Chapter 12

Thermometry, Thermal Expansion and Caloriemetry

Contents

Thermal expansion of solids has many practical applications.

An iron rim to be put on a wooden cart wheel is always of slightly smaller diameter than that of wheel because on heating it expands and in this red-hot state when it is placed over the wooden wheel and allow to cool then due to contraction it holds the wooden wheel tightly.

Thermometry, Thermal Expansion and Caloriemetry

12.1 Heat

The energy associated with configuration and random motion of the atoms and molecules with in a body is called internal energy and the part of this internal energy which is transferred from one body to the other due to temperature difference is called heat.

(1) As it is a type of energy, it is a scalar.

(2) Dimension : $[ML^2T^{-2}]$.

(3) Units : *Joule* (S.I.) and *calorie* (Practical unit)

One calorie is defined as the amount of heat energy required to raise the temperature of one *gm* of water through $1°C$ (more specifically from $14.5°C$ to $15.5°C$).

(4) As heat is a form of energy it can be transformed into others and *vice-versa*.

e.g. Thermocouple converts heat energy into electrical energy, resistor converts electrical energy into heat energy. Friction converts mechanical energy into heat energy. Heat engine converts heat energy into mechanical energy.

Here it is important that whole of mechanical energy *i.e.* work can be converted into heat but whole of heat can never be converted into work.

(5) When mechanical energy (work) is converted into heat, the ratio of work done (W) to heat produced (Q) always remains the same and constant, represented by J.

$$\frac{W}{Q} = J \qquad \text{or} \qquad W = JQ$$

J is called mechanical equivalent of heat and has value $4.2\ J/cal$. J is not a physical quantity but a conversion factor which merely express the equivalence between *Joule* and *calories*.

$$1\ calorie = 4.186\ Joule \simeq 4.12\ Joule$$

(6) Work is the transfer of mechanical energy irrespective of temperature difference, whereas heat is the transfer of thermal energy because of temperature difference only.

(7) Generally, the temperature of a body rises when heat is supplied to it. However the following two situations are also found to exist.

(i) When heat is supplied to a body either at its melting point or boiling point, the temperature of the body does not change. In this situation, heat supplied to the body is used up in changing its state.

(ii) When the liquid in a thermos flask is vigorously shaken or gas in a cylinder is suddenly compressed, the temperature of liquid or gas gets raised even without supplying heat. In this situation, work done on the system becomes a source of heat energy.

(8) The heat lost or gained by a system depends not only on the initial and final states, but also on the path taken up by the process *i.e.* heat is a path dependent and is taken to be positive if the system absorbs it and negative if releases it.

12.2 Temperature

Temperature is defined as the degree of hotness or coldness of a body. The natural flow of heat is from higher temperature to lower temperature.

Two bodies are said to be in thermal equilibrium with each other, when no heat flows from one body to the other. That is when both the bodies are at the same temperature.

(1) Temperature is one of the seven fundamental quantities with dimension $[\theta]$.

(2) It is a scalar physical quantity with S.I. unit kelvin.

(3) When heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls *i.e.* temperature can be regarded as the effect of cause "heat".

(4) According to kinetic theory of gases, temperature (macroscopic physical quantity) is a measure of average translational kinetic energy of a molecule (microscopic physical quantity).

Temperature $\propto$ kinetic energy $\qquad \left[\text{As } E = \frac{3}{2}RT\right]$

(5) Although the temperature of a body can to be raised without limit, it cannot be lowered without limit and theoretically limiting low temperature is taken to be zero of the kelvin scale.

(6) Highest possible temperature achieved in laboratory is about $10^8 K$ while lowest possible temperature attained is 10^{-8} K.

(7) Branch of physics dealing with production and measurement of temperatures close to $0K$ is known as cryogenics while that dealing with the measurement of very high temperature is called as pyrometry.

(8) Temperature of the core of the sun is 10^7 K while that of its surface is 6000 K.

(9) Normal temperature of human body is 310.15 K ($37°C = 98.6°F$).

(10) NTP or STP implies $273.15K$ ($0°C = 32°F$)

12.3 Scales of Temperature

The Kelvin temperature scale is also known as thermodynamic scale. The S.I. unit of temperature is *kelvin* and is defined as (1/273.16) of the temperature of the triple point of water. The triple point of water is that point on a *P-T* diagram where the three phases of water, the solid, the liquid and the gas, can coexist in equilibrium.

In addition to kelvin temperature scale, there are other temperature scales also like Celsius, Fahrenheit, Reaumer, Rankine *etc*.

To construct a scale of temperature, two fixed points are taken. First fixed point is the freezing point of water, it is called lower fixed point. The second fixed point is the boiling point of water, it is called upper fixed point.

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	$^\circ C$	$0^\circ C$	$100^\circ C$	100
Fahrenheit	$^\circ F$	$32^\circ F$	$212^\circ F$	180
Reaumer	$^\circ R$	$0^\circ R$	$80^\circ R$	80
Rankine	$^\circ Ra$	460 Ra	672 Ra	212
Kelvin	K	273.15 K	373.15 K	100

Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - \text{Lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{Lower fixed point (LFP)}} = \text{Constant for all scales}$$

$$\frac{C-0}{100} = \frac{F-32}{212-32} = \frac{K-273.15}{373.15-273.15} = \frac{R-0}{80-0} = \frac{Ra-460}{672-460}$$

or $$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} = \frac{R}{4} = \frac{Ra-460}{10.6}$$

12.4 Thermometry

An instrument used to measure the temperature of a body is called a thermometer.

The linear variation in some physical property of a substance with change of temperature is the basic principle of thermometry and these properties are defined as thermometric property (x) of the substance.

x may be (i) Length of liquid in capillary

(ii) Pressure of gas at constant volume.

(iii) Volume of gas at constant pressure.

(iv) Resistance of a given platinum wire.

In old thermometry, two arbitrarily fixed points ice and steam point (freezing point and boiling point at 1 *atm*) are taken to define the temperature scale. In celsius scale freezing point of water is assumed to be $0°C$ while boiling point $100°C$ and the temperature interval between these is divided into 100 equal parts.

So if the thermometric property at temperature $0°C$, $100°C$ and $T_c°C$ is x_0, x_{100} and x respectively then by linear variation ($y = mx + c$) we can say that

$0 = ax_0 + b$ .....(i) $100 = ax_{100} + b$ .....(ii) $T_c = ax + b$ .....(iii)

From these equations $\frac{T_c - 0}{100 - 0} = \frac{x - x_0}{x_{100} - x_0}$

$$\therefore \quad T_c = \frac{x - x_0}{x_{100} - x_0} \times 100\,°centigrade$$

In modern thermometry instead of two fixed points only one reference point is chosen (triple point of water 273.16 *K* at which ice, water and water vapours co-exist) the other is itself 0 *K* where the value of thermometric property is assumed to be zero.

So if the value of thermometric property at 0 *K*, 273.16 *K* and T_K *K* is 0, x_{Tr} and x respectively then by linear variation ($y = mx + c$) we can say that

$0 = a \times 0 + b$ .....(i) $273.16 = a \times x_{T_r} + b$ .....(ii) $T_K = a \times x + b$ .....(iii)

From these equation $\frac{T_K}{273.16} = \frac{x}{x_{Tr}}$

$$\therefore \quad T_K = 273.16 \left[\frac{x}{x_{Tr}}\right] kelvin$$

12.5 Thermometers

A thermometer is an instrument used to measure the temperature of a body. It works by absorbing some heat from the body, so the temperature recorded by it is lesser than the actual value unless the body is at constant temperature. Some common types of thermometers are :

(1) **Liquid thermometers :** In liquid thermometers mercury is preferred over other liquids as its expansion is large and uniform and it has high thermal conductivity and low specific heat.

(i) Range of temperature : $\underset{\text{(freezing point)}}{-50°}$ to $\underset{\text{(boiling point)}}{350\,°C}$

(ii) Upper limit of range of mercury thermometer can be raised upto $550°C$ by filling nitrogen in space over mercury under pressure (which elevates boiling point of mercury).

(iii) Mercury thermometer with cylindrical bulbs are more sensitive than those with spherical bulbs.

(iv) If alcohol is used instead of mercury then range of temperature measurement becomes $-80°C$ to $350°C$

(v) Formula : $T_c = \frac{l - l_0}{l_{100} - l_0} \times 100\,°C$

(2) **Gas thermometers :** These are of two types

(i) **Constant pressure gas thermometers**

(a) Principle $V \propto T_K$ (if P = constant)

(b) Formula : $T_c = \frac{V_t - V_0}{V_{100} - V_0} \times 100\,°\mathit{centigrade}$ or $T_K = 273.16 \frac{V}{V_{Tr}} \mathit{kelvin}$

(ii) **Constant volume gas thermometers**

(a) Principle $P \propto T_K$ (if V = constant)

(b) Formula : $T_c = \frac{P - P_0}{P_{100} - P_0} \times 100\,°\mathit{centigrade}$ or $T_K = 273.16 \frac{P}{P_{Tr}} \mathit{kelvin}$

(c) Range of temperature : Hydrogen gas thermometer – 200 to 500°C

Nitrogen gas thermometer – 200 to 1600°C

Helium gas thermometer – 268 to 500°C

(d) These are more sensitive and accurate than liquid thermometers as expansion of gases is more than that of liquids.

(3) **Resistance thermometers :** Resistance of metals varies with temperature according to relation.

$R = R_0(1 + \alpha T_c)$ where α is the temperature coefficient of resistance.

Usually platinum is used in resistance thermometers due to high melting point and large value of α.

(i) Formula : $T_c = \frac{R - R_0}{R_{100} - R_0} \times 100\,°\mathit{centigrade}$ or $T_K = 273.16 \frac{R}{R_{Tr}} \mathit{kelvin}$

(ii) Temperature range : Platinum resistance thermometer = – 200°C to 1200°C

Germanium resistance thermometer = 4 to 77 K

(4) **Thermoelectric thermometers :** These are based on "Seebeck effect" according to which when two distinct metals are joined to form a closed circuit called thermocouple and the difference in temperature is maintained between their junctions, an emf is developed. The emf is called thermo-emf and if one junction is at 0°C, it varies with temperature as $e = aT_c + bT_c^2$ where a and b are constants.

