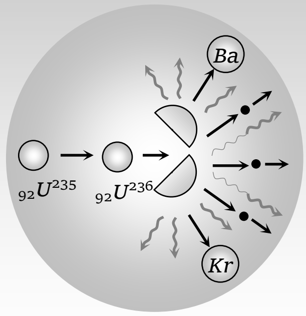
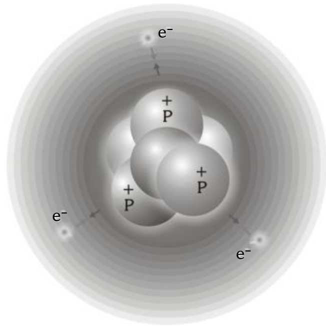


Nuclear Physics & Radioactivity

Rutherford's α-scattering experiment established that the mass of atom is concentrated with small positively charged region at the centre which is called 'nucleus'.



Nuclei are made up of proton and neutron. The number of protons in a nucleus (called the atomic number or proton number) is represented by the symbol Z. The number of neutrons (neutron number) is represented by N. The total number of neutrons and protons in a nucleus is called it's mass number A so $A = Z + N$.

Neutrons and proton, when described collectively are called ***nucleons.***

Nucleus contains two types of particles : Protons and neutrons

Nuclides are represented as $_ZX^A$; where X denotes the chemical symbol of the element.

Neutron

Neutron is a fundamental particle which is essential constituent of all nuclei except that of hydrogen atom. It was discovered by Chadwick.

(1) The charge of neutron : It is neutral

(2) The mass of neutron : 1.6750×10^{-27} kg

(3) It's spin angular momentum : $\frac{1}{2} \times \left(\frac{h}{2\pi}\right) J\text{-}s$

(4) It's magnetic moment : 9.57×10^{-27} $J/Tesla$

(5) It's half life : 12 minutes

(6) Penetration power : High

(7) Types : Neutrons are of two types slow neutron and fast neutron, both are fully capable of penetrating a nucleus and causing artificial disintegration.

> *A free neutron outside the nucleus is unstable and decays into proton and electron.*
>
> $$_0n^1 \rightarrow \underset{\text{Proton}}{{}_1H^1} + \underset{\text{Electron}}{{}_{-1}\beta^0} + \underset{\text{Antinutrino}}{\bar{\nu}}$$

Thermal neutrons

Fast neutrons can be converted into slow neutrons by certain materials called moderator's (Paraffin wax, heavy water, graphite) when fast moving neutrons pass through a moderator, they collide with the molecules of the moderator, as a result of this, the energy of moving neutron decreases while that of the molecules of the moderator increases. After sometime they both attains same energy. The neutrons are then in thermal equilibrium with the molecules of the moderator and are called thermal neutrons.

Note : ❑ Energy of thermal neutron is about 0.025 *eV* and speed is about 2.2 *km/s*.

Nucleus

(1) **Different types of nuclei**

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows

(i) **Isotopes :** The atoms of element having same atomic number but different mass number are called isotopes. All isotopes have the same chemical properties. The isotopes of some elements are the following

$$_1H^1,\ _1H^2,\ _1H^3 \quad _8O^{16},\ _8O^{17},\ _8O^{18} \quad _2He^3,\ _2He^4 \quad _{17}Cl^{35},\ _{17}Cl^{37} \quad _{92}U^{235},\ _{92}U^{238}$$

(ii) **Isobars :** The nuclei which have the same mass number (A) but different atomic number (Z) are called isobars. Isobars occupy different positions in periodic table so all isobars have different chemical properties. Some of the examples of isobars are

$$_1H^3 \text{ and } _2He^3,\quad _6C^{14} \text{ and } _7N^{14},\ _8O^{17} \text{ and } _9F^{17}$$

(iii) **Isotones :** The nuclei having equal number of neutrons are called isotones. For them both the atomic number (Z) and mass number (A) are different, but the value of (A – Z) is same. Some examples are

$$_4Be^9 \text{ and } _5B^{10},\ _6C^{13} \text{ and } _7N^{14},\ _8O^{18} \text{ and } _9F^{19},\ _3Li^7 \text{ and } _4Be^8,\ _1H^3 \text{ and } _2He^4$$

(iv) **Mirror nuclei :** Nuclei having the same mass number A but with the proton number (Z) and neutron number (A – Z) interchanged (or whose atomic number differ by 1) are called mirror nuclei for example.

$$_1H^3 \text{ and } _2He^3,\ _3Li^7 \text{ and } _4Be^7$$

(2) **Size of nucleus**

(i) Nuclear radius : Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where A is the mass number of nucleus *i.e.* $R \propto A^{1/3}$ $\Rightarrow R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}$ m = 1.2 *fm*.

Note : ❑ Heavier nuclei are bigger in size than lighter nuclei.

(ii) Nuclear volume : The volume of nucleus is given by $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$

(iii) Nuclear density : Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density } (\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi(R_0 A^{1/3})^3}$$

where m = Average of mass of a nucleon (= mass of proton + mass of neutron = 1.66×10^{-27} kg)

and mA = Mass of nucleus

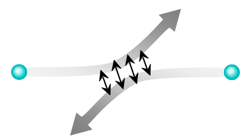
$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}\, kg/m^3$$

Note : ❑ ρ is independent of A, it means ρ is same of all atoms.

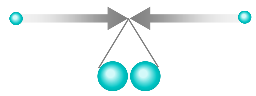
❑ Density of a nucleus is maximum at it's centre and decreases as we move outwards from the nucleus.

(3) **Nuclear force**

Forces that keep the nucleons bound in the nucleus are called nuclear forces.



At low speeds, electromagnetic repulsion prevents the collision of



At high speeds, nuclei come close enough for the strong force to bind them together.

(i) Nuclear forces are short range forces. These do not exist at large distances greater than 10^{-15} m.

(ii) Nuclear forces are the strongest forces in nature.

(iii) These are attractive force and causes stability of the nucleus.

(iv) These forces are charge independent.

(v) Nuclear forces are non-central force.

Nuclear forces are exchange forces

According to scientist Yukawa the nuclear force between the two nucleons is the result of the exchange of particles called mesons between the nucleons.

π - mesons are of three types – Positive π meson (π^+), negative π meson (π^-), neutral π *meson* (π^0)

The force between neutron and proton is due to exchange of charged meson between them *i.e.*

$$p \to \pi^+ + n, \quad n \to p + \pi^-$$

The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them *i.e.* $p \to p' + \pi^0$ and $n \to n' + \pi^0$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear forces.

Dog-Bone analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

(4) **Atomic mass unit (*amu*)**

The unit in which atomic and nuclear masses are measured is called atomic mass unit (*amu*)

1 *amu* (or 1*u*) $= \frac{1}{12}th$ of mass of ${}_6C^{12}$ atom $= 1.66 \times 10^{-27}$ *kg*

Masses of electron, proton and neutrons

Mass of electron (m_e) = 9.1×10^{-31} *kg* = 0.0005486 *amu*, Mass of proton (m_p) = 1.6726×10^{-27} *kg* = 1.007276 *amu*

Mass of neutron (m_n) = 1.6750×10^{-27} *kg* = 1.00865 *amu*, Mass of hydrogen atom ($m_e + m_p$) = 1.6729×10^{-27} *kg* = 1.0078 *amu*

Mass-energy equivalence

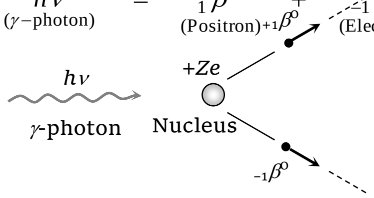
According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by $E = mc^2$

If m = 1 *amu*, $c = 3 \times 10^8$ *m/sec* then E = 931 *MeV* *i.e.* 1 *amu* is equivalent to 931 *MeV* or **1 *amu* (or 1 *u*) = 931 *MeV***

(5) **Pair production and pair-annihilation**

When an energetic γ-ray photon falls on a heavy substance. It is absorbed by some nucleus of the substance and an electron and a positron are produced. This phenomenon is called pair production and may be represented by the following equation

$$\underset{(\gamma-\text{photon})}{h\nu} = \underset{(\text{Positron})}{{}_{1}\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0}$$



The rest-mass energy of each of positron and electron is

$$E_0 = m_0c^2 = (9.1 \times 10^{-31}\ kg) \times (3.0 \times 10^8\ m/s)^2$$

$$= 8.2 \times 10^{-14}\ J = \mathbf{0.51\ MeV}$$

Hence, for pair-production it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02$ *MeV*. If the energy of γ-photon is less than this, it would cause photo-electric effect or Compton effect on striking the matter.

