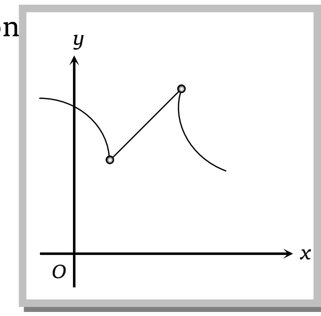
4.3 Maxima and Minima

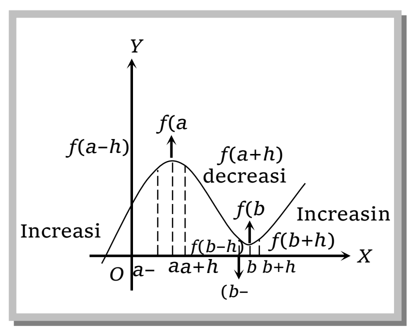
4.3.1 Introduction

In this chapter we shall study those points of the domain of a function where its graph changes its direction from upwards to downwards or from downwards to upwards. At such points the derivative of the function ... arily zero.



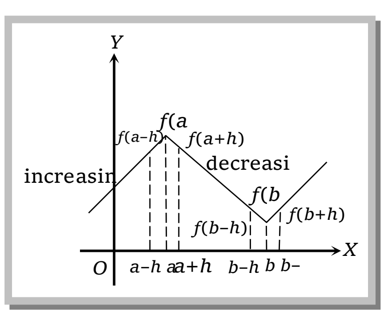
4.3.2 Maximum and Minimum Values of a Function

By the maximum / minimum value of function $f(x)$ we should mean local or regional maximum/minimum and not the greatest / least value attainable by the function. It is also possible in a function that local maximum at one point is smaller than local minimum at another point. Sometimes we use the word extreme for maxima and minima.

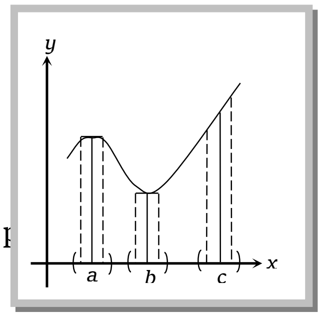


Definition: A function $f(x)$ is said to have a maximum at $x = a$ if $f(a)$ is greatest of all values in the suitably small neighbourhood of a where $x = a$ is an interior point in the domain of $f(x)$. Analytically this means $f(a) \geq f(a+h)$ and $f(a) \geq f(a-h)$ where $h \geq 0$. (very small quantity).

Similarly, a function $y = f(x)$ is said to have a minimum at $x = b$. If $f(b)$ is smallest of all values in the suitably small neighbourhood of b where $x = b$ is an interior point in the domain of $f(x)$. Analytically, $f(b) \leq f(b+h)$ and $f(b) \leq f(b-h)$ where $h \geq 0$. (very small quantity).



Hence we find that,



(i) $x = a$ is a maximum point of $f(x)$ $\begin{cases} f(a) - f(a+h) > 0 \\ f(a) - f(a-h) > 0 \end{cases}$

(ii) $x = b$ is a minimum point of $f(x)$ $\begin{cases} f(b) - f(b+h) < 0 \\ f(b) - f(b-h) < 0 \end{cases}$

(iii) $x = c$ is neither a maximum point nor a minimum p

$\left.\begin{cases} f(c) - f(c+h) \text{ and} \\ f(c) - f(c-h) \end{cases}\right\}$ have opposite signs .

4.3.3 Local Maxima and Local Minima

(1) **Local maximum :** A function $f(x)$ is said to attain a local maximum at $x = a$ if there exists a neighbourhood $(a-\delta, a+\delta)$ of a such that $f(x) < f(a)$ for all $x \in (a-\delta, a+\delta), x \neq a$

or $f(x) - f(a) < 0$ for all $x \in (a-\delta, a+\delta), x \neq a$.

In such a case $f(a)$ is called the local maximum value of $f(x)$ at $x = a$.

