at$ [As $a = g/2 = \frac{10}{2} = 5\,m/s^2$]

$v = 0 + 5 \times 0.4 = 2m/s = 200\ cm/sec.$

***Problem* 70.** In the above problem, the additional distance traversed by m_2 in coming to rest position will be

(a) 20 *cm* (b) 40 *cm* (c) 60 *cm* (d) 80 *cm*

Solution : (a) When m_2 mass acquired velocity 200 cm/sec it will move upward till its velocity becomes zero.

$$H = \frac{u^2}{2g} = \frac{(200)^2}{2 \times 100} = 20\ cm$$

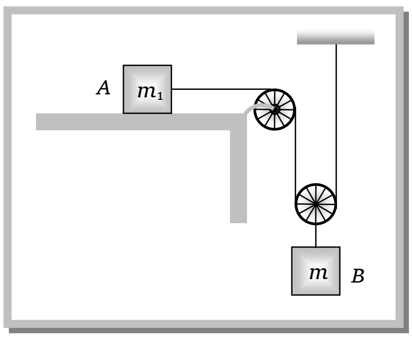
***Problem* 71.** The acceleration of block *B* in the figure will be

(a) $\frac{m_2 g}{(4m_1 + m_2)}$

(b) $\frac{2m_2 g}{(4m_1 + m_2)}$

(c) $\frac{2m_1 g}{(m_1 + 4m_2)}$

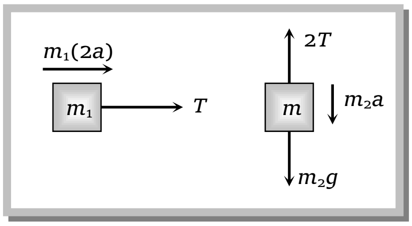
(d) $\frac{2m_1 g}{(m_1 + m_2)}$



Solution : (a) When the block m_2 moves downward with acceleration *a*, the acceleration of mass m_1 will be $2a$ because it covers double distance in the same time in comparison to m_2.

Let T is the tension in the string.

By drawing the free body diagram of A and B



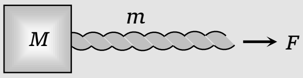
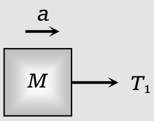
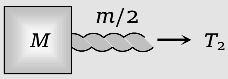
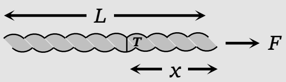
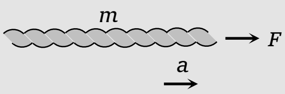
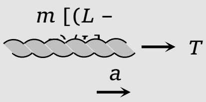
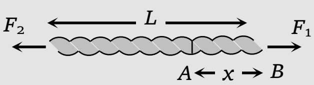
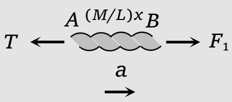
$$T = m_1 2a \qquad \text{........(i)}$$

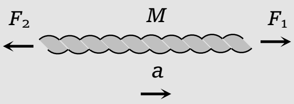
$$m_2g - 2T = m_2a \qquad \text{........(ii)}$$

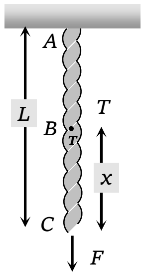
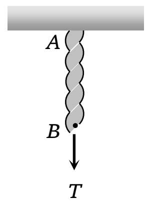
by solving (i) and (ii)

$$a = \frac{m_2g}{(4m_1 + m_2)}$$

4.19 Motion of Massive String

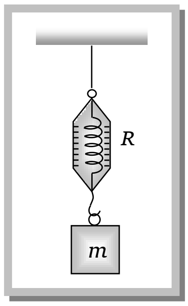
Condition	Free body diagram	Equation	Tension and acceleration
	 T_1 = force applied by the string on the block	$F = (M + m)a$ $T_1 = Ma$	$a = \frac{F}{M + m}$ $T_1 = M\frac{F}{(M + m)}$ $T_2 = \frac{(2M + m)}{2(M + m)}F$
	 T_2 = Tension at mid point of the rope	$T_2 = \left(M + \frac{m}{2}\right)a$	
 m = Mass of string T = Tension in string at a distance x from the end where the force is applied		$F = ma$	$a = F/m$ $T = \left(\frac{L - x}{L}\right)F$
		$T = m\left(\frac{L - x}{L}\right)a$	
		$F_1 - T = \frac{Mxa}{L}$	$a = \frac{F_1 - F_2}{M}$