The converse phenomenon pair-annihilation is also possible. Whenever an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons (energy) are produced. This phenomenon is called pair annihilation and is represented by the following equation.

$$\underset{\text{(Positron)}}{{}_{+1}\beta^{0}} + \underset{\text{(Electron)}}{{}_{-1}\beta^{0}} = \underset{(\gamma\text{-photon})}{h\nu} + \underset{(\gamma\text{-photon})}{h\nu}$$

(6) **Nuclear stability**

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ - EM waves. (This process is called radioactivity). The stability of nucleus is determined by many factors. Few such factors are given below :

(i) Neutron-proton ratio $\left(\dfrac{N}{Z}\text{Ratio}\right)$

The chemical properties of an atom are governed entirely by the number of protons (Z) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal ($N \approx Z$) *i.e.* $\dfrac{N}{Z} = 1$

Heavy nuclei are stable only when they have more neutrons than protons. Thus heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

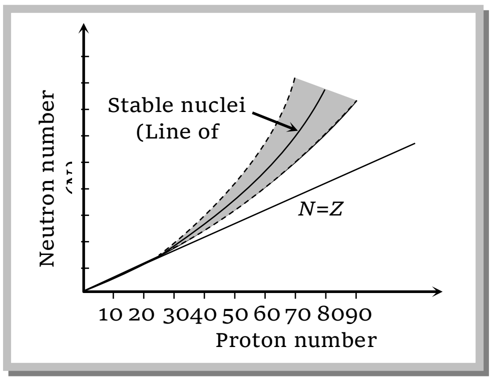
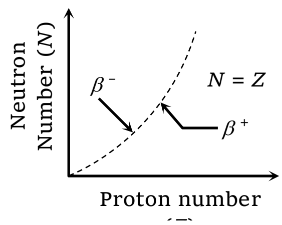


Figure shows a plot of N verses Z for the stable nuclei. For mass number upto about A = 40. For larger value of Z the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At *Bi* (Z = 83, A = 209), the neutron excess in $N - Z$ = 43. There are no stable nuclides with Z > 83.

Note : ❑ The nuclide ${}_{83}Bi^{209}$ is the heaviest stable nucleus.

❑ A nuclide above the line of stability *i.e.* having excess neutrons, decay through β^- emission (neutron changes into proton). Thus increasing atomic number Z and decreasing neutron number N. In β^- emission, $\frac{N}{Z}$ ratio decreases.



A nuclide below the line of stability have excess number of protons. It decays by β^+ emission, results in decreasing Z and increasing N. In β^+ emission, the $\frac{N}{Z}$ ratio increases.

(ii) Even or odd numbers of Z or N : The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

It is found that an even-even nucleus (even Z and even N) is more stable (60% of stable nuclide have even Z and even N).

An even-odd nucleus (even Z and odd N) or odd-even nuclide (odd Z and even N) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

Only five stable odd-odd nuclides are known : ${}_1H^2$, ${}_3Li^6$, ${}_5Be^{10}$, ${}_7N^{14}$ and ${}_{75}Ta^{180}$

(iii) Binding energy per nucleon : The stability of a nucleus is determined by value of it's binding energy per nucleon. In general higher the value of binding energy per nucleon, more stable the nucleus is

Mass Defect and Binding Energy

(1) **Mass defect (Δm)**

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state. This difference in masses is called mass defect. Hence mass defect

$$\Delta m = \text{Sum of masses of nucleons} - \text{Mass of nucleus}$$

$$= \{Zm_p + (A-Z)m_n\} - M = \{Zm_p + Zm_e + (A-Z)m_z\} - M'$$

where m_p = Mass of proton, m_n = Mass of each neutron, m_e = Mass of each electron

M = Mass of nucleus, Z = Atomic number, A = Mass number, M' = Mass of atom as a whole.

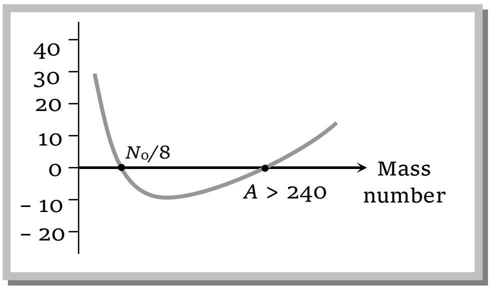
Note : ❑ The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

(2) **Packing fraction**

Mass defect per nucleon is called packing fraction

Packing fraction $(f) = \frac{\Delta m}{A} = \frac{M - A}{A}$ where M = Mass of nucleus, A = Mass number

Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, larger is the stability of the nucleus.



(i) Packing fraction may be of positive, negative or zero value.

(iii) At A = 16, $f \to$ Zero

(3) **Binding energy (B.E.)**

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart (or the same energy is released during the formation of the nucleus). This energy is called the binding energy of the nucleus.

or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect then according to Einstein's mass energy relation

Binding energy = $\Delta m \cdot c^2 = [\{m_p Z + m_n(A - Z)\} - M] \cdot c^2$

(This binding energy is expressed in *joule*, because Δm is measured in *kg*)

If Δm is measured in *amu* then binding energy = Δm *amu* = $[\{m_p Z + m_n(A - Z)\} - M]$ *amu* = $\Delta m \times 931$ *MeV*

(4) **Binding energy per nucleon**

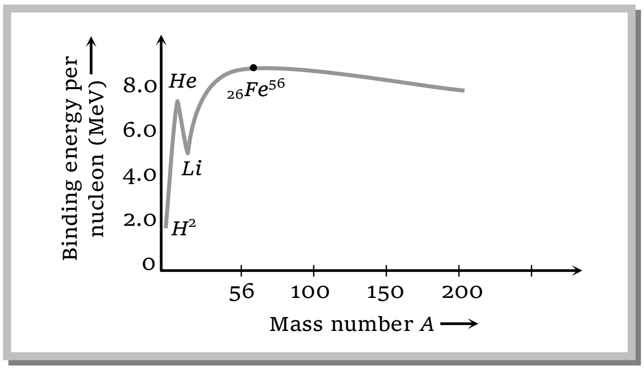
The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

Binding energy per nucleon $= \frac{\text{Total binding energy}}{\text{Mass number } (i.e. \text{ total number of nucleons})} = \frac{\Delta m \times 931}{A} \frac{MeV}{Nucleon}$

Binding energy per nucleon $\propto$ Stability of nucleus

Binding Energy Curve

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.* mass number *A*)

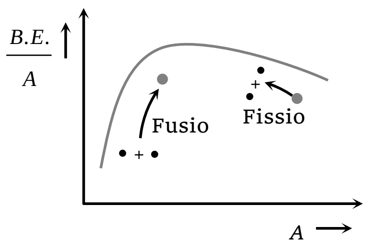


(1) Some nuclei with mass number A < 20 have large binding energy per nucleon than their neighbour nuclei. For example $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$. These nuclei are more stable than their neighbours.

(2) The binding energy per nucleon is maximum for nuclei of mass number A = 56 ($_{26}Fe^{56}$). It's value is 8.8 MeV per nucleon.

