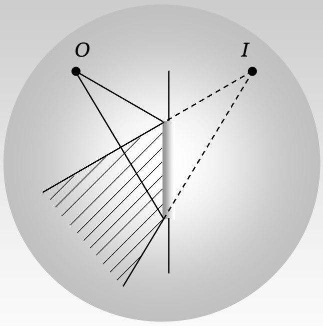
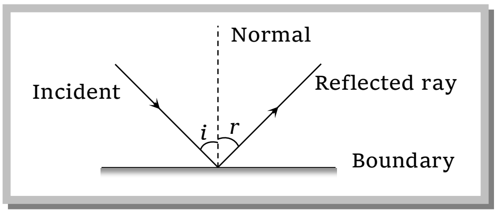


Reflection of Light

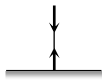
When a ray of light after incidenting on a boundary separating two media comes back into the same media, then this phenomenon, is called reflection of light.



- ⇒ $\angle i = \angle r$
- ⇒ After reflection, velocity, wave length and frequency of light remains same but intensity decreases
- ⇒ There is a phase change of π if reflection

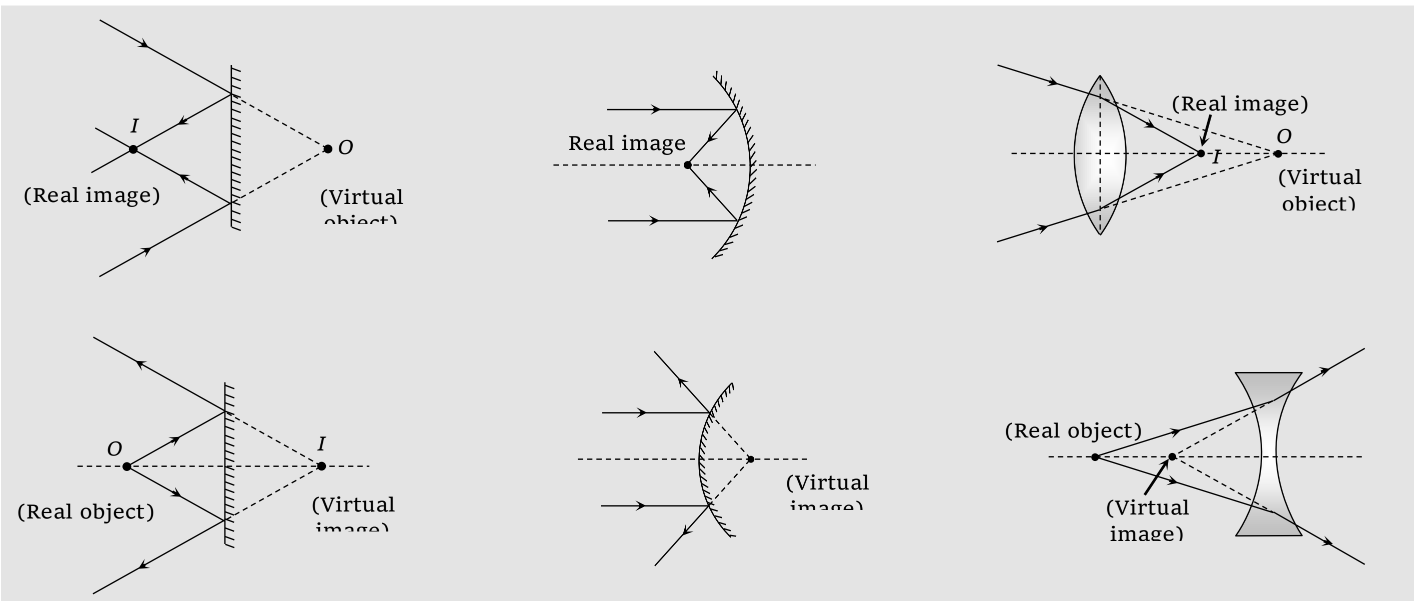
Note : ❑ After reflection velocity, wavelength and frequency of light remains same but intensity decreases.

❑ If light ray incident normally on a surface, after reflection it retraces the path.



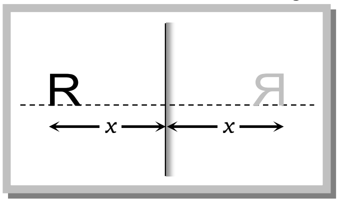
Real and virtual images

If light rays, after reflection or refraction, actually meets at a point then real image is formed and if they appears to meet virtual image is formed.