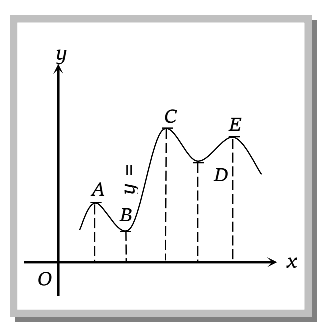
(2) **Local minimum:** A function $f(x)$ is said to attain a local minimum at $x = a$ if there exists a neighbourhood $(a-\delta, a+\delta)$ of a such that

$f(x) > f(a)$ for all $x \in (a-\delta, a+\delta), x \neq a$

or $f(x) - f(a) > 0$ for all $x \in (a-\delta, a+\delta), x \neq a$

The value of function at $x = a$ *i.e.,* $f(a)$ is called the local minimum value of $f(x)$ at $x = a$.

The points at which a function attains either the local maximum values or local minimum values are known as the extreme points or turning points and both local maximum and local minimum values are called the extreme values of $f(x)$. Thus, a function attains an extreme value at $x = a$ if $f(a)$ is either a local maximum value or a local minimum value. Consequently at an extreme point 'a' $f(x) - f(a)$ keeps the same sign for all values of *x* in a deleted *nbd* of *a*.



In fig. we observe that the *x*-coordinates of the points *A, C, E* are points of local maximum and the values at these points *i.e.,* their *y*-coordinates are the local maximum values of $f(x)$. The *x*-coordinates of points *B* and *D* are points of local minimum and their *y*-coordinates are the local minimum values of $f(x)$.

Note : ❑ By a local maximum (or local minimum) value of a function at a point $x = a$ we mean the greatest (or the least) value in the neighbourhood of point $x = a$ and not the absolute maximum (or the absolute minimum). In fact a function may have any number of points of local maximum (or local minimum) and even a local minimum value may be greater than a local maximum value. In fig. the minimum value at D is greater than the maximum value at A. Thus, a local maximum value may not be the greatest value and a local minimum value may not be the least value of the function in its domain.

❑ The maximum and minimum points are also known as extreme points.

❑ A function may have more than one maximum and minimum points.

❑ A maximum value of a function $f(x)$ in an interval $[a, b]$ is not necessarily its greatest value in that interval. Similarly, a minimum value may not be the least value of the function. A minimum value may be greater than some maximum value for a function.

❑ If a continuous function has only one maximum (minimum) point, then at this point function has its greatest (least) value.

❑ Monotonic functions do not have extreme points.

4.3.4 Conditions for Maxima and Minima of a Function

(1) **Necessary condition:** A point $x = a$ is an extreme point of a function $f(x)$ if $f'(a) = 0$, provided $f'(a)$ exists. Thus, if $f'(a)$ exists, then

$$x = a \text{ is an extreme point } \Rightarrow f'(a) = 0$$
$$\text{or}$$
$$f'(a) \neq 0 \Rightarrow x = a \text{ is not an extreme point}$$

But its converse is not true *i.e.,* $f'(a) = 0, x = a$ is not an extreme point.

For example if $f(x) = x^3$, then $f'(0) = 0$ but $x = 0$ is not an extreme point.

(2) **Sufficient condition:**

(i) The value of the function $f(x)$ at $x = a$ is maximum, if $f'(a) = 0$ and $f''(a) < 0$.

(ii) The value of the function $f(x)$ at $x = a$ is minimum if $f'(a) = 0$ and $f''(a) > 0$.

Note : ❑ If $f'(a) = 0$, $f''(a) = 0$, $f'''(a) \neq 0$ then $x = a$ is not an extreme point for the function $f(x)$.

❑ If $f'(a) = 0, f''(a) = 0, f'''(a) = 0$ then the sign of $f^{(iv)}$ (a) will determine the maximum and minimum value of function *i.e.,* $f(x)$ is maximum, if $f^{(iv)}(a) < 0$ and minimum if $f^{(iv)}(a) > 0$.