(3) For nuclei having A > 56, binding energy per nucleon gradually decreases for uranium (A = 238), the value of binding energy per nucleon drops to 7.5 MeV.

Note :
- ❑ When a heavy nucleus splits up into lighter nuclei, then binding energy per nucleon of lighter nuclei is more than that of the original heavy nucleus. Thus a large amount of energy is liberated in this process (nuclear fission).
- ❑ When two very light nuclei combines to form a relatively heavy nucleus, then binding energy per nucleon increases. Thus, energy is released in this process (nuclear fusion).



Nuclear Reactions

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. The general expression for the nuclear reaction is as follows.

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C} \longrightarrow \underset{\text{(Compound nucleus)}}{Y} + \underset{\text{(Product particles)}}{b} + \underset{\text{(Energy)}}{Q}$$

Here X and a are known as reactants and Y and b are known as products. This reaction is known as (a, b) reaction and can be represented as $X(a, b)\ Y$

(1) **Q value or energy of nuclear reaction**

The energy absorbed or released during nuclear reaction is known as Q-value of nuclear reaction.

Q-value = (Mass of reactants – mass of products)c^2 *Joules*

= (Mass of reactants – mass of products) *amu*

If $Q < 0$, The nuclear reaction is known as endothermic. (The energy is absorbed in the reaction)

If $Q > 0$, The nuclear reaction is known as exothermic (The energy is released in the reaction)

(2) **Law of conservation in nuclear reactions**

(i) Conservation of mass number and charge number : In the following nuclear reaction

$$_2He^4 + {}_7N^{14} \rightarrow {}_8O^{17} + {}_1H^1$$

Mass number (A) $\rightarrow$	Before the reaction	After the reaction
	4 +14 = 18	17 + 1 = 18
Charge number (Z) $\rightarrow$	2 + 7 = 9	8 + 1 = 9

(ii) Conservation of momentum : Linear momentum/angular momentum of particles before the reaction is equal to the linear/angular momentum of the particles after the reaction. That is $\Sigma p = 0$

(iii) Conservation of energy : Total energy before the reaction is equal to total energy after the reaction. Term Q is added to balance the total energy of the reaction.

(3) **Common nuclear reactions**

The nuclear reactions lead to artificial transmutation of nuclei. Rutherford was the first to carry out artificial transmutation of nitrogen to oxygen in the year 1919.

$$_2He^4 + {}_7N^{14} \rightarrow {}_9F^{18} \rightarrow {}_8O^{17} + {}_1H^1$$

It is called (α, p) reaction. Some other nuclear reactions are given as follows.

(p, n) reaction $\Rightarrow$ $\quad {}_1H^1 + {}_5B^{11} \rightarrow {}_6C^{12} \rightarrow {}_6C^{11} + {}_0n^1$

(p, α) reaction $\Rightarrow$ $\quad {}_1H^1 + {}_3Li^{11} \rightarrow {}_4Be^8 \rightarrow {}_2He^4 + {}_2He^4$

(p, γ) reaction $\Rightarrow$ $\quad {}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} \rightarrow {}_7N^{13} + \gamma$

(n, p) reaction $\Rightarrow$ $\quad {}_0n^1 + {}_7N^{14} \rightarrow {}_7N^{15} \rightarrow {}_6C^{14} + {}_1H^1$

(γ, n) reaction $\Rightarrow$ $\quad \gamma + {}_1H^2 \rightarrow {}_1H^1 + {}_0n^1$

Nuclear Fission and Fusion

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

The phenomenon of nuclear fission was discovered by scientist Ottohann and F. Strassman and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(1) **Fission reaction of U^{235}**

(i) Nuclear reaction :

$${}_{92}U^{235} + {}_0n^1 \rightarrow \underset{\text{(unstable nucleus)}}{{}_{92}U^{236}} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + Q$$

(ii) The energy released in U^{235} fission is about 200 *MeV* or 0.8 *MeV* per nucleon.

(iii) By fission of ${}_{92}U^{235}$, on an average 2.5 neutrons are liberated. These neutrons are called fast neutrons and their energy is about 2 *MeV* (for each). These fast neutrons can escape from the reaction so as to proceed the chain reaction they are need to slow down.

(iv) Fission of U^{235} occurs by slow neutrons only (of energy about 1*eV*) or even by thermal neutrons (of energy about 0.025 *eV*).

(v) 50 *kg* of U^{235} on fission will release $\approx 4 \times 10^{15}$ *J* of energy. This is equivalence to 20,000 tones of *TNT* explosion. The nuclear bomb dropped at Hiroshima had this much explosion power.

(vi) The mass of the compound nucleus must be greater than the sum of masses of fission products.

(vii) The $\dfrac{\text{Binding energy}}{\text{A}}$ of compound nucleus must be less than that of the fission products.

(viii) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments ${}_{56}Ba$ and ${}_{36}Kr$ are formed but they may be any stable isotopes of middle weight atoms.

Same other U^{235} fission reactions are

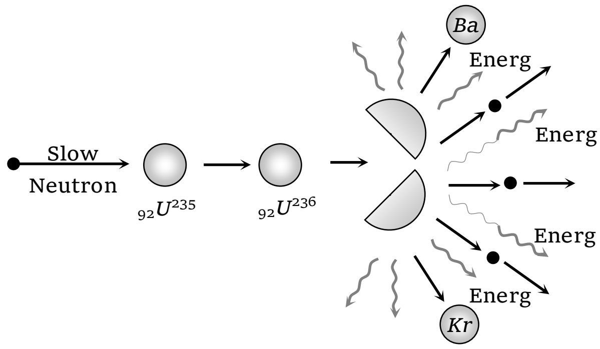
$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2{}_0n^1$$

$$\rightarrow {}_{57}La^{148} + {}_{35}Br^{85} + 3\ {}_{0}n^{1}$$

$\rightarrow$ Many more

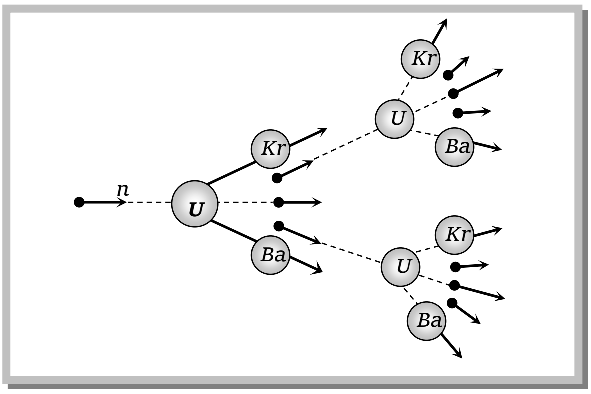
(ix) The neutrons released during the fission process are called prompt neutrons.

(x) Most of energy released appears in the form of kinetic energy of fission fragments.



(2) **Chain reaction**

In nuclear fission, three neutrons are produced along with the release of large energy. Under favourable conditions, these neutrons can cause further fission of other nuclei, producing large number of neutrons. Thus a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.



In the chain reaction, the number of nuclei undergoing fission increases very fast. So, the energy produced takes a tremendous magnitude very soon.

Difficulties in chain reaction

(i) Absorption of neutrons by U^{238}, the major part in natural uranium is the isotope U^{238} (99.3%), the isotope U^{235} is very little (0.7%). It is found that U^{238} is fissionable with fast neutrons, whereas U^{235} is fissionable with slow neutrons. Due to the large percentage of U^{238}, there is more possibility of collision of neutrons with U^{238}. It is found that the neutrons get slowed on coliding with U^{238}, as a result of it further fission of U^{238} is not possible (Because they are slow and they are absorbed by U^{238}). This stops the chain reaction.