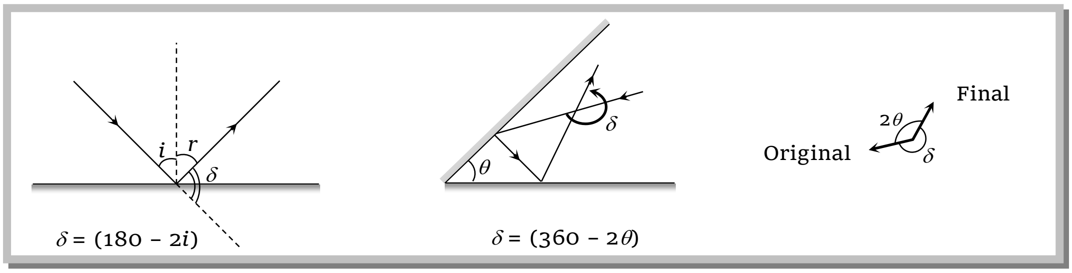


Plane Mirror

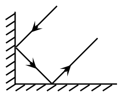
The image formed by a plane mirror is virtual, erect, laterally inverted, equal in size that of the object and at a distance equal to the distance of the object in front of the mirror.



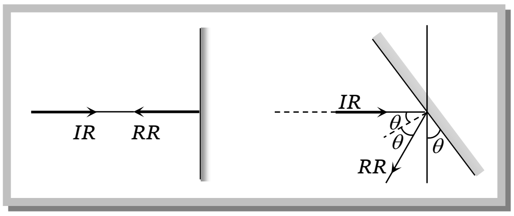
(1) **Deviation :** Deviation produced by a plane mirror and by two inclined plane mirrors.



Note : ❑ If two plane mirrors are inclined to each other at 90°, the emergent ray is anti-parallel to incident ray, if it suffers one reflection from each. Whatever be the angle to incidence.



(2) **Rotation :** If a plane mirror is rotated in the plane of incidence through angle θ, by keeping the incident ray fixed, the reflected ray turned through an angle 2θ.



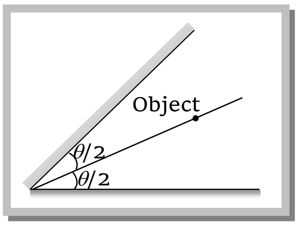
(3) **Images by two inclined plane mirrors :** When two plane mirrors are inclined to each other at an angle θ, then number of images (n) formed of an object which is kept between them.

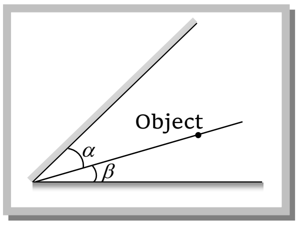
(i) $n = \left(\frac{360}{\theta} - 1\right)$; If $\frac{360}{\theta}$ = even integer

(ii) If $\frac{360}{\theta}$ = odd integer then there are two possibilities

(a) Object is placed symmetrically

(b) Object is placed asymmetrically





$$n = \left(\frac{360}{\theta} - 1\right)$$

$$n = \frac{360}{\theta}$$

Note : ❑ If $\theta = 0^\circ$ *i.e.* mirrors are parallel to each other so $n = \infty$ *i.e.* infinite images will be formed.

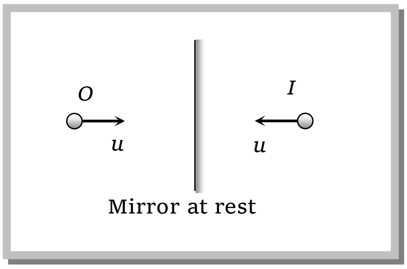
❑ If $\theta = 90^\circ$, $n = \frac{360}{90} - 1 = 3$

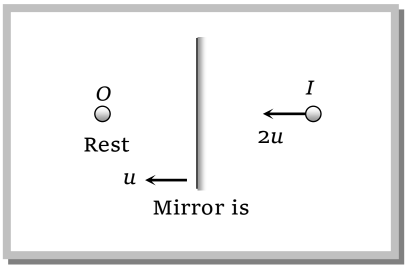
❑ If $\theta = 72^\circ$, $n = \frac{360}{72} - 1 = 4$ (If nothing is said object is supposed to be symmetrically placed).

(4) **Other important informations**

(i) When the object moves with speed u towards (or away) from the plane mirror then image also moves toward (or away) with speed u. But relative speed of image *w.r.t.* object is $2u$.

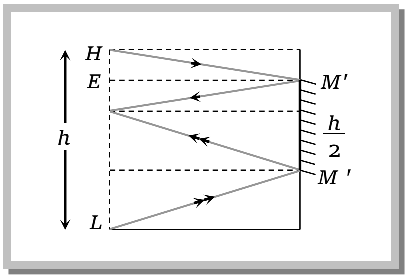
(ii) When mirror moves towards the stationary object with speed u, the image will move with speed $2u$.

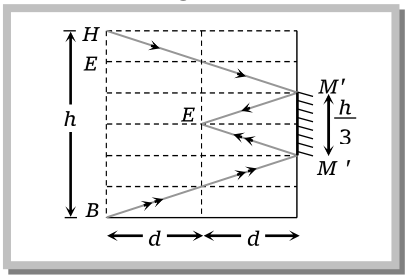




(iii) A man of height h requires a mirror of length at least equal to $h/2$, to see his own complete image.