Temperature range : Copper-iron thermocouple 0°C to 260°C

Iron-constantan thermocouple 0°C to 800°C

Tungsten-molybdenum thermocouple 2000°C to 3000°C

(5) **Pyrometers :** These are the devices used to measure the temperature by measuring the intensity of radiations received from the body. They are based on the fact that the amount of radiations emitted from a body per unit area per second is directly proportional to the fourth power of temperature (Stefan's law).

(i) These can be used to measure temperatures ranging from 800°*C* to 4000°*C*.

(ii) They cannot measure temperature below 800°*C* because the amount of radiations is too small to be measured.

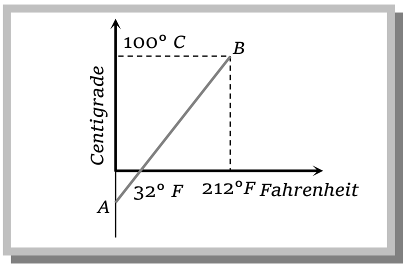
(6) **Vapour pressure thermometer :** These are used to measure very low temperatures. They are based on the fact that saturated vapour pressure *P* of a liquid depends on the temperature according to the relation

$$\log P = a + bT_K + \frac{c}{T_K}$$

The range of these thermometers varies from 120 *K* to 0.71 *K* for different liquid vapours.

Sample problems based on Thermometry

***Problem* 1.** The graph *AB* shown in figure is a plot of temperature of a body in degree celsius and degree Fahrenheit. Then



(a) Slope of line *AB* is 9/5

(b) Slope of line *AB* is 5/9

(c) Slope of line *AB* is 1/9

(d) Slope of line *AB* is 3/9

Solution : (b) Relation between Celsius and Fahrenheit scale of temperature is $\frac{C}{5} = \frac{F-32}{9}$

By rearranging we get, $C = \frac{5}{9}F - \frac{160}{9}$

By equating above equation with standard equation of line $y = mx + c$ we get $m = \frac{5}{9}$ and $c = \frac{-160}{9}$

i.e. Slope of the line *AB* is $\frac{5}{9}$.

***Problem* 2.** The freezing point on a thermometer is marked as 20° and the boiling point at as 150°. A temperature of 60°*C* on this thermometer will be read as

(a) 40° (b) 65° (c) 98° (d) 110°

Solution : (c) Temperature on any scale can be converted into other scale by $\frac{X - LFP}{UFP - LFP}$ = Constant for all scales

$$\therefore \frac{X-20°}{150°-20°} = \frac{C-0°}{100°-0°} \Rightarrow X = \frac{C\times 130°}{100°} + 20° = \frac{60°\times 130°}{100°} + 20° = 98°$$

***Problem* 3.** A thermometer is graduated in *mm*. It registers – 3*mm* when the bulb of thermometer is in pure melting ice and 22*mm* when the thermometer is in steam at a pressure of one *atm*. The temperature in °*C* when the thermometer registers 13*mm* is

(a) $\frac{13}{25}\times 100$ (b) $\frac{16}{25}\times 100$ (c) $\frac{13}{22}\times 100$ (d) $\frac{16}{22}\times 100$

Solution : (b) For a constant volume gas thermometer temperature in °*centigrade* is given as

$$T_c = \frac{P - P_0}{P_{100} - P_0}\times 100\ °C \qquad \Rightarrow T_c = \frac{13-(-3)}{22-(-3)}\times 100\ °C = \frac{16}{25}\times 100$$

***Problem* 4.** The temperature coefficient of resistance of a wire is 0.00125 *per* °*C*. At 300*K* its resistance is 1Ω. The resistance of wire will be 2Ω at

(a) 1154*K* (b) 1100*K* (c) 1400*K* (d) 1127*K*

Solution : (d) Resistance of wire varies with temperature as $R = R_0(1+\alpha T_c)$ where α is temperature coefficient of resistance

$$\Rightarrow \frac{R_{27}}{R_{T_c}} = \frac{R_0(1+27\alpha)}{R_0(1+\alpha T_c)} = \frac{1}{2} \Rightarrow T_c = \frac{1+54\alpha}{\alpha} = \frac{1+54\times 0.00125}{0.00125} = 854°C$$

$$\therefore \quad T_K = (854+273) = 1127K = 1127\ K.$$

12.6 Thermal Expansion

When matter is heated without any change in state, it usually expands. According to atomic theory of matter, a symmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration and hence energy of atoms increases, hence the average distance between the atoms increases. So the matter as a whole expands.

(1) Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.

(2) Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimension (volume expansion) while liquids and gases usually suffers change in volume only.

(3) The coefficient of linear expansion of the material of a solid is defined as the increase in its length per unit length per unit rise in its temperature.

$$\alpha = \frac{\Delta L}{L}\times\frac{1}{\Delta T}$$

Similarly the coefficient of superficial expansion $\beta = \frac{\Delta A}{A}\times\frac{1}{\Delta T}$

and coefficient of volume expansion $\gamma = \frac{\Delta V}{V}\times\frac{1}{\Delta T}$

The value of α, β and γ depends upon the nature of material. All have dimension $[\theta^{-1}]$ and unit per °*C*.

(4) As $\alpha = \frac{\Delta L}{L}\times\frac{1}{\Delta T}$, $\beta = \frac{\Delta A}{A}\times\frac{1}{\Delta T}$ and $\gamma = \frac{\Delta V}{V}\times\frac{1}{\Delta T}$

$\therefore$ $\Delta L = L\alpha\Delta T$, $\Delta A = A\beta\Delta T$ and $\Delta V = V\gamma\Delta T$

Final length $L' = L + \Delta L = L(1+\alpha\Delta T)$ .....(i)

Final area $A' = A + \Delta A = A(1 + \beta\Delta T)$ .....(ii)

Final volume $V' = V + \Delta V = V(1 + \gamma\Delta T)$ .....(iii)

(5) If L is the side of square plate and it is heated by temperature ΔT, then its side becomes L'.

The initial surface area $A = L^2$ and final surface $A' = L'^2$

$$\therefore \quad \frac{A'}{A} = \left(\frac{L'}{L}\right)^2 = \left(\frac{L(1+\alpha\Delta T)}{L}\right)^2 = (1+\alpha\Delta T)^2 = (1+2\alpha\Delta T)$$ [Using Binomial theorem]

or $A' = A(1 + 2\alpha\Delta T)$

Comparing with equation (ii) we get $\beta = 2\alpha$

Similarly for volumetric expansion $\frac{V'}{V} = \left(\frac{L'}{L}\right)^3 = \left(\frac{L(1+\alpha\Delta T)}{L}\right)^3 = (1+\alpha\Delta T)^3 = (1+3\alpha\Delta T)$ [Using Binomial theorem]

or $V' = V(1 + \gamma\Delta T)$

Comparing with equation (iii), we get $\gamma = 3\alpha$

So $\alpha : \beta : \gamma = 1 : 2 : 3$

(i) Hence for the same rise in temperature

Percentage change in area = 2 × percentage change in length.

Percentage change in volume = 3 × percentage change in length.

(ii) The three coefficients of expansion are not constant for a given solid. Their values depends on the temperature range in which they are measured.

(iii) The values of α, β, γ are independent of the units of length, area and volume respectively.

(iv) For anisotropic solids $\gamma = \alpha_x + \alpha_y + \alpha_z$ where α_x, α_y, and α_z represent the mean coefficients of linear expansion along three mutually perpendicular directions.

Material	**α[K^{-1} or $(°C)^{-1}$]**	**γ[K^{-1} or $(°C)^{-1}$]**
Steel	1.2×10^{-5}	3.6×10^{-5}
Copper	1.7×10^{-5}	5.1×10^{-5}
Brass	2.0×10^{-5}	6.0×10^{-5}
Aluminium	2.4×10^{-5}	7.2×10^{-5}

12.7 Variation of Density With Temperature

Most substances expand when they are heated, *i.e.*, volume of a given mass of a substance increases on heating, so the density should decrease $\left(\text{as } \rho \propto \frac{1}{V}\right)$.

$\rho = \frac{m}{V}$ or $\rho \propto \frac{1}{V}$ $\therefore$ $\frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V+\Delta V} = \frac{V}{V+\gamma V\Delta T} = \frac{1}{1+\gamma\Delta T}$ (For a given mass)

or $\rho' = \frac{\rho}{1+\gamma\Delta T} = \rho(1+\gamma\,\Delta T)^{-1} = \rho(1-\gamma\,\Delta T)$ [As γ is small $\therefore$ using Binomial theorem]

$\therefore$ $\rho' = \rho(1-\gamma\,\Delta T)$

Sample problems based on Thermal expansion of solid

***Problem* 5.** The design of a physical instrument requires that there be a constant difference in length of 10 *cm* between an iron rod and a copper cylinder laid side by side at all temperatures. If $\alpha_{Fe} = 11\times10^{-6}{}^\circ C^{-1}$ and $\alpha_{cu} = 17\times10^{-6}\,{}^\circ C^{-1}$, their lengths are

(a) 28.3 *cm*, 18.3 *cm* (b) 23.8 *cm*, 13.8 *cm* (c) 23.9 *cm*, 13.9 *cm* (d) 27.5 *cm*, 17.5 *cm*

Solution : (a) Since a constant difference in length of 10 *cm* between an iron rod and a copper cylinder is required therefore

$$L_{Fe} - L_{Cu} = 10\,cm \quad .....(i)$$

or $\Delta L_{Fe} - \Delta L_{Cu} = O \;\therefore\; \Delta L_{Fe} = \Delta L_{Cu}$

i.e., Linear expansion of iron rod = Linear expansion of copper cylinder

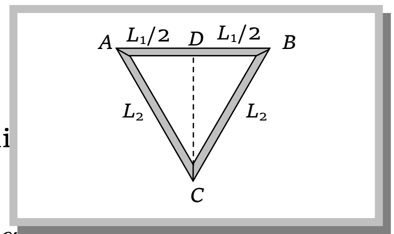
$$\Rightarrow L_{Fe}\times\alpha_{Fe}\times\Delta T = L_{Cu}\times\alpha_{Cu}\times\Delta T \Rightarrow \frac{L_{Fe}}{L_{Cu}} = \frac{\alpha_{Cu}}{\alpha_{Fe}} = \frac{17}{11} \qquad \therefore \frac{L_{Fe}}{L_{Cu}} = \frac{17}{11} \quad .....(ii)$$

From (i) and (ii) $L_{Fe} = 28.3cm,\ L_{Cu} = 18.3cm$.