Removal : (i) To sustain chain reaction ${}_{92}U^{235}$ is separated from the ordinary uranium. Uranium so obtained $\left({}_{92}U^{235}\right)$ is known as enriched uranium, which is fissionable with the fast and slow neutrons and hence chain reaction can be sustained.

(ii) If neutrons are slowed down by any method to an energy of about 0.3 eV, then the probability of their absorption by U^{238} becomes very low, while the probability of their fissioning U^{235} becomes high. This job is done by moderators. Which reduce the speed of neutron rapidly graphite and heavy water are the example of moderators.

(iii) Critical size : The neutrons emitted during fission are very fast and they travel a large distance before being slowed down. If the size of the fissionable material is small, the neutrons emitted will escape the fissionable material before they are slowed down. Hence chain reaction cannot be sustained.

Removal : The size of the fissionable material should be large than a critical size.

The chain reaction once started will remain steady, accelerate or retard depending upon, a factor called neutron reproduction factor (k). It is defined as follows.

$$k = \frac{\text{Rate of product ion of neutrons}}{\text{Rate of loss of neutrons}}$$

$\rightarrow$ If k = 1, the chain reaction will be steady. The size of the fissionable material used is said to be the critical size and it's mass, the critical mass.

$\rightarrow$ If k > 1, the chain reaction accelerates, resulting in an explosion. The size of the material in this case is super critical. (Atom bomb)

$\rightarrow$ If k < 1, the chain reaction gradually comes to a halt. The size of the material used us said to be sub-critical.

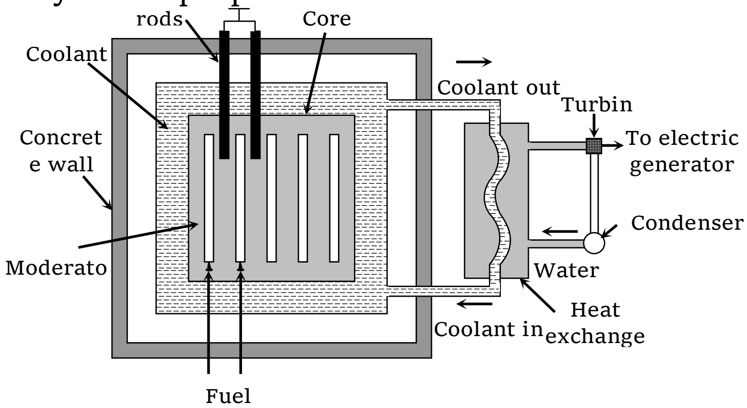
Types of chain reaction : Chain reactions are of following two types

Controlled chain reaction	**Uncontrolled chain reaction**
Controlled by artificial method	No control over this type of nuclear reaction
All neurons are absorbed except one	More than one neutron takes part into reaction
It's rate is slow	Fast rate
Reproduction factor k = 1	Reproduction factor k > 1
Energy liberated in this type of reaction is always less than explosive energy	A large amount of energy is liberated in this type of reaction
Chain reaction is the principle of nuclear reactors	Uncontrolled chain reaction is the principle of atom bomb.

Note : ❑ The energy released in the explosion of an atom bomb is equal to the energy released by 2000 tonn of TNT and the temperature at the place of explosion is of the order of 10^7 ^oC.

Nuclear Reactor

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.



(1) **Parts of nuclear reactor**

(i) **Fissionable material (Fuel) :** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}) Thorium isotope (Th^{232}) and Plutonium isotopes (Pu^{239}, Pu^{240} and Pu^{241}) are the most commonly used fuels in the reactor.

(ii) **Moderator :** Moderator is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O).

(iii) **Control Material :** Control material is used to control the chain reaction and to maintain a stable rate of reaction. This material controls the number of neutrons available for the fission. For example, cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.

(iv) **Coolant :** Coolant is a cooling material which removes the heat generated due to fission in the reactor. Commonly used coolants are water, CO_2 nitrogen *etc*.

(v) **Protective shield :** A protective shield in the form a concrete thick wall surrounds the core of the reactor to save the persons working around the reactor from the hazardous radiations.

Note : ❑ It may be noted that Plutonium is the best fuel as compared to other fissionable material. It is because fission in Plutonium can be initiated by both slow and fast neutrons. Moreover it can be obtained from U^{238}.

❑ Nuclear reactor is firstly devised by fermi.

❑ Apsara was the first Indian nuclear reactor.

(2) **Uses of nuclear reactor**

(i) In electric power generation.

(ii) To produce radioactive isotopes for their use in medical science, agriculture and industry.

(iii) In manufacturing of PU^{239} which is used in atom bomb.

(iv) They are used to produce neutron beam of high intensity which is used in the treatment of cancer and nuclear research.

Note : ❑ A type of reactor that can produce more fissile fuel than it consumes is the breeder reactor.

Nuclear fusion

In nuclear fusion two or more than two lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy

$$_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4\,MeV$$

$$_1H^3 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1 + 17.6\,MeV$$

or

$$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + 24\,MeV$$

For fusion high pressure ($\approx 10^6$ *atm*) and high temperature (of the order of 10^7 *K* to 10^8 *K*) is required and so the reaction is called thermonuclear reaction.

Fusion energy is greater then fission energy fission of one uranium atom releases about 200 *MeV* of energy. But the fusion of a deutron $({}_1H^2)$ and triton $({}_1H^3)$ releases about 17.6 *MeV* of energy. However the energy released per nucleon in fission is about 0.85 *MeV* but that in fusion is 4.4 *MeV*. So for the same mass of the fuel, the energy released in fusion is much larger than in fission.

Plasma : The temperature of the order of 10^8 *K* required for thermonuclear reactions leads to the complete ionisation of the atom of light elements. The combination of base nuclei and electron cloud is called plasma. The enormous gravitational field of the sun confines the plasma in the interior of the sun.

The main problem to carryout nuclear fusion in the laboratory is to contain the plasma at a temperature of $10^8 K$. No solid container can tolerate this much temperature. If this problem of containing plasma is solved, then the large quantity of deuterium present in sea water would be able to serve as in-exhaustible source of energy.

Note : ❑ To achieve fusion in laboratory a device is used to confine the plasma, called **Tokamak**.

Stellar Energy

Stellar energy is the energy obtained continuously from the sun and the stars. Sun radiates energy at the rate of about 10^{26} *joules* per *second*.

Scientist Hans Bethe suggested that the fusion of hydrogen to form helium (thermo nuclear reaction) is continuously taking place in the sun (or in the other stars) and it is the source of sun's (star's) energy.

The stellar energy is explained by two cycles

Proton-proton cycle	Carbon-nitrogen cycle
${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + Q_1$	${}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} + Q_1$
${}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + Q_2$	${}_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0$
${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2{}_1H^1 + Q_3$	${}_1H^1 + {}_6C^{13} \rightarrow {}_7N^{14} + Q_2$
$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\gamma + 26.7\,MeV$	${}_1H^1 + {}_7N^{14} \rightarrow {}_8O^{15} + Q_3$
	${}_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + Q_4$
	${}_1H^1 + {}_7N^{15} \rightarrow {}_6C^{12} + {}_2He^4$
	$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_1e^0 + 24.7\,MeV$

About 90% of the mass of the sun consists of hydrogen and helium.

Nuclear Bomb

Based on uncontrolled nuclear reactions.