M = Mass of uniform rod L = Length of rod		$F_1 - F_2 = Ma$	$T = F_1\left(1 - \frac{x}{L}\right) + F_2\left(\frac{x}{L}\right)$
 Mass of segment BC $= \left(\frac{M}{L}\right)x$		$T = \left(\frac{L-x}{L}\right)F$	$T = \left(\frac{L-x}{L}\right)F$

4.20 Spring Balance and Physical Balance

(1) **Spring balance :** When its upper end is fixed with rigid support and body of mass m hung from its lower end. Spring is stretched and the weight of the body can be measured by the reading of spring balance $R = W = mg$

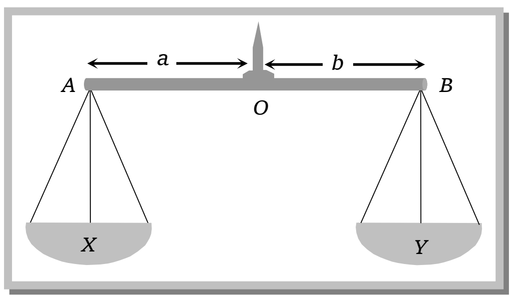


The mechanism of weighing machine is same as that of spring balance.

Effect of frame of reference : In inertial frame of reference the reading of spring balance shows the actual weight of the body but in non-inertial frame of reference reading of spring balance increases or decreases in accordance with the direction of acceleration

[for detail refer Article (4.13)]

(2) **Physical balance :** In physical balance actually we compare the mass of body in both the pans. Here we does not calculate the absolute weight of the body.



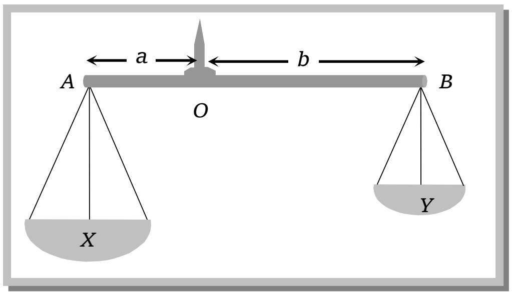
Here X and Y are the mass of the empty pan.

(i) Perfect physical balance :

Weight of the pan should be equal *i.e.* $X = Y$

and the needle must in middle of the beam *i.e.* $a = b$.

Effect of frame of reference : If the physical balance is perfect then there will be no effect of frame of reference (either inertial or non-inertial) on the measurement. It is always errorless.



(ii) False balance : When the masses of the pan are not equal then balance shows the error in measurement. False balance may be of two types

(a) If the beam of physical balance is horizontal (when the pans are empty) but the arms are not equal

$X > Y$ and $a < b$

For rotational equilibrium about point 'O'

$$Xa = Yb \quad ......(i)$$

In this physical balance if a body of weight W is placed in pan X then to balance it we have to put a weight W_1 in pan Y.

For rotational equilibrium about point 'O'

$$(X + W)a = (Y + W_1)b \quad .....(ii)$$

Now if the pans are changed then to balance the body we have to put a weight W_2 in pan X.