***Problem* 6.** Two rods of length L_2 and coefficient of linear expansion α_2 are connected freely to a third rod of length L_1 of coefficient of linear expansion α_1 to form an isosceles triangle. The arrangement is supported on the knife edge at the midpoint of L_1 which is horizontal. The apex of the isosceles triangle is to remain at a constant distance from the knife edge if

(a) $\frac{L_1}{L_2} = \frac{\alpha_2}{\alpha_1}$ (b) $\frac{L_1}{L_2} = \sqrt{\frac{\alpha_2}{\alpha_1}}$ (c) $\frac{L_1}{L_2} = 2\frac{\alpha_2}{\alpha_1}$ (d) $\frac{L_1}{L_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$

Solution : (d) The apex of the isosceles triangle to remain at a constant distance from the knife edge *DC* should remains constant before and after heating.



Before expansion : In triangle *ADC* $(DC)^2 = L_2^2 - \left(\frac{L_1}{2}\right)^2$ .....(i)

After expansion : $(DC)^2 = [L_2(1+\alpha_2 t)]^2 - \left[\frac{L_1}{2}(1+\alpha_1 t)\right]^2$ .....(ii)

Equating (i) and (ii) we get $L_2^2 - \left(\frac{L_1}{2}\right)^2 = [L_2(1+\alpha_2 t)]^2 - \left[\frac{L_1}{2}(1+\alpha_1 t)\right]$

$$\Rightarrow L_2^2 - \frac{L_1^2}{4} = L_2^2 + L_2^2\times 2\alpha_2\times t - \frac{L_1^2}{4} - \frac{L_1^2}{4}\times 2\alpha_1\times t$$ [Neglecting higher terms]

$$\Rightarrow \frac{L_1^2}{4}(2\alpha_1 t) = L_2^2(2\alpha_2 t) \Rightarrow \frac{L_1}{L_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$$

***Problem* 7.** A iron rod of length 50 *cm* is joined at an end to an aluminium rod of length 100 *cm*. All measurements refer to 20°C. The coefficients of linear expansion of iron and aluminium are $12\times10^{-6}/{}^\circ C$ and $24\times10^{-6}/{}^\circ C$ respectively. The average coefficient of composite system is

(a) $36\times10^{-6}/°C$ (b) $12\times10^{-6}/°C$ (c) $20\times10^{-6}/°C$ (d) $48\times10^{-6}/°C$

Solution : (c) Initially (at 20°*C*) length of composite system *L* = 50 + 100 = 150 *cm*

Length of iron rod at 100°*C* $=50[1+12\times10^{-6}\times(100-20)]=50.048\ cm$

Length of aluminum rod at 100°*C* $=100[1+24\times10^{-6}\times(100-20)]=100.192\ cm$

Finally (at 100°*C*) length of composite system *L*' = $50.048+100.192=150.24\ cm$

Change in length of the composite system Δ*L* = *L*' – *L* = 150.24 –150 = 0.24 *cm*

∴ Average coefficient of expansion at 100°*C* $\alpha=\frac{\Delta L}{L\times\Delta T}=\frac{0.24}{150\times(100-20)}=20\times10^{-6}/°C$

***Problem* 8.** A brass rod and lead rod each 80 *cm* long at 0°*C* are clamped together at one end with their free ends coinciding. The separation of free ends of the rods if the system is placed in a steam bath is ($\alpha_{brass}=18\times10^{-6}/°C$ and $\alpha_{lead}=28\times10^{-6}/°C$)

(a) 0.2 *mm* (b) 0.8 *mm* (c) 1.4 *mm* (d) 1.6 *mm*

Solution : (b) The Brass rod and the lead rod will suffer expansion when placed in steam bath.

∴ Length of brass rod at 100°*C* $L'_{brass}=L_{brass}(1+\alpha_{brass}\Delta T)=80[1+18\times10^{-6}\times100]$

and the length of lead rod at 100°*C* $L'_{lead}=L_{lead}(1+\alpha_{lead}\Delta T)=80[1+28\times10^{-6}\times100]$

Separation of free ends of the rods after heating = $L'_{lead}-L'_{brass}=80[28-18]\times10^{-4}=8\times10^{-2}\ cm=0.8 mm$

***Problem* 9.** The coefficient of apparent expansion of a liquid in a copper vessel is *C* and in a silver vessel *S*. The coefficient of volume expansion of copper is γ_C. What is the coefficient of linear expansion of silver

(a) $(C+\gamma_C+S)/3$ (b) $(C-\gamma_C+S)/3$ (c) $(C+\gamma_C-S)/3$ (d) $(C-\gamma_C-S)/3$

Solution : (c) Apparent coefficient of volume expansion for liquid $\gamma_{app}=\gamma_L-\gamma_s$ ∴ $\gamma_L=\gamma_{app}+\gamma_s$

where γ_s is coefficient of volume expansion for solid vessel.

When liquid is placed in copper vessel then $\gamma_L = C + \gamma_{copper}$ ....(i) [As $\gamma_{app.}$ for liquid in copper vessel = *C*]

When liquid is placed in silver vessel then $\gamma_L = S + \gamma_{silver}$ ....(ii)[As $\gamma_{app.}$ for liquid in silver vessel = *S*]

From equation (i) and (ii) we get $C + \gamma_{copper} = S + \gamma_{silver}$

∴ $\gamma_{silver}=C+\gamma_{copper}-S$

Coefficient of volume expansion = 3 × Coefficient of linear expansion

⇒ $\alpha_{\text{silver}}=\frac{\gamma_{\text{silver}}}{3}=\frac{C+\gamma_{\text{copper}}-S}{3}$

***Problem* 10.** A uniform solid brass sphere is rotating with angular speed ω_0 about a diameter. If its temperature is now increased by 100°*C*. What will be its new angular speed. (Given $\alpha_B=2.0\times10^{-5}\text{ per}°C$)

(a) $1.1\omega_0$ (b) $1.01\omega_0$ (c) $0.996\,\omega_0$ (d) $0.824\,\omega_0$

Solution : (c) Due to increase in temperature, radius of the sphere changes.

Let R_0 and R_{100} are radius of sphere at 0°C and 100°C $R_{100} = R_0[1+\alpha \times 100]$

Squaring both the sides and neglecting higher terms $R_{100}^2 = R_0^2[1+2\alpha \times 100]$

By the law of conservation of angular momentum $I_1\omega_1 = I_2\omega_2$

$$\Rightarrow \frac{2}{5}MR_0^2\omega_1 = \frac{2}{5}MR_{100}^2\omega_2 \quad \Rightarrow R_0^2\omega_1 = R_0^2[1+2\times 2\times 10^{-5}\times 100]\omega_2$$

$$\Rightarrow \omega_2 = \frac{\omega_1}{[1+4\times 10^{-3}]} = \frac{\omega_0}{1.004} = 0.996\,\omega_0$$

12.8 Expansion of Liquid

Liquids also expand on heating just like solids. Since liquids have no shape of their own, they suffer only volume expansion. If the liquid of volume V is heated and its temperature is raised by $\Delta\theta$ then

$V_L' = V(1+\gamma_L\Delta\theta)$ [γ_L = coefficient of real expansion or coefficient of volume expansion of liquid]

As liquid is always taken in a vessel for heating so if a liquid is heated, the vessel also gets heated and it also expands.

$V_S' = V(1+\gamma_S\Delta\theta)$ [γ_S = coefficient of volume expansion for solid vessel]

So the change in volume of liquid relative to vessel.

$V_L' - V_S' = V[\gamma_L - \gamma_S]\Delta\theta$

$\Delta V_{app} = V\gamma_{app}\Delta\theta$ [$\gamma_{app} = \gamma_L - \gamma_S =$ Apparent coefficient of volume expansion for liquid]

$\gamma_L > \gamma_S$	$\gamma_{app} > 0$	ΔV_{app} = positive	Level of liquid in vessel will rise on heating.
$\gamma_L < \gamma_S$	$\gamma_{app} < 0$	ΔV_{app} = negative	Level of liquid in vessel will fall on heating.
$\gamma_L = \gamma_S$	$\gamma_{app} = 0$	$\Delta V_{app} = 0$	level of liquid in vessel will remain same.

12.9 Effect of Temperature on Upthrust

The thrust on V volume of a body in a liquid of density σ is given by

$$Th = V\sigma g$$

Now with rise in temperature by $\Delta\theta$ C°, due to expansion, volume of the body will increase while density of liquid will decrease according to the relations $V' = V(1+\gamma_S\Delta\theta)$ and $\sigma' = \sigma/(1+\gamma_L\Delta\theta)$

So the thrust will become $Th' = V'\sigma' g$

$$\therefore \quad \frac{Th'}{Th} = \frac{V'\sigma' g}{V\sigma g} = \frac{(1+\gamma_S\Delta\theta)}{(1+\gamma_L\Delta\theta)}$$

and apparent weight of the body W_{app} = Actual weight – Thrust

As $\gamma_S < \gamma_L \therefore Th' < Th$ with rise in temperature thrust also decreases and apparent weight of body increases.