Atom bomb	Hydrogen bomb
Based on fission process it involves the fission of U^{235}	Based on fusion process. Mixture of deutron and tritium is used in it
In this critical size is important	There is no limit to critical size
Explosion is possible at normal temperature and pressure	High temperature and pressure are required
Less energy is released compared to hydrogen bomb	More energy is released as compared to atom bomb so it is more dangerous than atom bomb

Concepts

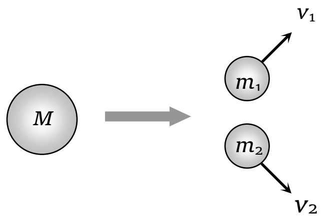
- *A test tube full of base nuclei will weight heavier than the earth.*
- *The nucleus of hydrogen contains only one proton. Therefore we may say that the proton is the nucleus of hydrogen atom.*
- *If the relative abundance of isotopes in an element has a ratio $n_1 : n_2$ whose atomic masses are m_1 and m_2 then atomic mass of the element is* $M = \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2}$

Example

***Example*: 1** A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio 8 : 1. The ratio of radii of the fragments is

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 2 : 1

Solution : (a)



By conservation of momentum $m_1v_1 = m_2v_2$

$\Rightarrow \frac{v_1}{v_2} = \frac{8}{1} = \frac{m_2}{m_1}$ ...... (i)

Also from $r \propto A^{1/3}$ $\Rightarrow \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$

***Example*: 2** The ratio of radii of nuclei $^{27}_{13}Al$ and $^{125}_{52}Te$ is approximately

(a) 6 : 10 (b) 13 : 52 (c) 40 : 177 (d) 14 : 7

Solution : (a) By using $r \propto A^{1/3}$ $\Rightarrow \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{8}{5} = \frac{6}{10}$

***Example*: 3** If Avogadro's number is 6×10^{23} then the number of protons, neutrons and electrons in 14 *g* of $_6C^{14}$ are respectively

(a) 36×10^{23}, 48×10^{23}, 36×10^{23} (b) 36×10^{23}, 36×10^{23}, 36×10^{21}

(c) 48×10^{23}, 36×10^{23}, 48×10^{21} (d) 48×10^{23}, 48×10^{23}, 36×10^{21}

Solution : (a) Since the number of protons, neutrons and electrons in an atom of $_6C^{14}$ are 6, 8 and 6 respectively. As 14 *gm* of $_6C^{14}$ contains 6×10^{23} atoms, therefore the numbers of protons, neutrons and electrons in 14 *gm* of $_6C^{14}$ are $6 \times 6 \times 10^{23} = 36 \times 10^{23}$, $8 \times 6 \times 10^{23} = 48 \times 10^{23}$, $6 \times 6 \times 10^{23} = 36 \times 10^{23}$.

***Example*: 4** Two Cu^{64} nuclei touch each other. The electrostatics repulsive energy of the system will be

(a) 0.788 *MeV* (b) 7.88 *MeV* (c) 126.15 *MeV* (d) 788 *MeV*

Solution : (c) Radius of each nucleus $R = R_0(A)^{1/3} = 1.2(64)^{1/3} = 4.8\ fm$

Distance between two nuclei (r) = $2R$

So potential energy $U = \frac{k \cdot q^2}{r} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19} \times 29)^2}{2 \times 4.8 \times 10^{-15} \times 1.6 \times 10^{-19}} = 126.15\ MeV.$

***Example*: 5** When $_{92}U^{235}$ undergoes fission. 0.1% of its original mass is changed into energy. How much energy is released if 1 *kg* of $_{92}U^{235}$ undergoes fission**[MP PET 1994; MP PMT/PET 1998; BHU 2001; BVP 20**

(a) $9 \times 10^{10}\ J$ (b) $9 \times 10^{11}\ J$ (c) $9 \times 10^{12}\ J$ (d) $9 \times 10^{13}\ J$

Solution : (d) By using $E = \Delta m \cdot c^2 \Rightarrow E = \left(\frac{0.1}{100} \times 1\right)(3 \times 10^8)^2 = 9 \times 10^{13}\ J$

***Example*: 6** 1 *g* of hydrogen is converted into 0.993 *g* of helium in a thermonuclear reaction. The energy released is

[EAMCET (Med.) 1995; CPMT 1999]

(a) $63 \times 10^7\ J$ (b) $63 \times 10^{10}\ J$ (c) $63 \times 10^{14}\ J$ (d) $63 \times 10^{20}\ J$

Solution : (b) $\Delta m = 1 - 0.993 = 0.007\ gm$

$\therefore E = \Delta mc^2 = 0.007 \times 10^{-3} \times (3 \times 10^8)^2 = 63 \times 10^{10}\ J$

***Example*: 7** The binding energy per nucleon of deuteron (^2_1H) and helium nucleus (^4_2He) is 1.1 *MeV* and 7 *MeV* respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is

[MP PMT 1992; Roorkee 1994; IIT-JEE 1996; AIIMS 1997; Haryana PMT 2000; Pb PMT 2001; CPMT 2001; AIEEE 2004]

(a) 13.9 *MeV* (b) 26.9 *MeV* (c) 23.6 *MeV* (d) 19.2 *MeV*

Solution : (c) ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + Q$

Total binding energy of helium nucleus = 4 × 7 = 28 *MeV*

Total binding energy of each deutron = 2 × 1.1 = 2.2 *MeV*

Hence energy released = 28 – 2 × 2.2 = 23.6 *MeV*

***Example*: 8** The masses of neutron and proton are 1.0087 *amu* and 1.0073 *amu* respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 *amu*. The binding energy of the helium nucleus will be [1 *amu*= 931 *MeV*]

(a) 28.4 *MeV* (b) 20.8 *MeV* (c) 27.3 *MeV* (d) 14.2 *MeV*

Solution : (a) Helium nucleus consist of two neutrons and two protons.

So binding energy $E = \Delta m$ *amu* = $\Delta m \times 931$ *MeV*

$\Rightarrow E = (2 \times m_p + 2m_n - M) \times 931$ *MeV* = (2 × 1.0073 + 2 × 1.0087 – 4.0015) × 931 = 28.4 *MeV*

***Example*: 9** A atomic power reactor furnace can deliver 300 *MW*. The energy released due to fission of each of uranium atom U^{238} is 170 *MeV*. The number of uranium atoms fissioned per hour will be **[UPSEAT 2000]**

(a) 5×10^{15} (b) 10×10^{20} (c) 40×10^{21} (d) 30×10^{25}

Solution : (c) By using $P = \frac{W}{t} = \frac{n \times E}{t}$ where n = Number of uranium atom fissioned and E = Energy released due to each fission so $300 \times 10^6 = \frac{n \times 170 \times 10^6 \times 1.6 \times 10^{-19}}{3600}$ $\Rightarrow n = 40 \times 10^{21}$

***Example*: 10** The binding energy per nucleon of O^{16} is 7.97 *MeV* and that of O^{17} is 7.75 *MeV*. The energy (in *MeV*) required to remove a neutron from O^{17} is

(a) 3.52 (b) 3.64 (c) 4.23 (d) 7.86

Solution : (c) $O^{17} \rightarrow O^{16} + {}_0n^1$

$\therefore$ Energy required = Binding of O^{17} – binding energy of O^{16} = 17 × 7.75 – 16 × 7.97 = 4.23 *MeV*

***Example*: 11** A gamma ray photon creates an electron-positron pair. If the rest mass energy of an electron is 0.5 *MeV* and the total kinetic energy of the electron-positron pair is 0.78 *MeV*, then the energy of the gamma ray photon must be

(a) 0.78 *MeV* (b) 1.78 *MeV* (c) 1.28 *MeV* (d) 0.28 *MeV*

Solution : (b) Energy of γ-rays photon = 0.5 + 0.5 +0.78 = 1.78 *MeV*

***Example*: 12** What is the mass of one Curie of U^{234} **[MNR 1985]**

(a) 3.7×10^{10} *gm* (b) 2.348×10^{23} *gm* (c) 1.48×10^{-11} *gm* (d) 6.25×10^{-34} *gm*

Solution : (c) 1 *curie* = 3.71×10^{10} disintegration/*sec* and mass of 6.02×10^{23} atoms of $U^{234} = 234\ gm$

$\therefore$ Mass of 3.71×10^{10} atoms $= \frac{234 \times 3.71 \times 10^{10}}{6.02 \times 10^{23}} = 1.48 \times 10^{-11} gm$

***Example*: 13** In the nuclear fusion reaction ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n,$ given that the repulsive potential energy between the two nuclei is $-7.7 \times 10^{-14} J$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23} J/K$]

(a) $10^9 K$ (b) $10^7 K$ (c) $10^5 K$ (d) $10^3 K$

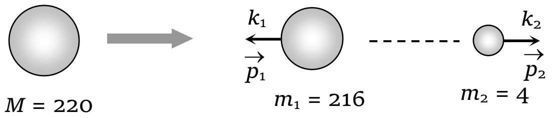
Solution : (a) Kinetic energy of molecules of a gas at a temperature T is 3/2 kT

$\therefore$ To initiate the reaction $\frac{3}{2}kT = 7.7 \times 10^{-14}\,J \;\Rightarrow T = 3.7 \times 10^9\,K$.