For rotational equilibrium about point 'O'

$$(X + W_2)a = (Y + W)b \quad ......(iii)$$

From (i), (ii) and (iii)

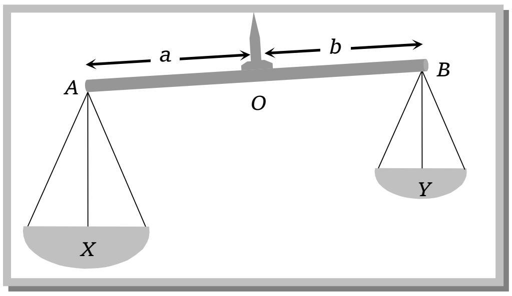
True weight $W = \sqrt{W_1 W_2}$

(b) If the beam of physical balance is not horizontal (when the pans are empty) and the arms are equal

i.e. $X > Y$ and $a = b$

In this physical balance if a body of weight W is placed in X Pan then to balance it.

We have to put a weight W_1 in Y Pan



For equilibrium $X + W = Y + W_1$ .....(i)

Now if pans are changed then to balance the body we have to put a weight W_2 in X Pan.

For equilibrium $X + W_2 = Y + W$ .....(ii)

From (i) and (ii)

True weight $W = \dfrac{W_1 + W_2}{2}$

4.21 Modification of Newton's Laws of Motion

According to Newton, direction and time *i.e.*, time and space are absolute. The velocity of observer has no effect on it. But, according to special theory of relativity Newton's laws are true, as long as we are dealing with velocities which are small compare to velocity of light. Hence the time and space measured by two observers in relative motion are not same. Some conclusions drawn by the special theory of relativity about mass, time and distance which are as follows :

(1) Let the length of a rod at rest with respect to an observer is L_0. If the rod moves with velocity v w.r.t. observer and its length is L, then $L = L_0\sqrt{1 - v^2/c^2}$

where, c is the velocity of light.

Now, as v increases L decreases, hence the length will appear shrinking.

(2) Let a clock reads T_0 for an observer at rest. If the clock moves with velocity v and clock reads T with respect to observer, then $T = \dfrac{T_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Hence, the clock in motion will appear slow.

(3) Let the mass of a body is m_0 at rest with respect to an observer. Now, the body moves with velocity v with respect to observer and its mass is m, then $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

m_0 is called the rest mass.

Hence, the mass increases with the increases of velocity.

Note :
- If $v << c$, *i.e.*, velocity of the body is very small *w.r.t.* velocity of light, then $m = m_0$. *i.e.*, in the practice there will be no change in the mass.
- If v is comparable to c, then $m > m_0$, *i.e.*, mass will increase.
- If $v = c$, then $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{v^2}}}$ or $m = \dfrac{m_0}{0} = \infty$. Hence, the mass becomes infinite, which is not possible, thus the speed cannot be equal to the velocity of light.
- The velocity of particles can be accelerated up to a certain limit. In cyclotron the speed of charged particles cannot be increased beyond a certain limit.

Sample problems (Miscellaneous)

***Problem* 72.** A body weighs 8 *gm*, when placed in one pan and 18*gm*, when placed in the other pan of a false balance. If the beam is horizontal (when both the pans are empty), the true weight of the body is

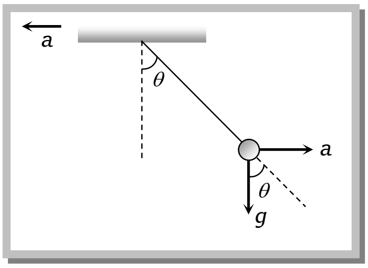
(a) 13 *gm* (b) 12 *gm* (c) 15.5 *gm* (d) 15 *gm*

Solution : (b) For given condition true weight = $\sqrt{W_1W_2} = \sqrt{8 \times 18}$ =12 *gm*.

Problem **73.** A plumb line is suspended from a ceiling of a car moving with horizontal acceleration of *a*. What will be the angle of inclination with vertical

(a) $\tan^{-1}(a/g)$ (b) $\tan^{-1}(g/a)$ (c) $\cos^{-1}(a/g)$ (d) $\cos^{-1}(g/a)$

Solution : (a) From the figure



$$\tan\theta = \frac{a}{g}$$

$$\theta = \tan^{-1}(a/g)$$

Problem **74.** A block of mass $5kg$ is moving horizontally at a speed of 1.5 *m/s*. A perpendicular force of 5 *N* acts on it for 4 *sec*. What will be the distance of the block from the point where the force started acting **[Pb PMT 2002]**