(iv) To see complete wall behind himself a person requires a plane mirror of at least one third the height of wall. It should be noted that person is standing in the middle of the room.

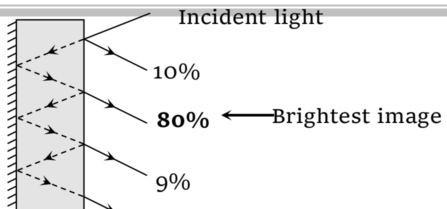




Concepts

☞ *The reflection from a denser medium causes an additional phase change of π or path change of $\lambda/2$ while reflection from rarer medium doesn't cause any phase change.*

☞ *We observe number of images in a thick plane mirror, out of them only second is brightest.*



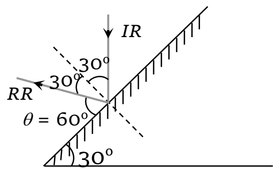
☞ *To find the location of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.*

Example

Example: 1 A plane mirror makes an angle of 30° with horizontal. If a vertical ray strikes the mirror, find the angle between mirror and reflected ray

(a) 30° (b) 45° (c) 60° (d) 90°

Solution : (c) Since angle between mirror and normal is 90° and reflected ray (RR) makes an angle of 30° with the normal so required angle will be $\theta = 60^o$.



Example: 2 Two vertical plane mirrors are inclined at an angle of 60° with each other. A ray of light travelling horizontally is reflected first from one mirror and then from the other. The resultant deviation is

(a) 60° (b) 120° (c) 180° (d) 240°

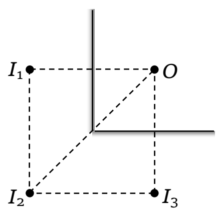
Solution : (d) By using $\delta = (360 - 2\theta)$ $\Rightarrow$ $\delta = 360 - 2 \times 60 = 240^o$

Example: 3 A person is in a room whose ceiling and two adjacent walls are mirrors. How many images are formed

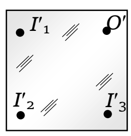
[AFMC 2002]

(a) 5 (b) 6 (c) 7 (d) 8

Solution : (c) The walls will act as two mirrors inclined to each other at 90° and so sill form $\frac{360}{90} - 1 = 3$ images of the person. Now these images with object (Person) will act as objects for the ceiling mirror and so ceiling will form 4 images as shown. Therefore total number of images formed = 3 + 4 = 7



Three images by



Four images by

Note : ❑ The person will see only six images of himself $(I_1, I_2, I_3, I_1^{'}, I_2^{'}, I_3^{'})$

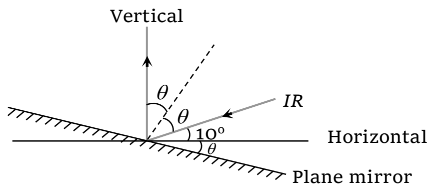
***Example*: 4** A ray of light makes an angle of 10° with the horizontal above it and strikes a plane mirror which is inclined at an angle θ to the horizontal. The angle θ for which the reflected ray becomes vertical is

(a) 40° (b) 50° (c) 80° (d) 100°

Solution : (a) From figure

$\theta + \theta + 10 = 90$

$\Rightarrow \theta = 40^o$



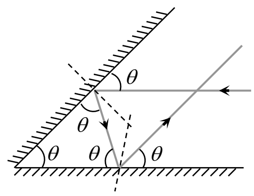
***Example*: 5** A ray of light incident on the first mirror parallel to the second and is reflected from the second mirror parallel to first mirror. The angle between two mirrors is

(a) 30° (b) 60° (c) 75° (d) 90°

Solution : (b) From geometry of figure

$\theta + \theta + \theta = 180^o$

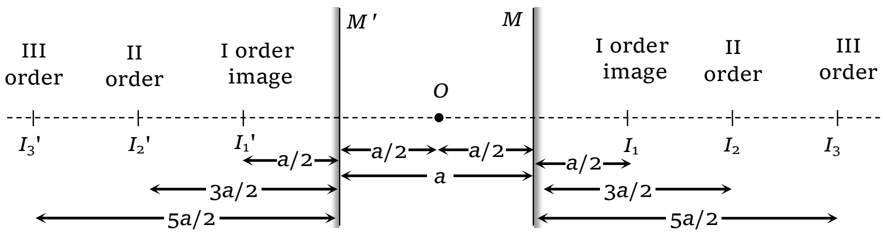
$\Rightarrow \theta = 60^o$



***Example*: 6** A point object is placed mid-way between two plane mirrors distance 'a' apart. The plane mirror forms an infinite number of images due to multiple reflection. The distance between the *n*th order image formed in the two mirrors is

(a) *na* (b) 2*na* (c) *na*/2 (d) n^2 *a*

Solution : (b)