Example: 14 A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV. Calculate the kinetic energy of the α-particle

(a) 4.4 MeV (b) 5.4 MeV (c) 5.6 MeV (d) 6.5 MeV

Solution : (b)



Q-value of the reaction is 5.5 eV *i.e.* $k_1 + k_2 = 5.5\,MeV$ ......(i)

By conservation of linear momentum $p_1 = p_2 \Rightarrow \sqrt{2(216)k_1} = \sqrt{2(4)k_2} \;\Rightarrow\; k_2 = 54\,k_1$ ......(ii)

On solving equation (i) and (ii) we get $k_2 = 5.4\,MeV$.

Example: 15 Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}^{20}_{10}Ne$ nucleus and M_2 the mass of a ${}^{40}_{20}Ca$ nucleus. Then

(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$ (c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

Solution : (c, d) Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less then the sum of masses of it's constituent particles ${}^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of ${}^{20}_{10}Ne$ nucleus $M_1 < 10\,(m_p + m_n)$

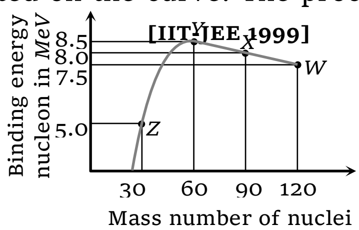
Also heavier the nucleus, more is he mass defect thus $20\,(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

or $\quad 10\,(m_p + m_n) > M_2 - M_1$

$\Rightarrow \quad M_2 < M_1 + 10\,(m_p + m_n) \;\Rightarrow\; M_2 < M_1 + M_1 \;\Rightarrow\; M_2 < 2M_1$

Tricky example: 1

Binding energy per nucleon *vs* mass number curve for nuclei is shown in the figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is **[IIT-JEE 1999]**



(a) $Y \rightarrow 2Z$

(b) $W \rightarrow X + Z$

(c) $W \rightarrow 2Y$

(d) $X \rightarrow Y + Z$

Solution : (c) Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleon) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (c) this happens.

Given $W \rightarrow 2Y$

Binding energy of reactants = 120 × 7.5 = 900 MeV

and binding energy of products = 2 (60 × 8.5) = 1020 MeV > 900 MeV

Radioactivity

The phenomenon of spontaneous emission of radiatons by heavy elements is called radioactivity. The elements which shows this phenomenon are called radioactive elements.

(1) Radioactivity was discovered by Henery Becquerel in uranium salt in the year 1896.

(2) After the discovery of radioactivity in uranium, Piere Curie and Madame Curie discovered a new radioactive element called radium (which is 10^6 times more radioactive than uranium)

(3) Some examples of radio active substances are : Uranium, Radium, Thorium, Polonium, Neptunium *etc.*

(4) Radioactivity of a sample cannot be controlled by any physical (pressure, temperature, electric or magnetic field) or chemical changes.

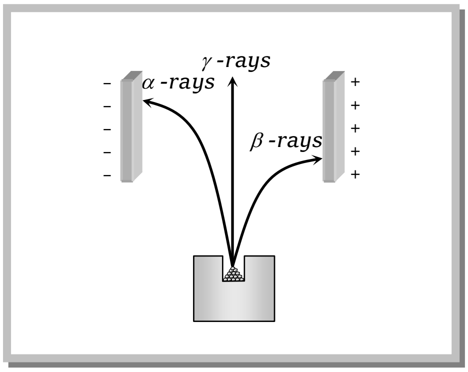
(5) All the elements with atomic number (Z) > 82 are naturally radioactive.

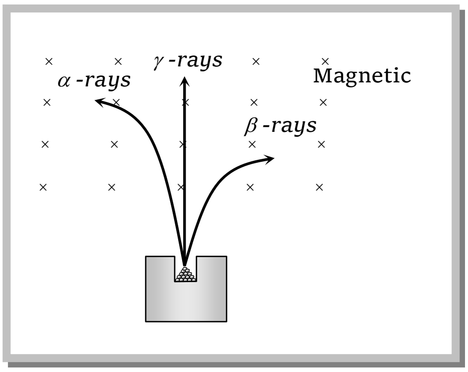
(6) The conversion of lighter elements into radioactive elements by the bombardment of fast moving particles is called artificial or induced radioactivity.

(7) Radioactivity is a nuclear event and not atomic. Hence electronic configuration of atom don't have any relationship with radioactivity.

Nuclear radiatons

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts





(i) Radiations which deflects towards negative plate are called α-rays (stream of positively charged particles)

(ii) Radiations which deflects towards positive plate are called β particles (stream of negatively charged particles)

(iii) Radiations which are undeflected called *γ-rays*. (E.M. waves or photons)

Note :
- ❑ Exactly same results were obtained when these radiations were subjected to magnetic field.
- ❑ No radioactive substance emits both α and β particles simultaneously. Also γ-rays are emitted after the emission of α or β-particles.
- ❑ β-particles are not orbital electrons they come from nucleus. The neutron in the nucleus decays into proton and an electron. This electron is emitted out of the nucleus in the form of β-rays.

Properties of α, β and γ-rays

Features	α- particles	β - particles	γ - rays
1. Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electron ($-\beta^0$ or β^-)	Photons (E.M. waves)
2. Charge	+ 2e	– e	Zero
3. Mass 4 m_p (m_p = mass of proton = 1.87×10^{-27}	4 m_p	m_e	Massless
4. Speed	$\approx 10^7$ m/s	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 MeV to 9 MeV	All possible values between a minimum certain value to 1.2 MeV	Between a minimum value to 2.23 MeV
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 cm of iron (or Pb) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	$_ZX^A \xrightarrow{\alpha-decay}$	$_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$	$_zX^A \rightarrow {}_zX^a + \gamma$

	$_{Z-2}Y^{A-4} + {_2}He^4$ $_ZX^A \xrightarrow{n_\alpha} {_{Z'}}Y^{A'}$ $\Rightarrow \boldsymbol{n_\alpha = \frac{A'-A}{4}}$	$_ZX^A \xrightarrow{n_\beta} {_{Z'}}X^A$ $\Rightarrow \boldsymbol{n_\beta = (2n_\alpha - Z + Z')}$	

Radioactive Disintegration

(1) **Law of radioactive disintegration**

According to Rutherford and Soddy law for radioactive decay is as follows.

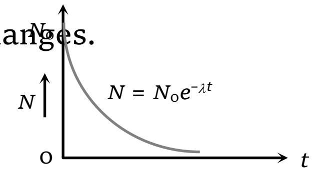
"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant" *i.e.* $-\frac{dN}{dt} \propto N \quad \Rightarrow \frac{dN}{dt} = -\lambda N$. It can be proved that $\boldsymbol{N = N_0 e^{-\lambda t}}$

This equation can also be written in terms of mass *i.e.* $M = M_0 e^{-\lambda t}$

where N = Number of atoms remains undecayed after time t, N_0 = Number of atoms present initially (*i.e.* at t = 0), M = Mass of radioactive nuclei at time t, M_0 = Mass of radioactive nuclei at time t = 0, $N_0 - N$ = Number of disintegrated nucleus in time t

$\frac{dN}{dt}$ = rate of decay, λ = Decay constant or disintegration constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

Note : ❑ λ depends only on the nature of substance. It is independent of time and any physical or chemical changes.