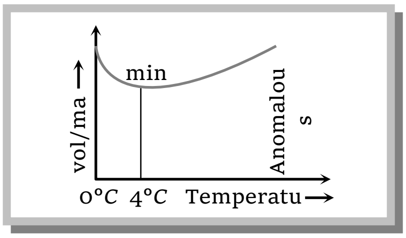
12.10 Anomalous Expansion of Water

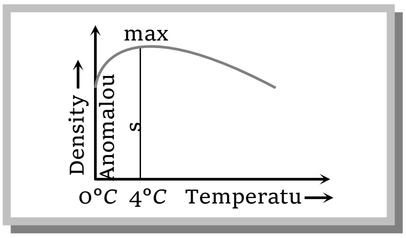
(1) Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°*C*. In the range 0°*C* to 4°*C*, water contracts on heating and expands on cooling, *i.e.* γ is negative. This behaviour of water in the range from 0°*C* to 4°*C* is called anomalous expansion.

(2) The anomalous behaviour of water arises due to the fact that water has three types of molecules, *viz.*, $H_2O, (H_2O)_2$ and $(H_2O)_3$ having different volume per unit mass and at different temperatures their properties in water are different.

(3) At 4°*C*, density of water is maximum while its specific volume is minimum.

During winter when the water at the surface of a lake cools below 4°*C* by cool air, it expands and becomes lighter than water below. Therefore the water cooled below 4°*C* stays on the surface and freezes when the temperature of surroundings falls below 0°*C*. Thus the lake freezes first at the surface and water in contact with ice has temperature 0°*C* while at the bottom of the lake 4°*C* [as density of water at 4°*C* is maximum] and fish and other aquatic animals remain alive in this water.





Sample problems based on Thermal expansion of liquid

***Problem* 11.** A glass flask of volume one *litre* at 0°*C* is filled, level full of mercury at this temperature. The flask and mercury are now heated to 100°*C*. How much mercury will spill out, if coefficient of volume expansion of mercury is $1.82 \times 10^{-4}/°C$ and linear expansion of glass is $0.1 \times 10^{-4}/°C$ respectively **[MNR 1994; CEEE 1994]**

(a) 21.2 *cc* (b) 15.2 *cc* (c) 1.52 *cc* (d) 2.12 *cc*

Solution : (c) Due to volume expansion of both liquid and vessel, the change in volume of liquid relative to container is given by $\Delta V = V[\gamma_L - \gamma_S]\Delta\theta$

Given *V* = 1000 *cc*, $\alpha_g = 0.1\times10^{-4}/°C$ $\therefore \gamma_g = 3\alpha_g = 3 \times 0.1 \times 10^{-4}/°C = 0.3 \times 10^{-4}/°C$

$\therefore \Delta V = 1000\,[1.82 \times 10^{-4} - 0.3 \times 10^{-4}] \times 100 = 15.2\ cc$

***Problem* 12.** Liquid is filled in a flask up to a certain point. When the flask is heated, the level of the liquid

(a) Immediately starts increasing (b) Initially falls and then rises

(c) Rises abruptly (d) Falls abruptly

Solution : (b) Since both the liquid and the flask undergoes volume expansion and the flask expands first therefore the level of the liquid initially falls and then rises.

***Problem* 13.** The absolute coefficient of expansion of a liquid is 7 times that the volume coefficient of expansion of the vessel. Then the ratio of absolute and apparent expansion of the liquid is

(a) $\frac{1}{7}$ (b) $\frac{7}{6}$ (c) $\frac{6}{7}$ (d) None of these

Solution : (b) Apparent coefficient of Volume expansion $\gamma_{app.} = \gamma_L - \gamma_s = 7\gamma_s - \gamma_s = 6\gamma_s$ (given $\gamma_L = 7\gamma_s$)

Ratio of absolute and apparent expansion of liquid $\frac{\gamma_L}{\gamma_{app.}} = \frac{7\gamma_s}{6\gamma_s} = \frac{7}{6}$.

Problem **14.** In cold countries, water pipes sometimes burst, because

(a) Pipe contracts (b) Water expands on freezing

(c) When water freezes, pressure increases (d) When water freezes, it takes heat from pipes

Solution : (b) In anomalous expansion, water contracts on heating and expands on cooling in the range $0°C$ to $4°C$. Therefore water pipes sometimes burst, in cold countries.

Problem **15.** A solid whose volume does not change with temperature floats in a liquid. For two different temperatures t_1 and t_2 of the liquid, fractions f_1 and f_2 of the volume of the solid remain submerged in the liquid. The coefficient of volume expansion of the liquid is equal to

(a) $\frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$ (b) $\frac{f_1 - f_2}{f_1 t_1 - f_2 t_2}$ (c) $\frac{f_1 + f_2}{f_2 t_1 + f_1 t_2}$ (d) $\frac{f_1 + f_2}{f_1 t_1 + f_2 t_2}$

Solution : (a) As with the rise in temperature, the liquid undergoes volume expansion therefore the fraction of solid submerged in liquid increases.

Fraction of solid submerged at $t_1 °C = f_1$ = Volume of displaced liquid $= V_0(1 + \gamma t_1)$ .....(i)

and fraction of solid submerged at $t_2 °C = f_2$ = Volume of displaced liquid $= V_0(1 + \gamma t_2)$ .....(ii)

From (i) and (ii) $\frac{f_1}{f_2} = \frac{1 + \gamma t_1}{1 + \gamma t_2} \Rightarrow \gamma = \frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$

12.11 Expansion of Gases

Gases have no definite shape, therefore gases have only volume expansion. Since the expansion of container is negligible in comparison to the gases, therefore gases have only real expansion.

Coefficient of volume expansion : At constant pressure, the unit volume of a given mass of a gas, increases with $1°C$ rise of temperature, is called coefficient of volume expansion.

$$\alpha = \frac{\Delta V}{V} \times \frac{1}{\Delta T} \quad \therefore \text{ Final volume } V' = V(1 + \alpha \Delta T)$$

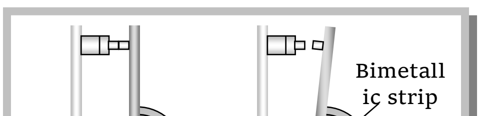
Coefficient of pressure expansion : $\beta = \frac{\Delta P}{P} \times \frac{1}{\Delta T}$ $\therefore$ Final pressure $P' = P(1 + \beta \Delta T)$

For an ideal gas, coefficient of volume expansion is equal to the coefficient of pressure expansion.

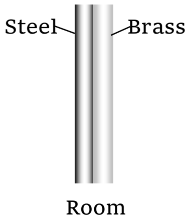
i.e. $$\alpha = \beta = \frac{1}{273} °C^{-1}$$

12.12 Application of Thermal Expansion

(1) **Bi-metallic strip :** Two strips of equal lengths but of different materials (different coefficient of linear expansion) when join together, it is called "bi-metallic strip", and can be



used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metal. The strip will bend with metal of greater α on outer side *i.e.* convex side.



(2) **Effect of temperature on the time period of a simple pendulum :** A pendulum clock keeps proper time at temperature θ. If temperature is increased to $\theta'(>\theta)$ then due to linear expansion, length of pendulum and hence its time period will increase.

Time period $T = 2\pi\sqrt{\frac{L}{g}}$ $\Rightarrow \frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L(1+\alpha\,\Delta\theta)}{L}} = \sqrt{(1+\alpha\,\Delta\theta)}$

$$T' = T\left(1+\frac{1}{2}\alpha\,\Delta\theta\right) = T + \frac{1}{2}\alpha\;\Delta\theta\;T \quad \text{or} \quad \frac{T'-T}{T} = \frac{1}{2}\alpha\,\Delta\theta$$

$\therefore$ $$\frac{\Delta T}{T} = \frac{1}{2}\alpha\,\Delta\theta$$

(i) Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time.

Loss of time in a time period $\Delta T = \frac{1}{2}\alpha\;\Delta\theta\;T$

$\therefore$ Loss of time in any given time interval t can be given by $\Delta t = \frac{1}{2}\alpha\;\Delta\theta\;t$.

(ii) The clock will lose time *i.e.* will become slow if $\theta' > \theta$ (in summer)

and will gain time *i.e.* will become fast if $\theta' < \theta$ (in winter).

(iii) The gain or loss in time is independent of time period T and depends on the time interval t.

(iv) Time lost by the clock in a day (t = 86400 *sec*)

$$\Delta t = \frac{1}{2}\alpha\;\Delta\theta\;t = \frac{1}{2}\alpha\;\Delta\theta\,(86400) = 43200\;\alpha\;\Delta\theta\;sec$$

(v) Since coefficient of linear expansion (α) is very small for invar, hence pendulums are made of invar to show the correct time in all seasons.