4.3.5 Working rule for Finding Maxima and Minima

(1) Find the differential coefficient of $f(x)$ with respect to x, *i.e.,* $f'(x)$ and equate it to zero.

(2) Find differential real values of x by solving the equation $f'(x)=0$. Let its roots be *a, b, c*......

(3) Find the value of $f''(x)$ and substitute the value of $a_1, a_2, a_3......$ in it and get the sign of $f''(x)$ for each value of x.

(4) If $f''(a)<0$ then the value of $f(x)$ is maximum at $x=a$ and if $f''(a)>0$ then value of $f(x)$ will be minimum at $x=a$. Similarly by getting the signs of $f''(x)$ at other points *b, c*.....we can find the points of maxima and minima.

Example: 1 What are the minimum and maximum values of the function $x^5-5x^4+5x^3-10$ **[DCE 1999; Rajasthan PET 1995]**

(a) $-37, -9$ (b) 10, 0

(c) It has 2 minimum and 1 maximum values (d) It has 2 maximum and 1 minimum values

Solution: (a) $y=x^5-5x^4+5x^3-10$

$\therefore \frac{dy}{dx}=5x^4-20x^3+15x^2=5x^2(x^2-4x+3)=5x^2(x-3)(x-1)$

$\frac{dy}{dx}=0$, gives $x=0,1,3$ ......(i)

Now, $\frac{d^2y}{dx^2}=20x^3-60x^2+30x=10x(2x^2-6x+3)$ and $\frac{d^3y}{dx^3}=10(6x^2-12x+3)$

For $x=0: \frac{dy}{dx}=0, \frac{d^2y}{dx^2}=0, \frac{d^3y}{dx^3}\neq 0$, $\therefore$ Neither minimum nor maximum

For $x=1$, $\frac{d^2y}{dx^2}=-10$ =negative, $\therefore$ Maximum value $y_{\text{max.}}=-9$

For $x=3$, $\frac{d^2y}{dx^2}=90$ =positive, $\therefore$ Minimum value $y_{\text{min.}}=-37$.

Example: 2 The maximum value of $\sin x(1+\cos x)$ will be at **[UPSEAT 1999]**

(a) $x=\frac{\pi}{2}$ (b) $x=\frac{\pi}{6}$ (c) $x=\frac{\pi}{3}$ (d) $x=\pi$

Solution: (c) $y=\sin x(1+\cos x)=\sin x+\frac{1}{2}\sin 2x$

$\therefore \frac{dy}{dx}=\cos x+\cos 2x$ and $\frac{d^2y}{dx^2}=-\sin x-2\sin 2x$

On putting $\frac{dy}{dx}=0, \cos x+\cos 2x=0 \Rightarrow \cos x=-\cos 2x=\cos(\pi-2x) \Rightarrow x=\pi-2x$

$\therefore x=\frac{\pi}{3}$, $\therefore \left(\frac{d^2y}{dx^2}\right)_{x=\pi/3}=-\sin\left(\frac{1}{3}\pi\right)-2\sin\left(\frac{2}{3}\pi\right)=\frac{-\sqrt{3}}{2}-2.\frac{\sqrt{3}}{2}=\frac{-3\sqrt{3}}{2}$ which is negative.

$\therefore$ at $x=\frac{\pi}{3}$ the function is maximum.

Example: 3 If $y=a\log x+bx^2+x$ has its extremum value at $x=1$ and $x=2$, then $(a,b)=$

(a) $\left(1, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, 2\right)$ (c) $\left(2, -\frac{1}{2}\right)$ (d) $\left(-\frac{2}{3}, -\frac{1}{6}\right)$

Solution: (d) $\frac{dy}{dx} = \frac{a}{x} + 2bx + 1 \Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = a + 2b + 1 = 0 \Rightarrow a = -2b - 1$

and $\left(\frac{dy}{dx}\right)_{x=2} = \frac{a}{2} + 4b + 1 = 0 \Rightarrow \frac{-2b-1}{2} + 4b + 1 = 0 \Rightarrow -b + 4b + \frac{1}{2} = 0 \Rightarrow 3b = \frac{-1}{2} \Rightarrow b = \frac{-1}{6}$ and $a = \frac{1}{3} - 1 = \frac{-2}{3}$.