(a) 10 *m* (b) 8 *m* (c) 6 *m* (d) 2 *m*

Solution : (a) In the given problem force is working in a direction perpendicular to initial velocity. So the body will move under the effect of constant velocity in horizontal direction and under the effect of force in vertical direction.

$$S_x = u_x \times t = 1.5 \times 4 = 6m$$

$$S_y = u_y t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(F/m)t^2 = \frac{1}{2}(5/5)(4)^2 = 8m$$

$$\therefore\ S = \sqrt{S_x^2 + S_y^2} = \sqrt{36+64} = \sqrt{100} = 10\,m$$

Problem **75.** The velocity of a body of rest mass m_0 is $\frac{\sqrt{3}}{2}c$ (where c is the velocity of light in vacuum). Then mass of this body is

(a) $(\sqrt{3}/2)m_0$ (b) $(1/2)m_0$ (c) $2m_0$ (d) $(2/\sqrt{3})m_0$

Solution : (c) From Einstein's formula $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{\left(\sqrt{\frac{3}{2}}C\right)^2}{C^2}}} = \frac{m_0}{\sqrt{1-\frac{3}{4}}} = 2m_0$

Problem **76.** Three weights *W*, 2*W* and 3*W* are connected to identical springs suspended form rigid horizontal rod. The assembly of the rod and the weights fall freely. The positions of the weights from the rod are such that

[Roorkee 1999]

(a) 3 *W* will be farthest (b) *W* will be farthest

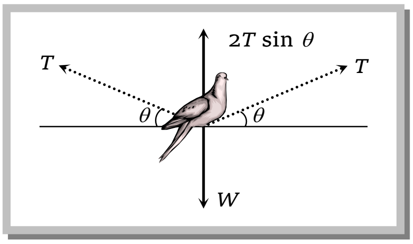
(c) All will be at the same distance (d) 2 *W* will be farthest

Solution : (c) For *W*, 2*W*, 3*W* apparent weight will be zero because the system is falling freely. So there will be no extension in any spring *i.e.* the distances of the weight from the rod will be same.

Problem **77.** A bird is sitting on stretched telephone wires. If its weight is *W* then the additional tension produced by it in the wires will be

(a) $T = W$ (b) $T > W$ (c) $T < W$ (d) $T = 0$

Solution : (b) For equilibrium



$$2T\sin\theta = W$$

$$T = \frac{W}{2\sin\theta}$$

θ lies between 0 to 90° *i.e.* $\sin\theta < 1$

$\therefore$ $T > W$

***Problem* 78.** With what minimum acceleration can a fireman slides down a rope while breaking strength of the rope is $\frac{2}{3}$ his weight

(a) $\frac{2}{3}g$ (b) g (c) $\frac{1}{3}g$ (d) Zero

Solution : (c) When fireman slides down, Tension in the rope $T = m(g-a)$

For critical condition $m(g - a) = 2/3\ mg$ $\Rightarrow mg - ma = \frac{2}{3}mg$ $\therefore$ $a = \frac{g}{3}$

So, this is the minimum acceleration by which a fireman can slides down on a rope.

***Problem* 79.** A car moving at a speed of 30 *kilometres* per hour's is brought to a halt in 8 *metres* by applying brakes. If the same car is moving at 60 *km*. per hour, it can be brought to a halt with same braking power in

(a) 8 *metres* (b) 16 *metres* (c) 24 *metres* (d) 32 *metres*

Solution : (d) From $v^2 = u^2 - 2as$

$$0 = u^2 - 2as$$

$s = \frac{u^2}{2a}$ $\Rightarrow$ $s \propto u^2$ (if a = constant)

$\frac{s_2}{s_1} = \left(\frac{u_2}{u_1}\right)^2 = \left(\frac{60}{30}\right)^2 = 4$ $\Rightarrow$ $s_2 = 4s_1 = 4\times 8 = 32$ *metres*.