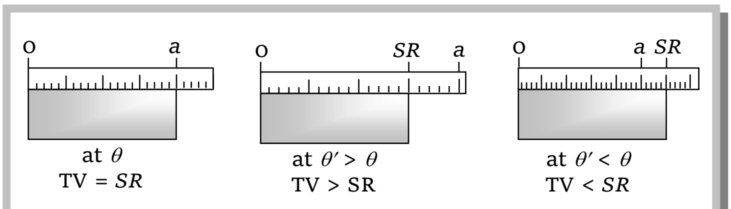
(3) **Thermal stress in a rigidly fixed rod :** When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports. If the change in temperature of a rod of length L is $\Delta\theta$ then

Thermal strain $= \frac{\Delta L}{L} = \alpha \Delta \theta$ $\left[\text{As } \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta \theta}\right]$

So Thermal stress $= Y \alpha \Delta \theta$ $\left[\text{As } Y = \frac{\text{stress}}{\text{strain}}\right]$

or Force on the supports $F = YA\,\alpha\,\Delta\theta$

(4) **Error in scale reading due to expansion or contraction :** If a scale gives correct reading at temperature θ, at temperature $\theta'(>\theta)$ due to linear expansion of scale, the scale will expand and scale reading will be lesser than true value so that,

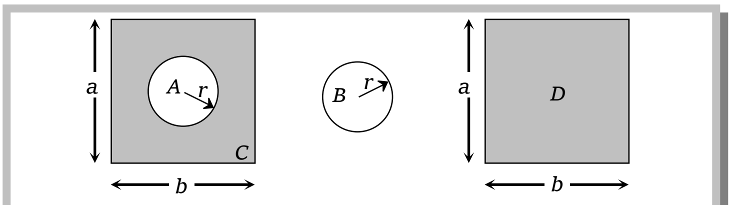


True value = Scale reading $[1 + \alpha(\theta' - \theta)]$

i.e. $\text{TV} = SR\,[1 + \alpha\,\Delta\theta]$ with $\Delta\theta = (\theta' - \theta)$

However, if $\theta' < \theta$, due to contractions of scale, scale reading will be more than true value, so true value will be lesser than scale reading and will still be given by equation with $\Delta\theta = (\theta' - \theta)$ negative.

(5) **Expansion of cavity :** Thermal expansion of an isotropic object may be imagined as a photographic enlargement. So if there is a hole A in a plate C (or cavity A inside a body C), the area of hole (or volume of cavity) will increase when body expands on heating, just as if the hole (or cavity) were solid B of the same material. Also the expansion of area (or volume) of the body C will be independent of shape and size of hole (or cavity), *i.e.*, will be equal to that of D.



Expansion of A = Expansion of Expansion of C = Expansion of

Note : ❑ A solid and hollow sphere of same radius and material, heated to the same temperature then expansion of both will be equal because thermal expansion of isotropic solids is similar to true photographic enlargement. It means the expansion of cavity is same as if it has been a solid body of the same material. But if same heat is given to the two spheres, due to lesser mass, rise in temperature of hollow sphere will be more $\left\{\text{As}\left(\Delta\theta = \frac{a}{mc}\right)\right\}$. Hence its expansion will be more.

(6) **Practical application**

(i) When rails are laid down on the ground, space is left between the ends of two rails.

(ii) The transmission cable are not tightly fixed to the poles.

(iii) Pendulum of wall clock and balance wheel of wrist watch are made of invar (an alloy which have very low value of coefficient of expansion).

(iv) Test tubes, beakers and crucibles are made of pyrex-glass or silica because they have very low value of coefficient of linear expansion.

(v) The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel.

(vi) A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

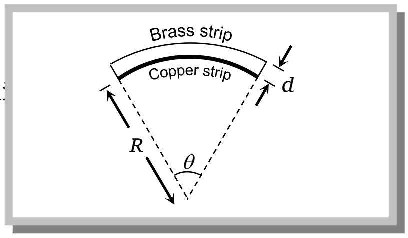
Sample problems based on Application of thermal expansion

***Problem* 16.** A bimetallic strip is formed out of two identical strips, one of copper and other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is **[IIT-JEE (Screening) 1999]**

(a) Proportional to ΔT
(b) Inversely proportional to ΔT
(c) Proportional to $|\alpha_B - \alpha_C|$
(d) Inversely proportional to $|\alpha_B - \alpha_C|$

Solution : (b, d) On heating, the strip undergoes linear expansion

So after expansion length of brass strip $L_B = L_0(1 + \alpha_B \Delta T)$ and length of copper strip $L_C = L_0(1 + \alpha_C \Delta T)$



From the figure $L_B = (R + d)\theta$ ......(i)

and $L_c = R\theta$ ......(ii) [As angle = Arc/Rad

Dividing (i) by (ii) $\frac{R+d}{R} = \frac{L_B}{L_C} = \frac{1+\alpha_B\Delta T}{1+\alpha_C\Delta T}$

$\Rightarrow 1 + \frac{d}{R} = (1+\alpha_B\Delta T)(1+\alpha_C\Delta T)^{-1} = (1+\alpha_B\Delta T)(1-\alpha_C\Delta T) = 1 + (\alpha_B - \alpha_C)\Delta T$

$\Rightarrow \frac{d}{R} = (\alpha_B - \alpha_C)\Delta T$ or $R = \frac{d}{(\alpha_B - \alpha_C)\Delta T}$ [Using Binomial theorem and neglecting higher terms]

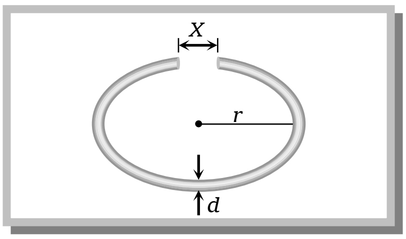
So we can say $R \propto \frac{1}{(\alpha_B - \alpha_C)}$ and $R \propto \frac{1}{\Delta T}$

***Problem* 17.** Two metal strips that constitute a thermostat must necessarily differ in their

(a) Mass
(b) Length
(c) Resistivity
(d) Coefficient of linear expansion

Solution : (d) Thermostat is used in electric apparatus like refrigerator, Iron *etc* for automatic cut off. Therefore for metallic strips to bend on heating their coefficient of linear expansion should be different.

***Problem* 18.** A cylindrical metal rod of length L_0 is shaped into a ring with a small gap as shown. On heating the system



(a) x decreases, r and d increase

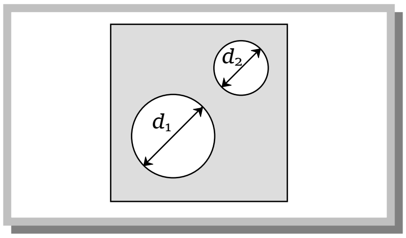
(b) x and r increase, d decreases

(c) x, r and d all increase

(d) Data insufficient to arrive at a conclusion

Solution : (c) On heating the system; x, r, d all increases, since the expansion of isotropic solids is similar to true photographic enlargement

Problem 19. Two holes of unequal diameters d_1 and d_2 $(d_1 > d_2)$ are cut in a metal sheet. If the sheet is heated



(a) Both d_1 and d_2 will decrease

(b) Both d_1 and d_2 will increase

(c) d_1 will increase, d_2 will decrease

(d) d_1 will decrease, d_2 will increase

Solution : (b) If the sheet is heated then both d_1 and d_2 will increase since the thermal expansion of isotropic solid is similar to true photographic enlargement.

Problem 20. An iron tyre is to be fitted onto a wooden wheel 1.0 m in diameter. The diameter of the tyre is 6 mm smaller than that of wheel. The tyre should be heated so that its temperature increases by a minimum of

(Coefficient of volume expansion of iron is $3.6 \times 10^{-5}/°C$)

(a) 167°C (b) 334°C (c) 500°C (d) 1000°C

Solution : (c) Initial diameter of tyre = (1000 – 6) mm = 994 mm, so initial radius of tyre $R = \frac{994}{2} = 497\ mm$

and change in diameter ΔD = 6 mm so $\Delta R = \frac{6}{2} = 3\,mm$

After increasing temperature by ΔT tyre will fit onto wheel

Increment in the length (circumference) of the iron tyre

$$\Delta L = L \times \alpha \times \Delta T = L \times \frac{\gamma}{3} \times \Delta T \qquad \left[\text{As } \alpha = \frac{\gamma}{3}\right]$$

$$\Rightarrow \quad 2\pi\,\Delta R = 2\pi\,R\left(\frac{\gamma}{3}\right)\Delta T \quad \Rightarrow \quad \Delta T = \frac{3}{\gamma}\frac{\Delta R}{R} = \frac{3 \times 3}{3.6 \times 10^{-5} \times 497}$$ [As ΔR = 3 mm and R = 497 mm]

$$\Rightarrow \quad \Delta T = 500^o\,C$$

Problem 21. A clock with a metal pendulum beating seconds keeps correct time at 0°C. If it loses 12.5 seconds a day at 25°C, the coefficient of linear expansion of metal of pendulum is

(a) $\frac{1}{86400}$ per ^oC (b) $\frac{1}{43200}$ per °C (c) $\frac{1}{14400}$ per °C (d) $\frac{1}{28800}$ per °C

Solution : (a) Loss of time due to heating a pendulum is given as

$$\Delta T = \frac{1}{2}\alpha\Delta\theta T \quad \Rightarrow 12.5 = \frac{1}{2} \times \alpha \times (25 - 0)°C \times 86400 \quad \Rightarrow \alpha = \frac{1}{86400}\ \text{per °C}$$

Problem **22.** A wire of length L_0 is supplied heat to raise its temperature by T. If γ is the coefficient of volume expansion of the wire and Y is the Young's modulus of the wire then the energy density stored in the wire is

(a) $\frac{1}{2}\gamma^2T^2Y$ (b) $\frac{1}{3}\gamma^2T^2Y^3$ (c) $\frac{1}{18}\frac{\gamma^2T^2}{Y}$ (d) $\frac{1}{18}\gamma^2T^2Y$

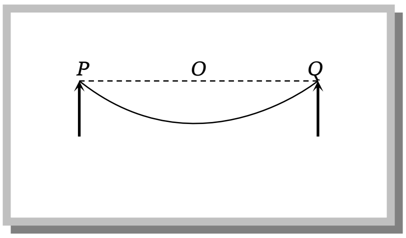
Solution : (d) Due to heating the length of the wire increases. ∴ Longitudinal strain is produced ⇒ $\frac{\Delta L}{L} = \alpha \times \Delta T$

Elastic potential energy per unit volume $E = \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} \times Y \times (\text{Strain})^2$

$$\Rightarrow \quad E = \frac{1}{2} \times Y \times \left(\frac{\Delta L}{L}\right)^2 = \frac{1}{2} \times Y \times \alpha^2 \times \Delta T^2$$

or $E = \frac{1}{2} \times Y \times \left(\frac{\gamma}{3}\right)^2 \times T^2 = \frac{1}{18}\gamma^2 YT^2$ [As $\gamma = 3\alpha$ and $\Delta T = T$ (given)]