From above figure it can be proved that seperation between *n*th order image formed in the two mirrors = 2*na*

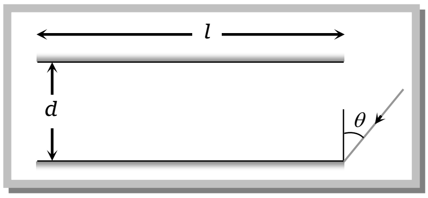
***Example*: 7** Two plane mirrors *P* and *Q* are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of θ at a point just inside one end of *A*. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is

(a) $\frac{l}{d\tan\theta}$

(b) $\frac{d}{l\tan\theta}$

(c) $ld\tan\theta$

(d) None of these

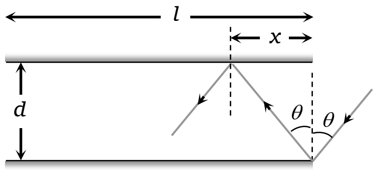


Solution : (a) Suppose n = Total number of reflection light ray undergoes before exist out.

x = Horizontal distance travelled by light ray in one reflection.

So $nx = l$ also $\tan\theta = \frac{x}{d}$

$\Rightarrow n = \frac{l}{d\tan\theta}$



***Example*: 8** A plane mirror and a person are moving towards each other with same velocity v. Then the velocity of the image is

(a) v (b) $2v$ (c) $3v$ (d) $4v$

Solution : (c) If mirror would be at rest, then velocity of image should be $2v$. but due to the motion of mirror, velocity of image will be $2v + v = 3v$.

***Example*: 9** A ray reflected successively from two plane mirrors inclined at a certain angle undergoes a deviation of 300°. The number of images observable are

(a) 10 (b) 11 (c) 12 (d) 13

Solution : (b) By using $\delta = (360 - 2\theta) \Rightarrow 300 = 360 - 2\theta$

$\Rightarrow \theta = 30^o$. Hence number of images $= \frac{360}{30} - 1 = 11$

Tricky example: 1

A small plane mirror placed at the centre of a spherical screen of radius R. A beam of light is falling on the mirror. If the mirror makes n revolution. per second, the speed of light on the screen after reflection from the mirror will be

(a) $4\pi nR$ (b) $2\pi nR$ (c) $\frac{nR}{2\pi}$ (d) $\frac{nR}{4\pi}$

Solution : (a) When plane mirror rotates through an angle θ, the reflected ray rotates through an angle 2θ. So spot on the screen will make $2n$ revolution per second

$\therefore$ Speed of light on screen $v = \omega R = 2\pi(2n)R = 4\pi nR$

Tricky example: 2

A watch shows time as 3 : 25 when seen through a mirror, time appeared will be

[RPMT 1997; JIPMER 2001, 2002]

(a) 8 : 35 (b) 9 : 35 (c) 7 : 35 (d) 8 : 25

Solution : (a) For solving this type of problems remember

Actual time = 11 : 60 – given time

So here Actual time = 11 : 60 – 3 : 25 = 8 : 35

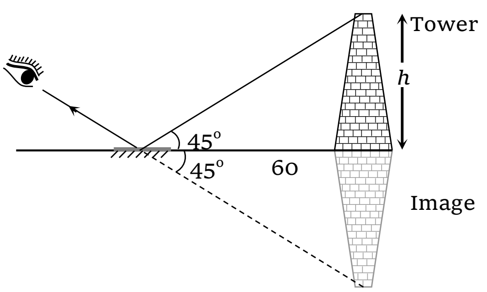
Tricky example: 3

When a plane mirror is placed horizontally on a level ground at a distance of 60 *m* from the foot of a tower, the top of the tower and its image in the mirror subtend an angle of 90^o at the eye. The height of the tower will be

(a) 30 *m* (b) 60 *m* (c) 90 *m* (d) 120 *m*

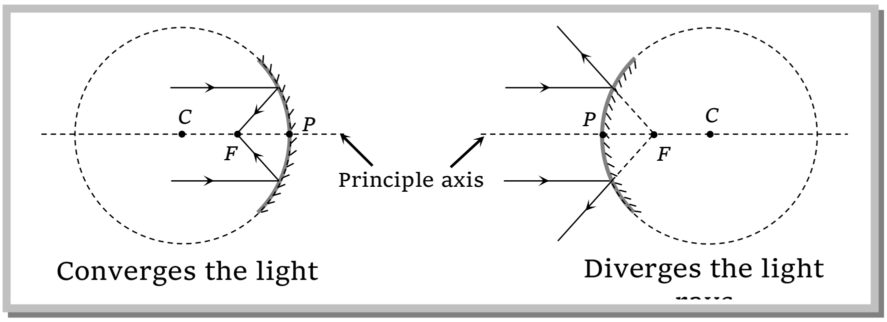
Solution : (b) Form the figure it is clear that $\frac{h}{60} = \tan 45^o$

$\Rightarrow h = 60\ m$



Curved Mirror

It is a part of a transparent hollow sphere whose one surface is polished.