(2) **Activity**

It is defined as the rate of disintegration (or count rate) of the substance (or the number of atoms of any material decaying per second) *i.e.* $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

where A_0 = Activity of t = 0, A = Activity after time t

Units of activity (Radioactivity)

It's units are *Becqueral* (Bq), Curie (Ci) and Rutherford (Rd)

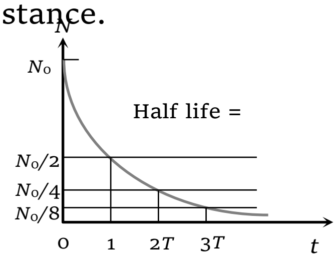
1 *Becquerel* = 1 *disintegration/sec*, 1 *Rutherford* = 10^6 *dis/sec*, 1 *Curie* = 3.7×10^{11} *dis/sec*

Note : ❑ Activity per *gm* of a substance is known as specific activity. The specific activity of 1 *gm* of radium – 226 is 1 *Curie*.

- 1 *millicurie* = 37 *Rutherford*
- The activity of a radioactive substance decreases as the number of undecayed nuclei decreases with time.
- Activity $\propto \dfrac{1}{\text{Half life}}$

(3) **Half life ($T_{1/2}$)**

Time interval in which the mass of a radioactive substance or the number of it's atom reduces to half of it's initial value is called the half life of the substance.



i.e. if $N = \dfrac{N_0}{2}$ then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda(T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$

Time (t)	Number of undecayed atoms (N) (N_0 = Number of initial atoms)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed ($N_0 - N$) $/N_0$ probability of decay
$t = 0$	N_0	1 (100%)	0
$t = T_{1/2}$	$\dfrac{N_0}{2}$	$\dfrac{1}{2}$ (50%)	$\dfrac{1}{2}$ (50%)
$t = 2(T_{1/2})$	$\dfrac{1}{2}\times\dfrac{N_0}{2} = \dfrac{N_0}{(2)^2}$	$\dfrac{1}{4}$ (25%)	$\dfrac{3}{4}$ (75%)
$t = 3(T_{1/2})$	$\dfrac{1}{2}\times\dfrac{N_0}{(2)} = \dfrac{N_0}{(2)^3}$	$\dfrac{1}{8}$ (12.5%)	$\dfrac{7}{8}$ (87.5%)
$t = 10\ (T_{1/2})$	$\dfrac{N_0}{(2)^{10}}$	$\left(\dfrac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = n\ (N_{1/2})$	$\dfrac{N}{(2)^2}$	$\left(\dfrac{1}{2}\right)^n$	$\left\{1-\left(\dfrac{1}{2}\right)^n\right\}$

Useful relation

After n half-lives, number of undecayed atoms $N = N_0\left(\dfrac{1}{2}\right)^n = N_0\left(\dfrac{1}{2}\right)^{t/T_{1/2}}$

(4) **Mean (or average) life (τ)**

The time for which a radioactive material remains active is defined as mean (average) life of that material.

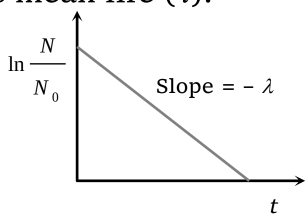
Other definitions

(i) It is defined as the sum of lives of all atoms divided by the total number of atoms

$$i.e.\ \tau = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}} = \frac{1}{\lambda}$$

(ii) From $N = N_0 e^{-\lambda t} \Rightarrow \dfrac{\ln \frac{N}{N_0}}{t} = -\lambda$ slope of the line shown in the graph

i.e. the magnitude of inverse of slope of $\ln \dfrac{N}{N_0}$ $vs\ t$ curve is known as mean life (τ).



(iii) From $N = N_0 e^{-\lambda t}$

If $t = \dfrac{1}{\lambda} = \tau \Rightarrow N = N_0 e^{-1} = N_0 \left(\dfrac{1}{e}\right) = 0.37 N_0 = 37\%$ of N_0.

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\dfrac{1}{e}$ times or 0.37 times or 37% of original number of atoms.

or

It is the time in which number of decayed atoms ($N_0 - N$) becomes $\left(1 - \dfrac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(iv) From $T_{1/2} = \dfrac{0.693}{\lambda} \Rightarrow \dfrac{1}{\lambda} = \tau = \dfrac{1}{0.693}.(t_{1/2}) = 1.44\,(T_{1/2})$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

Note : ❑ Half life and mean life of a substance doesn't change with time or with pressure, temperature *etc.*

Radioactive Series

If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Mass number	**Series (Nature)**	**Parent**	**Stable and product**	**Integer n**	**Number of lost particles**
$4n$	Thorium (natural)	${}_{90}Th^{232}$	${}_{82}Pb^{208}$	52	$\alpha = 6, \beta = 4$
$4n + 1$	Neptunium (Artificial)	${}_{93}Np^{237}$	${}_{83}Bi^{209}$	52	$\alpha = 8, \beta = 5$
$4n + 2$	Uranium (Natural)	${}_{92}U^{238}$	${}_{82}Pb^{206}$	51	$\alpha = 8, \beta = 6$
$4n + 3$	Actinium (Natural)	${}_{89}Ac^{227}$	${}_{82}Pb^{207}$	51	$\alpha = 7, \beta = 4$

Note : ❑ The $4n + 1$ series starts from $_{94}PU^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.

❑ The $4n + 3$ series actually starts from $_{92}U^{235}$.

Successive Disintegration and Radioactive Equilibrium

Suppose a radioactive element A disintegrates to form another radioactive element B which intern disintegrates to still another element C; such decays are called successive disintegration.

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$$

Rate of disintegration of $A = \frac{dN_1}{dt} = -\lambda_1 N_1$ (which is also the rate of formation of B)

Rate of disintegration of $B = \frac{dN_2}{dt} = -\lambda_2 N_2$

$\therefore$ Net rate of formation of B = Rate of disintegration of A – Rate of disintegration of B

$= \lambda_1 N_1 - \lambda_2 N_2$

Equilibrium

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to it's rate of production from the previous member.

i.e. $\lambda_1 N_1 = \lambda_2 N_2 \quad \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_2} = \frac{\tau_2}{\tau_1} = \frac{(T_{1/2})}{(T_{1/2})_1}$

Note : ❑ In successive disintegration if N_0 is the initial number of nuclei of A at $t = 0$ then number of nuclei of product B at time t is given by $N_2 = \frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$ where $\lambda_1 \lambda_2$ – decay constant of A and B.

Uses of radioactive isotopes

(1) **In medicine**

(i) For testing blood-chromium - 51

(ii) For testing blood circulation - Na - 24

(iii) For detecting brain tumor- Radio mercury - 203

(iv) For detecting fault in thyroid gland - Radio iodine - 131

(v) For cancer - cobalt - 60

(vi) For blood - Gold - 189

(vii) For skin diseases - Phospohorous - 31

(2) **In Archaeology**

(i) For determining age of archaeological sample (carbon dating

(ii) For determining age of meteorites - K^{40}

(iii) For determining age of earth-Lead isotopes

(3) **In agriculture**

(i) For protecting potato crop from earthworm- CO^{60} (ii) For artificial rains - *AgI* (iii) As fertilizers - P^{32}

(4) **As tracers** - (Tracer) : Very small quantity of radioisotopes present in a mixture is known as tracer

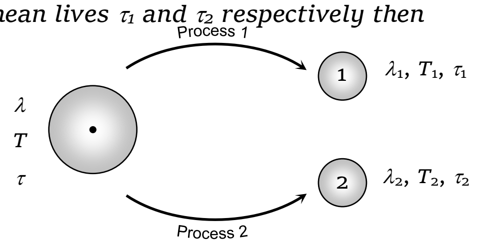
(i) Tracer technique is used for studying biochemical reaction in tracer and animals.