Problem **23.** Span of a bridge is 2.4 *km*. At 30°*C* a cable along the span sags by 0.5 *km*. Taking $\alpha = 12 \times 10^{-6}\ per\ ^oC$, change in length of cable for a change in temperature from 10°*C* to 42°*C* is



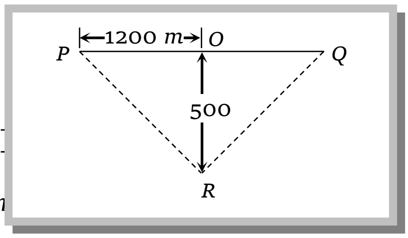
(a) 9.9 *m*

(b) 0.099 *m*

(c) 0.99 *m*

(d) 0.4 *km*

Solution : (c) Span of bridge = 2400 *m* and Bridge sags by 500 *m* at 30°



From the figure $L_{PRQ} = 2\sqrt{1200^2 + 500^2} = 2600\ m$

But $L = L_0(1 + \alpha\,\Delta t)$ [Due to linear expansion]

⇒ $2600 = L_0(1 + 12 \times 10^{-6} \times 30)$ ∴ Length of the cable $L_0 = 2599\ m$

Now change in length of cable due to change in temperature from 10°*C* to 42°*C*

$$\Delta L = 2599 \times 12 \times 10^{-6} \times (42 - 10) = 0.99m$$

12.13 Thermal Capacity and Water Equivalent

(1) **Thermal capacity :** It is defined as the amount of heat required to raise the temperature of the whole body (mass *m*) through 0°*C* or 1*K*.

Thermal capacity $= mc = \mu C = \frac{Q}{\Delta T}$

The value of thermal capacity of a body depends upon the nature of the body and its mass.

Dimension : $[ML^2T^{-2}\theta^{-1}]$, Unit : *cal/°C* (practical) *Joule/k* (S.I.)

(2) **Water Equivalent :** Water equivalent of a body is defined as the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by *W*.

If m = Mass of the body, c = Specific heat of body, ΔT = Rise in temperature.

Then heat given to body $\Delta Q = mc\Delta T$ ..... (i)

If same amount of heat is given to W *gm* of water and its temperature also rises by ΔT

Then heat given to water $\Delta Q = W \times 1 \times \Delta T$ [As $c_{\text{water}} = 1$] ..... (ii)

From equation (i) and (ii) $\Delta Q = mc\Delta T = W \times 1 \times \Delta T$

$\therefore$ Water equivalent (W) = mc *gm*

Unit : *Kg* (S.I.) Dimension : $[ML^0T^0]$

Note :
- Unit of thermal capacity is *J/kg* while unit of water equivalent is *kg*.
- Thermal capacity of the body and its water equivalent are numerically equal.
- If thermal capacity of a body is expressed in terms of mass of water it is called water-equivalent of the body.

12.14 Specific Heat

(1) **Gram specific heat :** When heat is given to a body and its temperature increases, the heat required to raise the temperature of unit mass of a body through 1°*C* (or *K*) is called specific heat of the material of the body.

If Q heat changes the temperature of mass m by ΔT

Specific heat $c = \dfrac{Q}{m\Delta T}$.

Units : *Calorie/gm* × °*C* (practical), *J/kg* × *K* (S.I.) Dimension : $[L^2T^{-2}\theta^{-1}]$

(2) **Molar specific heat :** Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one gram mole of the substance through a unit degree it is represented by (capital) *C*.

By definition, one mole of any substance is a quantity of the substance, whose mass M grams is numerically equal to the molecular mass M.

$\therefore$ Molar specific heat $= M \times \text{Gram specific heat}$

or $C = M\,c$

$$C = M\frac{Q}{m\Delta T} = \frac{1}{\mu}\frac{Q}{\Delta T} \qquad \left[\text{As } c = \frac{Q}{m\Delta T} \text{ and } \mu = \frac{m}{M}\right]$$

$$\therefore \quad C = \frac{Q}{\mu\Delta T}$$

Units : *calorie/mole* × °*C* (practical); *J/mole* × *kelvin* (S.I.) Dimension : $[ML^2T^{-2}\theta^{-1}\mu^{-1}]$

Important points

(1) Specific heat for hydrogen is maximum $(3.5\ cal/gm \times {}^oC)$ and for water, it is $1 cal/gm \times °C$.

For all other substances, the specific heat is less than $1 cal/gm \times °C$ and it is minimum for radon and actinium $(\simeq 0.022\ cal/gm \times °C)$.

(2) Specific heat of a substance also depends on the state of the substance *i.e.* solid, liquid or gas.

For example, $c_{\text{ice}} = 0.5\, cal/gm \times °C$ (Solid), $c_{\text{water}} = 1\, cal/gm \times °C$ (Liquid) and $c_{\text{steam}} = 0.47\ cal/gm \times °C$ (Gas)

(3) The specific heat of a substance when it melts or boils at constant temperature is infinite.

As $C = \dfrac{Q}{m\Delta T} = \dfrac{Q}{m \times 0} = \infty$ [As ΔT = 0]

(4) The specific heat of a substance when it undergoes adiabatic changes is zero.

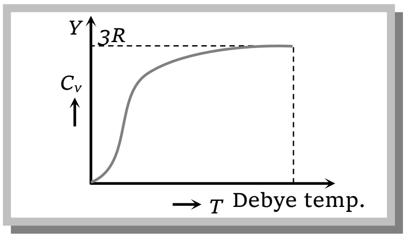
As $C = \dfrac{Q}{m\Delta T} = \dfrac{0}{m\Delta T} = 0$ [As Q = 0]

(5) Specific heat of a substance can also be negative. Negative specific heat means that in order to raise the temperature, a certain quantity of heat is to be withdrawn from the body.

Example. Specific heat of saturated vapours.

12.15 Specific Heat of Solids

When a solid is heated through a small range of temperature, its volume remains more or less constant. Therefore specific heat of a solid may be called its specific heat at constant volume C_v.



From the graph it is clear that at T = 0, C_v tends to zero

With rise in temperature, C_v increases and becomes constant = $3R$

= 6 *cal/mole* × *kelvin* = 25 *J/mole* × *kelvin*

at some particular temperature (Debye Temperature)

For most of the solids, Debye temperature is close to room temperature.

(1) **Specific heat of some solids at room temperature and atmospheric pressure**

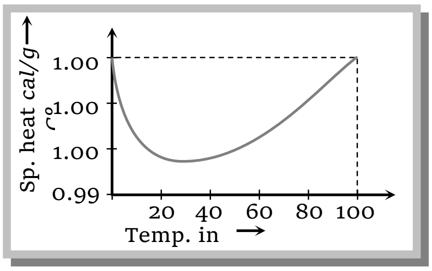
Substance	**Specific heat (J-kg^{-1} K^{-1})**	**Molar specific heat (J-g $mole^{-1}$ K^{-1})**
Aluminium	900.0	24.4
Copper	386.4	24.5
Silver	236.1	25.5
Lead	127.7	26.5

Tungsten	134.4	24.9

(2) **Dulong and Petit law :** Average molar specific heat of all metals at room temperature is constant, being nearly equal to $3R = 6\ cal.\ mole^{-1}\ K^{-1} = 25\ J\ mole^{-1}\ K^{-1}$, where R is gas constant for one mole of the gas. This statement is known as Dulong and Petit law.

12.16 Specific Heat of Water

The variation of specific heat with temperature for water is shown in the figure. Usually this temperature dependence of specific heat is neglected.



From the graph :

Temperature (°C)	0	15	35	50	100
Specific heat (cal/ gm × °C)	1.008	1.000	0.997	0.998	1.006

As specific heat of water is very large; by absorbing or releasing large amount of heat its temperature changes by small amount. This is why, it is used in hot water bottles or as coolant in radiators.

Note : ❑ When specific heats are measured, the values obtained are also found to depend on the conditions of the experiment. In general measurements made at constant pressure are different from those at constant volume. For solids and liquids this difference is very small and usually neglected. The specific heat of gases are quite different under constant pressure condition (c_P) and constant volume (c_V). In the chapter "Kinetic theory of gases" we have discussed this topic in detail.

Sample problems based on Specific heat, thermal capacity and water equivalent

Problem 24. Two spheres made of same substance have diameters in the ratio 1 : 2. Their thermal capacities are in the ratio of

(a) 1 : 2 (b) 1 : 8 (c) 1 : 4 (d) 2 : 1

Solution : (b) Thermal capacity = Mass × Specific heat

Due to same material both spheres will have same specific heat

$$\therefore \text{ Ratio of thermal capacity } = \frac{m_1}{m_2} = \frac{V_1\rho}{V_2\rho} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{2}\right)^3 = 1:8$$

Problem 25. When 300 J of heat is added to 25 gm of sample of a material its temperature rises from 25°C to 45°C. the thermal capacity of the sample and specific heat of the material are respectively given by

(a) 15 $J/°C$, 600 J/kg °C (b) 600 $J/°C$, 15 $J°/kg$ °C (c) 150 $J/°C$, 60 J/kg °C(d)

Solution : (a) Thermal capacity = $mc = \frac{Q}{\Delta T} = \frac{300}{45-25} = \frac{300}{20} = 15\ J/°C$

Specific heat = $\frac{\text{Thermal capacity}}{\text{Mass}} = \frac{15}{25\times 10^{-3}} = 600\ J/kg°C$

***Problem* 26.** The specific heat of a substance varies with temperature $t(°C)$ as

$$c = 0.20 + 0.14\,t + 0.023\ t^2\ (cal/gm\,°C)$$

The heat required to raise the temperature of 2 *gm* of substance from 5°*C* to 15°*C* will be

(a) 24 *calorie* (b) 56 *calorie* (c) 82 *calorie* (d) 100 *calorie*

Solution : (c) Heat required to raise the temperature of *m gm* of substance by dT is given as

$$dQ = mc\ dT \Rightarrow Q = \int mc\ dT$$

∴ To raise the temperature of 2 *gm* of substance from 5°*C* to 15°*C* is

$$Q = \int_5^{15} 2\times(0.2 + 0.14\,t + 0.023\,t^2)dT = 2\times\left[0.2t + \frac{0.14\,t^2}{2} + \frac{0.023\,t^3}{3}\right]_5^{15} = 82\ calorie$$

12.17 Latent Heat

(1) When a substance changes from one state to another state (say from solid to liquid or liquid to gas or from liquid to solid or gas to liquid) then energy is either absorbed or liberated. This heat energy is called latent heat.