(1) **Some definitions :**

(i) **Pole (P)** : Mid point of the mirror

(ii) Centre of curvature (C) : Centre of the sphere of which the mirror is a part.

(iii) Radius of curvature (R) : Distance between pole and centre of curvature.

($R_{concave} = -ve$, $R_{convex} = +ve$, $R_{plane} = \infty$)

(iv) Principle axis : A line passing through P and C.

(v) Focus (F) : An image point on principle axis for which object is at ∞

(vi) Focal length (f) : Distance between P and F.

(vii) Relation between f and R : $f = \frac{R}{2}$ ($f_{concare} = -ve$, $f_{convex} = +ve$, $f_{plane} = \infty$)

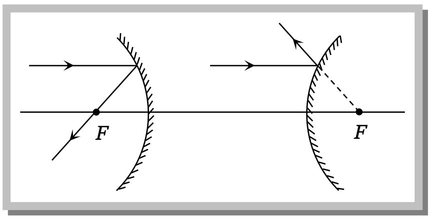
(viii) Power : The converging or diverging ability of mirror

(ix) Aperture : Effective diameter of light reflecting area. Intensity of image $\propto$ Area $\propto$ (Aperture)^2

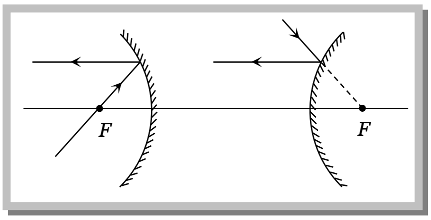
(x) Focal plane : A plane passing from focus and perpendicular to principle axis.

(2) **Rules of image formation and sign convention** :

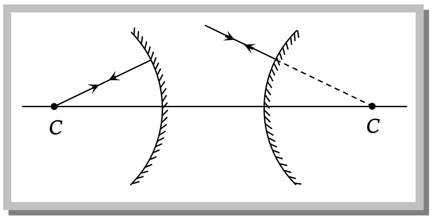
Rule (i)



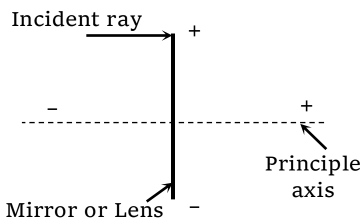
Rule (ii)



Rule (iii)



(3) **Sign conventions :**



(i) All distances are measured from the pole.

(ii) Distances measured in the direction of incident rays are taken as positive while in the direction opposite of incident rays are taken negative.

(iii) Distances above the principle axis are taken positive and below the principle axis are taken negative.

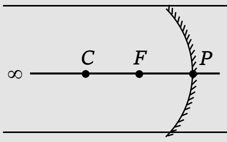
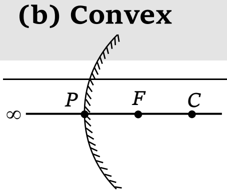
Note : ❑Same sign convention are also valid for lenses.

Use following sign while solving the problem :

Concave mirror		**Convex mirror**
Real image (u ≥ f)	**Virtual image (u< f)**	

Distance of object	$u \to -$	$u \to -$	$u \to -$
Distance of image	$v \to -$	$v \to +$	$v \to +$
Focal length	$f \to -$	$f \to -$	$f \to +$
Height of object	$O \to +$	$O \to +$	$O \to +$
Height of image	$I \to -$	$I \to +$	$I \to +$
Radius of curvature	$R \to -$	$R \to -$	$R \to +$
Magnification	$m \to -$	$m \to +$	$m \to +$

(4) **Position, size and nature of image formed by the spherical mirror**

Mirror	**Location of the object**	**Location of the image**	**Magnification, Size of the image**	**Nature**	
				Real / virtual	**Erect / inverted**
(a) Concave	At infinity $i.e.\ u = \infty$	At focus $i.e.\ v = f$	$m << 1$, diminished	Real	inverted
	Away from centre of curvature ($u > 2f$)	Between f and $2f$ $i.e.$ $f < v < 2f$	$m < 1$, diminished	Real	inverted
	At centre of curvature $u = 2f$	At centre of curvature $i.e.\ v = 2f$	$m = 1$, same size as that of the object	Real	inverted
	Between centre of curvature and focus : $F < u < 2f$	Away from the centre of curvature $v > 2f$	$m > 1$, magnified	Real	inverted
	At focus $i.e.\ u = f$	At infinity $i.e.\ v = \infty$	$m = \infty$, magnified	Real	inverted
	Between pole and focus $u < f$	$v > u$	$m > 1$ magnified	Virtual	erect
(b) Convex	At infinity $i.e.\ u = \infty$	At focus $i.e.$, $v = f$	$m < 1$, diminished	Virtual	erect
	Anywhere between infinity and pole	Between pole and focus	$m < 1$, diminished	Virtual	erect

Note :
- ❑ In case of convex mirrors, as the object moves away from the mirror, the image becomes smaller and moves closer to the focus.
- ❑ Images formed by mirrors do not show chromatic aberration.
- ❑ For convex mirror maximum image distance is it's focal length.