(5) **In industries**

(i) For detecting leakage in oil or water pipe lines (ii) For determining the age of planets.

Concept

☞ *If a nuclide can decay simultaneously by two different process which have decay constant λ_1 and λ_2, half life T_1 and T_2 and mean lives τ_1 and τ_2 respectively then*



$$\Rightarrow \lambda = \lambda_1 + \lambda_2$$

$$\Rightarrow T = \frac{T_1 T_2}{T_1 + T_2}$$

$$\Rightarrow \tau = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

Example

***Example*: 16** When ${}_{90}Th^{228}$ transforms to ${}_{83}Bi^{212}$, then the number of the emitted α–and β–particles is, respectively

[MP PET 2002]

(a) $8\alpha, 7\beta$ (b) $4\alpha, 7\beta$ (c) $4\alpha, 4\beta$ (d) $4\alpha, 1\beta$

Solution : (d) ${}_{Z=90}Th^{A=228} \rightarrow {}_{Z'=83}Bi^{A'=212}$

Number of α-particles emitted $n_\alpha = \frac{A - A'}{4} = \frac{228 - 212}{4} = 4$

Number of β-particles emitted $n_\beta = 2n_\alpha - Z + Z' = 2 \times 4 - 90 + 83 = 1.$

***Example*: 17** A radioactive substance decays to 1/16th of its initial activity in 40 days. The half-life of the radioactive substance expressed in days is

(a) 2.5 (b) 5 (c) 10 (d) 20

Solution : (c) By using $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^{40/T_{1/2}} \Rightarrow T_{1/2} = 10$ days.

***Example*: 18** A sample of radioactive element has a mass of 10 *gm* at an instant *t* = 0. The approximate mass of this element in the sample after two mean lives is

(a) 2.50 *gm* (b) 3.70 *gm* (c) 6.30 *gm* (d) 1.35 *gm*

Solution : (d) By using $M = M_0 e^{-\lambda t} \Rightarrow M = 10\,e^{-\lambda(2\tau)} = 10\,e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.359$ *gm*

***Example*: 19** The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16[th] of its initial value is

(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

Solution : (a) By using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{t/100} \Rightarrow t = 400\ \mu\ sec.$

***Example*: 20** The mean lives of a radioactive substance for α and β emissions are 1620 years and 405 years respectively. After how much time will the activity be reduced to one fourth

(a) 405 year (b) 1620 year (c) 449 year (d) None of these

Solution : (c) $\lambda_\alpha = \frac{1}{1620}$ *per year* and $\lambda_\beta = \frac{1}{405}$ *per year* and it is given that the fraction of the remained activity $\frac{A}{A_0} = \frac{1}{4}$

Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ *per year*

We know that $A = A_0 e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda}\log_e \frac{A_0}{A} \Rightarrow t = \frac{1}{\lambda}\log_e 4 = \frac{2}{\lambda}\log_e 2$ = 324 × 2 × 0.693 = 449 *years.*

***Example*: 21** At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances **[RPMT 1996]**

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

Solution : (a) By using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2} \times \frac{(1/2)^{n_1}}{(1/2)^{n_2}} = \frac{2}{1} \times \frac{\left(\frac{1}{2}\right)^{\frac{2\times 24}{12}}}{\left(\frac{1}{2}\right)^{\frac{2\times 24}{16}}} = \frac{1}{1}$

***Example*: 22** From a newly formed radioactive substance (Half-life 2 *hours*), the intensity of radiation is 64 times the permissible safe level. The minimum time after which work can be done safely from this source is

[IIT 1983; SCRA 1996]

(a) 6 *hours* (b) 12 *hours* (c) 24 *hours* (d) 128 *hours*

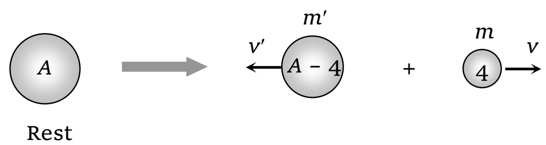
Solution : (b) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{A}{A_0} = \frac{1}{64} = \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^n \Rightarrow n = 6$

$\Rightarrow \frac{t}{T_{1/2}} = 6 \Rightarrow t = 6 \times 2 = 12$ *hours.*

***Example*: 23** nucleus of mass number *A*, originally at rest, emits an α-particle with speed *v*. The daughter nucleus recoils with a speed

(a) $2v/(A+4)$ (b) $4v/(A+4)$ (c) $4v/(A-4)$ (d) $2v/(A-4)$

Solution : (c)



According to conservation of momentum $4v = (A-4)v' \Rightarrow v' = \frac{4v}{A-4}$.

***Example*: 24** The counting rate observed from a radioactive source at t = 0 second was 1600 counts per second and at t = 8 seconds it was 100 counts per second. The counting rate observed as counts per second at t = 6 seconds will be

(a) 400 (b) 300 (c) 200 (d) 150

Solution : (c) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow 100 = 1600\left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow T_{1/2} = 2\ sec$

Again by using the same relation the count rate at t = 6 *sec* will be $A = 1600\left(\frac{1}{2}\right)^{6/2} = 200$.

***Example*: 25** The kinetic energy of a neutron beam is 0.0837 *eV*. The half-life of neutrons is 693*s* and the mass of neutrons is 1.675×10^{-27} *kg*. The fraction of decay in travelling a distance of 40*m* will be

(a) 10^{-3} (b) 10^{-4} (c) 10^{-5} (d) 10^{-6}

Solution : (c) $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times0.0837\times1.6\times10^{-19}}{1.675\times10^{-27}}} = 4\times10^3\ m/sec$

$\therefore$ Time taken by neutrons to travel a distance of 40 m $\Delta t' = \frac{40}{4\times10^3} = 10^{-2}\ sec$

$\therefore \frac{dN}{dt} = \lambda N \Rightarrow \frac{dN}{N} = \lambda\, dt$

$\therefore$ Fraction of neutrons decayed in Δt *sec* in $\frac{\Delta N}{N} = \lambda\,\Delta t = \frac{0.693}{T}\Delta t = \frac{0.693}{693}\times10^{-2} = 10^{-5}$

***Example*: 26** The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) 77/80 (b) 71/80 (c)31/32 (d) 15/16

Solution : (c) By using $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow t = \frac{T_{1/2}\log_e\left(\frac{N_0}{N}\right)}{\log_e(2)} \Rightarrow t \propto \log_e\frac{N_0}{N} \Rightarrow \frac{t_1}{t_2} = \frac{\left(\log_e\frac{N_0}{N}\right)_1}{\left(\log_e\frac{N_0}{N}\right)_2}$

Hence $\frac{6}{10} = \frac{\log_e(8/1)}{\log_e(N_0/N)} \Rightarrow \log_e\frac{N_0}{N} = \frac{10}{6}\log_e(8) = \log_e 32 \Rightarrow \frac{N_0}{N} = 32$.

So fraction that decays $= 1 - \frac{1}{32} = \frac{31}{32}$.

Half-life of a substance is 20 minutes. What is the time between 33% decay and 67% decay **[AIIMS 2000]**

(a) 40 minutes (b) 20 minutes (c) 30 minutes (d) 25 minutes

Solution : (b) Let N_0 be the number of nuclei at beginning

$\therefore$ Number of undecayed nuclei after 33% decay = 0.67 N_0

and number of undecayed nuclei after 67% of decay = 0.33 N_0

$\because$ 0.33 $N_0 \simeq \frac{0.67\,N_0}{2}$ and in the half-life time the number of undecayed nuclei becomes half.