(2) No change in temperature is involved when the substance changes its state. That is, phase transformation is an isothermal change. Ice at 0°*C* melts into water at 0°*C*. Water at 100°*C* boils to form steam at 100°*C*.

(3) The amount of heat required to change the state of the mass *m* of the substance is written as : $\Delta Q = mL$, where *L* is the latent heat. Latent heat is also called as Heat of Transformation.

(4) Unit : *cal/gm* or *J/kg* and Dimension : $[L^2T^{-2}]$

(5) Any material has two types of latent heats

(i) Latent heat of fusion : The latent heat of fusion is the heat energy required to change 1 *kg* of the material in its solid state at its melting point to 1 *kg* of the material in its liquid state. It is also the amount of heat energy released when at melting point 1 *kg* of liquid changes to 1 *kg* of solid. For water at its normal freezing temperature or melting point (0°*C*), the latent heat of fusion (or latent heat of ice) is

$$L_F = L_{\text{ice}} \approx 80\ cal/g \approx 60\ kJ/mol \approx 336\ kilo\ joule/kg\ .$$

(ii) Latent heat of vaporisation : The latent heat of vaporisation is the heat energy required to change 1 *kg* of the material in its liquid state at its boiling point to 1 *kg* of the material in its gaseous state. It is also the amount of heat energy released when 1 *kg* of vapour changes into 1 *kg* of liquid. For water at its normal boiling point or condensation temperature (100°*C*), the latent heat of vaporisation (latent heat of steam) is

$$L_V = L_{\text{steam}} \approx 540\ cal/g \approx 40.8\ kJ/mol \approx 2260\ kilo\ joule/kg$$

(6) In the process of melting or boiling, heat supplied is used to increase the internal potential energy of the substance and also in doing work against external pressure while internal kinetic energy remains constant. This is the reason that internal energy of steam at 100°*C* is more than that of water at 100°*C*.

(7) It is more painful to get burnt by steam rather than by boiling water at same temperature. This is so because when steam at 100°*C* gets converted to water at 100°*C*, then it gives out 536 *calories* of heat. So, it is clear that steam at 100°*C* has more heat than water at 100°*C* (*i.e.*, boiling of water).

(8) In case of change of state if the molecules come closer, energy is released and if the molecules move apart, energy is absorbed.

(9) Latent heat of vaporisation is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, there is a large increase in volume. Hence more amount of heat is required. But when a solid gets converted to a liquid, then the increase in volume is negligible. Hence very less amount of heat is required. So, latent heat of vaporisation is more than the latent heat of fusion.

(10) After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So, in the mountains, when snow falls, one does not feel too cold, but when ice melts, he feels too cold.

(11) There is more shivering effect of ice-cream on teeth as compared to that of water (obtained from ice). This is because, when ice-cream melts down, it absorbs large amount of heat from teeth.

(12) Freezing mixture : If salt is added to ice, then the temperature of mixture drops down to less than 0°*C*. This is so because, some ice melts down to cool the salt to 0°*C*. As a result, salt gets dissolved in the water formed and saturated solution of salt is obtained; but the ice point (freeing point) of the solution formed is always less than that of pure water. So, ice cannot be in the solid state with the salt solution at 0°*C*. The ice which is in contact with the solution, starts melting and it absorbs the required latent heat from the mixture, so the temperature of mixture falls down.

Sample problems based on Latent heat

***Problem* 27.** Work done in converting one gram of ice at – 10°*C* into steam at 100°*C* is

[MP PET /PMT 1988; EAMCET (Med.) 1995; MP PMT 2003]

(a) 3045 *J* (b) 6056 *J* (c) 721 *J* (d) 616 *J*

Solution : (a) Work done in converting 1*gm* of ice at – 10°*C* to steam at 100°*C*

= Heat supplied to raise temperature of 1*gm* of ice from – 10°*C* to 0°*C* $[m \times c_{ice} \times \Delta T]$

+ Heat supplied to convert 1 *gm* ice into water at 0°*C* $[m \times L_{ice}]$

+ Heat supplied to raise temperature of 1*gm* of water from 0°*C* to 100°*C* $[m \times c_{water} \times \Delta T]$

+ Heat supplied to convert 1 *gm* water into steam at 100°*C* $[m \times L_{vapour}]$

$= [m \times c_{ice} \times \Delta T] + [m \times L_{ice}] + [m \times c_{water} \times \Delta T] + [m \times L_{vapour}]$

$= [1 \times 0.5 \times 10] + [1 \times 80] + [1 \times 1 \times 100] + [1 \times 540] = 725 \; calorie = 725 \times 4.2 = 3045 \; J$

***Problem* 28.** 2 *kg* of ice at – 20°*C* is mixed with 5 *kg* of water at 20°*C* in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 *kcal/kg per °C* and 0.5 *kcal/kg/°C* while the latent heat of fusion of ice is 80 *kcal/kg*

[IIT-JEE (Screening) 2003]

(a) 7 *kg* (b) 6 *kg* (c) 4 *kg* (d) 2 *kg*

Solution : (b) Initially ice will absorb heat to raise it's temperature to 0°*C* then it's melting takes place

If m = Initial mass of ice, m' = Mass of ice that melts and m_w = Initial mass of water

By Law of mixture Heat gain by ice = Heat loss by water

$\Rightarrow \quad m \times c \times (20) + m' \times L = m_w c_w [20]$

$\Rightarrow \quad 2 \times 0.5(20) + m' \times 80 = 5 \times 1 \times 20 \Rightarrow m' = 1 kg$

So final mass of water = Initial mass of water + Mass of ice that melts = 5 + 1= 6 *kg*.

***Problem* 29.** If mass energy equivalence is taken into account, when water is cooled to form ice, the mass of water should

[AIEEE 2002]

(a) Increase (b) Remain unchanged

(c) Decrease (d) First increase then decrease

Solution : (b) When water is cooled at 0°*C* to form ice then 80 *calorie/gm* (latent heat) energy is released. Because potential energy of the molecules decreases. Mass will remain constant in the process of freezing of water.

***Problem* 30.** Compared to a burn due to water at 100°*C*, a burn due to steam at 100°*C* is

(a) More dangerous (b) Less dangerous (c) Equally dangerous (d) None of these

Solution : (a) Steam at 100°*C* contains extra 540 *calorie/gm* energy as compare to water at 100°*C*. So it's more dangerous to burn with steam then water.

***Problem* 31.** Latent heat of ice is 80 *calorie/gm*. A man melts 60 *g* of ice by chewing in 1 *minute*. His power is

(a) 4800 *W* (b) 336 *W* (c) 1.33 *W* (d) 0.75 *W*

Solution : (b) Work done by man = Heat absorbed by ice = mL = 60 × 80 = 4800 *calorie* = 20160 *J*

$\therefore$ Power $= \frac{W}{t} = \frac{20160}{60} = 336 W$

12.18 Principle of Caloriemetry

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

Heat lost = Heat gained

i.e. principle of caloriemetry represents the law of conservation of heat energy.

(1) Temperature of mixture (T) is always ≥ lower temperature (T_L) and ≤ higher temperature (T_H), *i.e.*,

$$T_L \le T \le T_H$$

i.e., the temperature of mixture can never be lesser than lower temperatures (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Furthermore usually rise in temperature of one body is not equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

(2) When temperature of a body changes, the body releases heat if its temperature falls and absorbs heat when its temperature rises. The heat released or absorbed by a body of mass m is given by, $Q = mc\,\Delta T$

where c is specific heat of the body and ΔT change in its temperature in ^oC or K.

(3) When state of a body changes, change of state takes place at constant temperature [m.pt. or b.pt.] and heat released or absorbed is given by, $Q = mL$

where L is latent heat. Heat is absorbed if solid converts into liquid (at m.pt.) or liquid converts into vapours (at b.pt.) and is released if liquid converts into solid or vapours converts into liquid.

(4) If two bodies A and B of masses m_1 and m_2, at temperatures T_1 and T_2 $(T_1 > T_2)$ and having gram specific heat c_1 and c_2 when they are placed in contact.

Heat lost by A = Heat gained by B

or $m_1c_1(T_1 - T) = m_2c_2(T - T_2)$ [where T = Temperature of equilibrium]

$$\therefore \quad T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

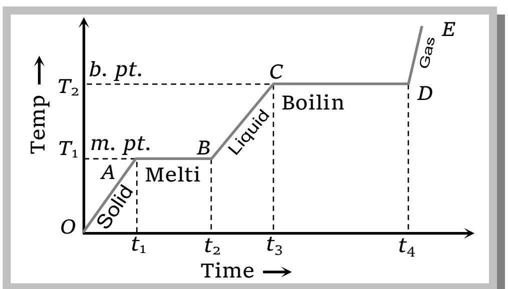
(i) If bodies are of same material $c_1 = c_2$ then $T = \dfrac{m_1T_1 + m_2T_2}{m_1 + m_2}$

(ii) If bodies are of same mass $(m_1 = m_2)$ then $T = \dfrac{T_1c_1 + T_2c_2}{c_1 + c_2}$

(iii) If bodies are of same material and of equal masses $(m_1 = m_2, c_1 = c_2)$ then $T = \dfrac{T_1 + T_2}{2}$

12.19 Heating curve

If to a given mass (m) of a solid, heat is supplied at constant rate P and a graph is plotted between temperature



and time, the graph is as shown in figure and is called heating curve. From this curve it is clear that

(1) In the region *OA* temperature of solid is changing with time so,

$$Q = mc_S \Delta T$$

or $$P\,\Delta t = mc_S \Delta T \qquad [\text{as } Q = P\Delta t]$$

But as $(\Delta T/\Delta t)$ is the slope of temperature-time curve

$$c_S \propto (1/\text{slope of line } OA)$$

i.e. specific heat (or thermal capacity) is inversely proportional to the slope of temperature-time curve.