- ❑ In concave mirror, minimum distance between a real object and it's real image is zero.
  (*i.e.* when $u = v = 2f$)

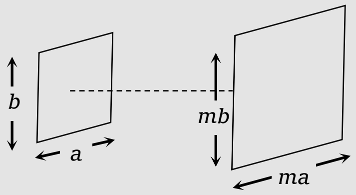
Mirror formula and magnification

For a spherical mirror if u = Distance of object from pole, v = distance of image from pole, f = Focal length, R = Radius of curvature, O = Size of object, I = size of image, m = magnification (or linear magnification), m_s = Areal magnification, A_o = Area of object, A_i = Area of image

(1) **Mirror formula :** $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$; (use sign convention while solving the problems).

Note : ❑ **Newton's formula :** If object distance (x_1) and image distance (x_2) are measured from focus instead of pole then $f^2 = x_1 x_2$

(2) **Magnification :** m = $\frac{\text{Size of object}}{\text{Size of image}}$

Linear magnification		**Areal magnification**
Transverse	**Longitudinal**	
When a object is placed perpendicular to the principle axis, then linear magnification is called lateral or transverse magnification. It is given by $m = \frac{I}{O} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$ (* Always use sign convention while solving the problems)	When object lies along the principle axis then its longitudinal magnification $m = \frac{I}{O} = \frac{-(v_2 - v_1)}{(u_2 - u_1)}$ If object is small; $m = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2$ Also Length of image = $\left(\frac{v}{u}\right)^2 \times \text{Length of object } (L_0)$ $(L_i) = \left(\frac{f}{u-f}\right)^2 . L_o$	 If a $2D$-object is placed with it's plane perpendicular to principle axis It's Areal magnification $M_s = \frac{\text{Area of image } (A_i)}{\text{Area of object } (A_o)} = \frac{ma \times mb}{ab} = m^2$ $\Rightarrow m_s = m^2 = \frac{A_i}{A_o}$

Note : ❑ Don't put the sign of quantity which is to be determined.

- ❑ If a spherical mirror produces an image 'm' times the size of the object (m = magnification) then u, v and f are given by the followings

$$u = \left(\frac{m-1}{m}\right) f, \quad v = -(m-1) f \quad \text{and} \quad f = \left(\frac{m}{m-1}\right) u$$ (use sign convention)

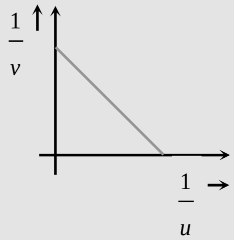
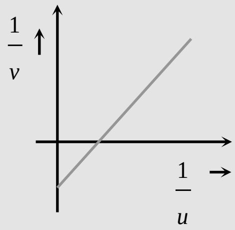
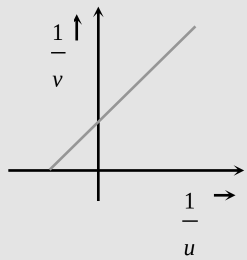
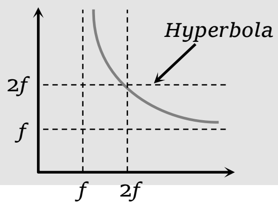
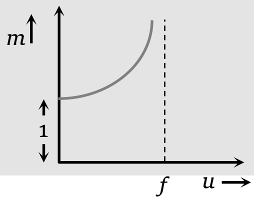
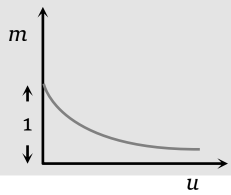
(3) **Uses of mirrors**

(i) **Concave mirror :** Used as a shaving mirror, In search light, in cinema projector, in telescope, by E.N.T. specialists etc.

(ii) **Convex mirror :** In road lamps, side mirror in vehicles *etc*.

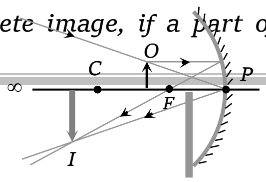
Note : ❑Field of view of convex mirror is more than that of concave mirror.