(2) In the region *AB* temperature is constant, so it represents change of state, *i.e.*, melting of solid with melting point T_1. At *A* melting starts and at *B* all solid is converted into liquid. So between *A* and *B* substance is partly solid and partly liquid. If L_F is the latent heat of fusion.

$$Q = mL_F \text{ or } L_F = \frac{P(t_2 - t_1)}{m} \quad [\text{as } Q = P(t_2 - t_1)]$$

or $$L_F \propto \text{length of line } AB$$

i.e. Latent heat of fusion is proportional to the length of line of zero slope. [In this region specific heat $\propto \frac{1}{\tan 0} = \infty$]

(3) In the region *BC* temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line *BC*

i.e., $$c_L \propto (1/\text{slope of line } BC)$$

(4) In the region *CD* temperature is constant, so it represents the change of state, *i.e.*, boiling with boiling point T_2. At *C* all substance is in liquid state while at *D* in vapour state and between *C* and *D* partly liquid and partly gas. The length of line *CD* is proportional to latent heat of vaporisation

i.e., $L_V \propto$ Length of line *CD* [In this region specific heat $\propto \frac{1}{\tan 0} = \infty$]

(5) The line *DE* represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

Sample problems based on Caloriemetry

***Problem* 32.** 50 *g* of copper is heated to increase its temperature by 10°*C*. If the same quantity of heat is given to 10 *g* of water, the rise in its temperature is (Specific heat of copper $= 420\ Joule\text{-}kg^{-1}\,{}^\circ C^{-1}$) **[EAMCET (Med.) 2000]**

(a) 5°C (b) 6°C (c) 7°C (d) 8°C

Solution : (a) Same amount of heat is supplied to copper and water so $m_c c_c \Delta T_c = m_\omega c_\omega \Delta T_\omega$

$$\Rightarrow \Delta T_\omega = \frac{m_c c_c \Delta T_c}{m_\omega c_\omega} = \frac{50 \times 10^{-3} \times 420 \times 10}{10 \times 10^{-3} \times 4200} = 5°C$$

***Problem* 33.** Two liquids A and B are at 32°C and 24°C. When mixed in equal masses the temperature of the mixture is found to be 28°C. Their specific heats are in the ratio of

(a) 3 : 2 (b) 2 : 3 (c) 1 : 1 (d) 4 : 3

Solution : (c) Heat lost by A = Heat gained by B

$\Rightarrow m_A \times c_A \times (T_A - T) = m_B \times c_B \times (T - T_B)$ Since $m_A = m_B$ and Temperature of the mixture (T) = 28°C

$$\therefore c_A \times (32 - 28) = c_B \times (28 - 24) \Rightarrow \frac{c_A}{c_B} = 1 : 1$$

***Problem* 34.** 22 g of CO_2 at 27°C is mixed with 16g of O_2 at 37°C. The temperature of the mixture is **[CBSE PMT 1**

(a) 27°C (b) 30.5°C (c) 32°C (d) 37°C

Solution : (c) Heat lost by CO_2 = Heat gained by O_2

If μ_1 and μ_2 are the number of moles of carbon di-oxide and oxygen respectively and C_{v_1} and C_{v_2} are the specific heats at constant volume then $\mu_1 C_{v_1} \Delta T_1 = \mu_2 C_{v_2} \Delta T_2$

$\Rightarrow \frac{22}{44} \times 3R \times (T - 27) = \frac{16}{32} \times \frac{5R}{2}(37 - T) \Rightarrow T = 31.5°C \approx 32°C$ (where T is temperature of mixture)

***Problem* 35.** A beaker contains 200 *gm* of water. The heat capacity of the beaker is equal to that of 20 *gm* of water. The initial temperature of water in the beaker is 20°C. If 440 *gm* of hot water at 92°C is poured in it, the final temperature (neglecting radiation loss) will be nearest to

(a) 58°C (b) 68°C (c) 73°C (d) 78°C

Solution : (b) Heat lost by hot water = Heat gained by cold water in beaker + Heat absorbed by beaker

$\Rightarrow$ 440 (92 – T) = 200 × (T – 20) + 20 × (T – 20) $\Rightarrow T$ = 68°C

***Problem* 36.** A liquid of mass m and specific heat c is heated to a temperature 2T. Another liquid of mass m/2 and specific heat 2c is heated to a temperature T. If these two liquids are mixed, the resulting temperature of the mixture is

[EAMCET 1992]

(a) (2/3)T (b) (8/5)T (c) (3/5)T (d) (3/2)T

Solution : (d) Temperature of mixture is given by $T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2} = \frac{m.c.2T + \frac{m}{2}.2.c.T}{m.c. + \frac{m}{2}.2c} = \frac{3}{2}T$

***Problem* 37.** Three liquids with masses m_1, m_2, m_3 are thoroughly mixed. If their specific heats are c_1, c_2, c_3 and their temperatures T_1, T_2, T_3 respectively, then the temperature of the mixture is

(a) $\frac{c_1T_1 + c_2T_2 + c_3T_3}{m_1c_1 + m_2c_2 + m_3c_3}$ (b) $\frac{m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3}{m_1c_1 + m_2c_2 + m_3c_3}$

(c) $\frac{m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3}{m_1T_1 + m_2T_2 + m_3T_3}$ (d) $\frac{m_1T_1 + m_2T_2 + m_3T_3}{c_1T_1 + c_2T_2 + c_3T_3}$

Solution : (b) Let the final temperature be T °C.

Total heat supplied by the three liquids in coming down to 0°C = $m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3$ ..... (i)

Total heat used by three liquids in raising temperature from 0°C to T°C = $m_1c_1T + m_2c_2T + m_3c_3T$ .....(ii)

By equating (i) and (ii) we get $(m_1c_1 + m_2c_2 + m_3c_3)T = m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3$

$\Rightarrow \quad T = \frac{m_1c_1T_1 + m_2c_2T_2 + m_3c_3T_3}{m_1c_1 + m_2c_2 + m_3c_3}$.

***Problem* 38.** In an industrial process 10 *kg* of water per hour is to be heated from 20°C to 80°C. To do this steam at 150°C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°C. how many *kg* of steam is required per hour.

(Specific heat of steam = 1 *calorie* per *gm*°C, Latent heat of vaporisation = 540 *cal*/*gm*)

(a) 1 *gm* (b) 1 *kg* (c) 10 *gm* (d) 10 *kg*

Solution : (b) Heat required by 10 *kg* water to change its temperature from 20°C to 80°C in one hour is

$Q_1 = (mc\Delta T)_{water} = (10\times 10^3)\times 1\times (80-20) = 600\times 10^3$ *calorie*

In condensation (i) Steam release heat when it looses it's temperature from 150°C to 100°C. $[mc_{steam}\Delta T]$

(ii) At 100°C it converts into water and gives the latent heat. $[mL]$

(iii) Water release heat when it looses it's temperature from 100°C to 90°C. $[ms_{water}\Delta T]$

If m *gm* steam condensed per hour, then heat released by steam in converting water of 90°C

$Q_2 = (mc\Delta T)_{steam} + mL_{steam} + (ms\Delta T)_{water} = m[1\times(150-100) + 540 + 1\times(100-90)] = 600\, m$ *calorie*

According to problem $Q_1 = Q_2 \Rightarrow 600\times 10^3$ *cal* = 600 m *cal* $\Rightarrow m = 10^3$ *gm* = 1 *kg*.

***Problem* 39.** A caloriemeter contains 0.2*kg* of water at 30°C. 0.1 *kg* of water at 60°C is added to it, the mixture is well stirred and the resulting temperature is found to be 35°C. The thermal capacity of the caloriemeter is

(a) 6300 J/K (b) 1260 J/K (c) 4200 J/K (d) None of these

Solution : (b) Let X be the thermal capacity of calorimeter and specific heat of water = 4200 J/kg-K

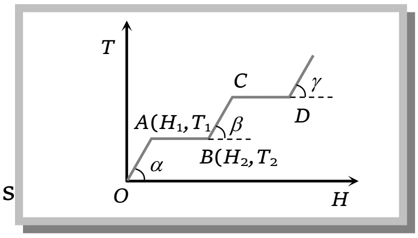
Heat lost by 0.1 *kg* of water = Heat gained by water in calorimeter + Heat gained by calorimeter

$\Rightarrow \quad 0.1 \times 4200 \times (60-35) = 0.2 \times 4200 \times (35-30) + X(35-30)$

$$10500 = 4200 + 5X \Rightarrow X = 1260\ J/K$$

***Problem* 40.** The graph shows the variation of temperature (T) of one *kilogram* of a material with the heat (H) supplied to it. At O, the substance is in the solid state

From the graph, we can conclude that



(a) T_2 is the melting point of the solid

(b) BC represents the change of state from solid to liquid

(c) $(H_2 - H_1)$ represents the latent heat of fusion of the subs

(d) $(H_3 - H_1)$ represents the latent heat of vaporization of the liquid

Solution : (c) Since in the region AB temperature is constant therefore at this temperature phase of the material changes from solid to liquid and $(H_2 - H_1)$ heat will be absorb by the material. This heat is known as the heat of melting of the solid.

Similarly in the region CD temperature is constant therefore at this temperature phase of the material changes from liquid to gas and $(H_4 - H_3)$ heat will be absorb by the material. This heat as known as the heat of vaporisation of the liquid.