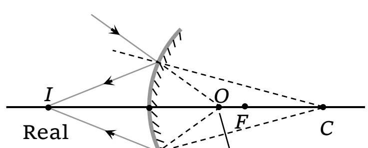
Different graphs

<table>
<tr><th colspan="3">Graph between $\frac{1}{v}$ and $\frac{1}{u}$</th></tr>
<tr><td>(a) Real image formed by concave mirror
</td><td>(b) Virtual image formed by concave mirror
</td><td>(c) Virtual image formed by convex mirror
</td></tr>
<tr><td>Graph between u and v for real image of concave mirror</td><td>Graph between u and m for virtual image by concave mirror</td><td>Graph between u and m for virtual image by convex mirror.</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Concepts

- *Focal length of a mirror is independent of material of mirror, medium in which it is placed, wavelength of incident light*
- *Divergence or Convergence power of a mirror does not change with the change in medium.*
- *If an object is moving at a speed v_o towards a spherical mirror along it's axis then speed of image away from mirror is* $v_i = -\left(\frac{f}{u-f}\right)^2 . v_o$ *(use sign convention)*
- *When object is moved from focus to infinity at constant speed, the image will move faster in the beginning and slower later on, towards the mirror.*
- *As every part of mirror forms a complete image, if a part of the mirror is obstructed, full image will be formed but intensity will be reduced.*





☞ *Can a convex mirror form real images?*

yes if (distance of virtual object) u < f (focal length)

Example

***Example*: 10** A convex mirror of focal length f forms an image which is $1/n$ times the object. The distance of the object from the mirror is

(a) $(n - 1) f$ (b) $\left(\frac{n-1}{n}\right) f$ (c) $\left(\frac{n+1}{n}\right) f$ (d) $(n + 1) f$

Solution : (a) By using $m = \frac{f}{f-u}$

Here $m = +\frac{1}{n}$, $f \to +f$ So, $+\frac{1}{n} = \frac{+f}{+f-u}$ $\Rightarrow$ $u = -(n-1)f$

***Example*: 11** An object 5 *cm* tall is placed 1 *m* from a concave spherical mirror which has a radius of curvature of 20 *cm*. The size of the image is

(a) 0.11 *cm* (b) 0.50 *cm* (c) 0.55 *cm* (d) 0.60 *cm*

Solution : (c) By using $\frac{I}{O} = \frac{f}{f-u}$

Here $O + 5\, cm$, $f = -\frac{R}{2} = -10\ cm$, $u = -1\, m = -100\ cm$

So, $\frac{I}{+5} = \frac{-10}{-10-(-100)}$ $\Rightarrow$ I = – 0.55 *cm*.

***Example*: 12** An object of length 2.5 *cm* is placed at a distance of 1.5 f from a concave mirror where f is the magnitude of the focal length of the mirror. The length of the object is perpendicular to the principle axis. The length of the image is

(a) 5 *cm,* erect (b) 10 *cm,* erect (c) 15 *cm,* erect (d) 5 *cm,* inverted

Solution : (d) By using $\frac{I}{O} = \frac{f}{f-u}$; where I = ? , O = + 2.5 *cm*. $f \to -f$, u = – 1.5 f

$\therefore$ $\frac{I}{+2.5} = \frac{-f}{-f-(-1.5f)}$ $\Rightarrow$ $I = -5\, cm$. (Negative sign indicates that image is inverted.)

***Example*: 13** A convex mirror has a focal length f. A real object is placed at a distance f in front of it from the pole produces an image at

(a) Infinity (b) f (c) $f/2$ (d) $2f$

Solution : (c) By using $\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \Rightarrow \frac{1}{+f}=\frac{1}{v}+\frac{1}{(-f)} \Rightarrow v=\frac{f}{2}$

***Example*: 14** Two objects A and B when placed one after another infront of a concave mirror of focal length 10 *cm* from images of same size. Size of object A is four times that of B. If object A is placed at a distance of 50 *cm* from the mirror, what should be the distance of B from the mirror

(a) 10 *cm* (b) 20 *cm* (c) 30 *cm* (d) 40 *cm*

Solution : (b) By using $\frac{I}{O}=\frac{f}{f-u} \Rightarrow \frac{I_A}{I_B}\times\frac{O_B}{O_A}=\frac{f-u_B}{f-u_A} \Rightarrow \frac{1}{1}\times\frac{1}{4}=\frac{-10-u_B}{-10-(-50)} \Rightarrow u_B=-20\,cm$.

***Example*: 15** A square of side 3 *cm* is placed at a distance of 25 *cm* from a concave mirror of focal length 10 *cm*. The centre of the square is at the axis of the mirror and the plane is normal to the axis. The area enclosed by the image of the wire is

(a) 4 cm^2 (b) 6 cm^2 (c) 16 cm^2 (d) 36 cm^2

Solution : (a) By using $m^2=\frac{A_i}{A_o}$; where $m=\frac{f}{f-u}$

Hence from given values $m=\frac{-10}{-10-(-25)}=\frac{-2}{3}$ and $A_o=9\,cm^2$ $\therefore A_i=\left(\frac{-2}{3}\right)^2\times 9=4\,cm^2$

***Example*: 16** A convex mirror of focal length 10 *cm* is placed in water. The refractive index of water is 4/3. What will be the focal length of the mirror in water

(a) 10 *cm* (b) 40/3 *cm* (c) 30/4 *cm* (d) None of these

Solution : (a) No change in focal length, because f depends only upon radius of curvature R.

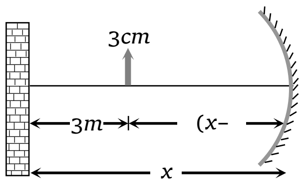
***Example*: 17** A candle flame 3 *cm* is placed at distance of 3 *m* from a wall. How far from wall must a concave mirror be placed in order that it may form an image of flame 9 *cm* high on the wall

(a) 225 *cm* (b) 300 *cm* (c) 450 *cm* (d) 650 *cm*

Solution : (c) Let the mirror be placed at a distance x from wall

By using

$\frac{I}{O}=\frac{-v}{u} \Rightarrow \frac{-9}{+3}=\frac{-(-x)}{-(x-3)} \Rightarrow x=-4.5m=-450\,cm.$



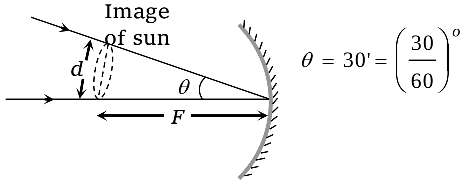
***Example*: 18** A concave mirror of focal length 100 *cm* is used to obtain the image of the sun which subtends an angle of 30'. The diameter of the image of the sun will be

(a) 1.74 *cm* (b) 0.87 *cm* (c) 0.435 *cm* (d) 100 *cm*

Solution : (b) Diameter of image of sun $d=f\theta$

$\Rightarrow d=100\times\left(\frac{30}{60}\right)\times\frac{\pi}{180}$

$\Rightarrow d=0.87\,cm$.

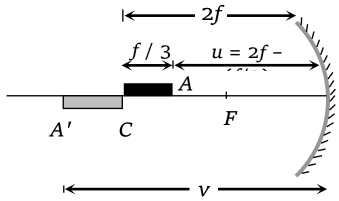


***Example*: 19** A thin rod of length $f/3$ lies along the axis of a concave mirror of focal length f. One end of its magnified image touches an end of the rod. The length of the image is

(a) f (b) $\frac{1}{2}f$ (c) $2f$ (d) $\frac{1}{4}f$

Solution : (b) If end A of rod acts an object for mirror then it's image will be A' and if $u = 2f - \frac{f}{3} = \frac{5f}{3}$

So by using $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $\Rightarrow \frac{1}{-f} = \frac{1}{v} + \frac{1}{-\frac{5f}{3}}$ $\Rightarrow v = -\frac{5}{2}f$



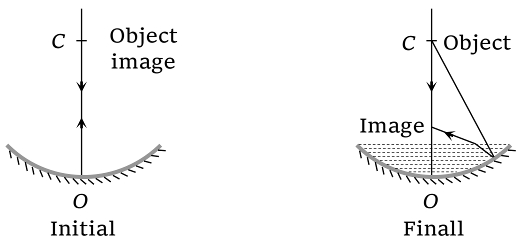
$\therefore$ Length of image $= \frac{5}{2}f - 2f = \frac{f}{2}$

***Example*: 20** A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be

(a) Real, and will remain at C (b) Real, and located at a point between C and ∞

(c) Virtual and located at a point between C and O (d) Real, and located at a point between C and O

Solution : (d)



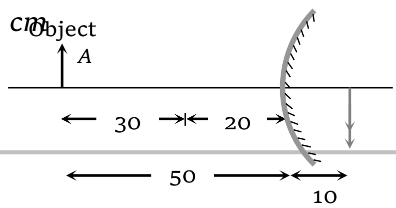
Tricky example: 4

An object is placed infront of a convex mirror at a distance of 50 *cm*. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and plane mirror is 30 *cm,* it is found that there is no parallel between the images formed by two mirrors. Radius of curvature of mirror will be

(a) 12.5 *cm* (b) 25 *cm* (c) $\frac{50}{3}$ *cm* (d) 18 *cm*

Solution : (b) Since there is no parallel, it means that both images (By plane mirror and convex mirror) coinciding each other.

According to property of plane mirror it will form image at a distance of 30 *cm* behind it. Hence for convex mirror $u = -50$ *cm,* $v = +10$ *cm*



By using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\Rightarrow \frac{1}{f} = \frac{1}{+10} + \frac{1}{-50} = \frac{4}{50}$

$\Rightarrow \quad f = \frac{25}{2} cm \qquad \Rightarrow R = 2f = 25\, cm.$

Tricky example: 5

A convergent beam of light is incident on a convex mirror so as to converge to a distance 12 *cm* from the pole of the mirror. An inverted image of the same size is formed coincident with the virtual object. What is the focal length of the mirror

(a) 24 *cm* (b) 12 *cm* (c) 6 *cm* (d) 3 *cm*

Solution : (c) Here object and image are at the same position so this position must be centre of curvature

$\therefore \quad R = 12\ cm \quad \Rightarrow f = \frac{R}{2}$