Example: 4 Maximum value of $\left(\frac{1}{x}\right)^x$ is **[DCE 1999; Karnataka CET 1999; UPSEAT 2003]**

(a) $(e)^e$ (b) $(e)^{1/e}$ (c) $(e)^{-e}$ (d) $\left(\frac{1}{e}\right)^e$

Solution: (b) $f(x) = \left(\frac{1}{x}\right)^x \Rightarrow f'(x) = \left(\frac{1}{x}\right)^x \left(\log\frac{1}{x} - 1\right)$

$f'(x) = 0 \Rightarrow \log\frac{1}{x} = 1 = \log e \Rightarrow \frac{1}{x} = e \Rightarrow x = \frac{1}{e}$. Therefore, maximum value of function is $e^{1/e}$.

Example: 5 Maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is

(a) 0 (b) 12 (c) 16 (d) 32

Solution: (b) $y = f(x) = -x^3 + 3x^2 + 9x - 27$

The slope of this curve $f'(x) = -3x^2 + 6x + 9$

Let $g(x) = f'(x) = -3x^2 + 6x + 9$

Differentiate with respect to *x*, $g'(x) = -6x + 6$

Put $g'(x) = 0 \Rightarrow x = 1$

Now, $g''(x) = -6 < 0$ and hence at $x = 1, g(x)$

(Slope) will have maximum value.

$\therefore [g(1)]_{\text{max.}} = -3 \times 1 + 6 + 9 = 12$.

Example: 6 The function $f(x) = \int_1^x t(e^t - 1)(t-1)(t-2)^3(t-3)^5 dt$ has a local minimum at $x =$ **[IIT1999]**

(a) 0 (b) 1 (c) 2 (d) 3

Solution: (b, d) $f(x) = \int_{-1}^x t(e^t - 1)(t-1)(t-2)^3(t-3)^5 dt$, $\therefore$ $f'(x) = x(e^x - 1)(x-1)(x-2)^3(x-3)^5$

For local minima, slope *i.e.*, $f'(x)$ should change sign from – *ve* to +*ve*

$f'(x) = 0 \Rightarrow x = 0, 1, 2, 3$

If $x = 0 - h$, where *h* is a very small number, then $f'(x) = (-)(-)(-1)(-1)(-1) = -ve$

If $x = 0 + h$, $f'(x) = (+)(+)(-)(-1)(-1) = -ve$

Hence at $x = 0$ neither maxima nor minima.

If $x = 1 - h$, $f'(x) = (+)(+)(-)(-1)(-1) = -ve$

If $x = 1 + h$, $f'(x) = (+)(+)(+)(-1)(-1) = +ve$

Hence, at $x = 1$ there is a local minima.

If $x = 2 - h$, $f'(x) = (+)(+1)(+)(-)(-) = +ve$

If $x = 2+h$, $f'(x) = (+)(+)(+)(+)(-1) = -ve$

Hence at $x = 2$ there is a local maxima.

If $x = 3-h$, $f'(x) = (+)(+)(+)(+)(-) = -ve$

If $x = 3+h$, $f'(x) = (+)(+)(+)(+)(+) = +ve$

Hence at $x = 3$ there is a local minima.

Example: 7 If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(a) 3 (b) 1 (c) 2 (d) $\frac{1}{2}$

Solution: (c) $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$f'(x) = 6x^2 - 18ax + 12a^2$

$f''(x) = 12x - 18a$

For maximum and minimum, $6x^2 - 18ax + 12a^2 = 0 \Rightarrow x^2 - 3ax + 2a^2 = 0$

$x = a$ or $x = 2a$ at $x = a$ maximum and at $x = 2a$ minimum

$\because\ p^2 = q$

$a^2 = 2a \Rightarrow a = 2$ or $a = 0$ but $a > 0$, therefore $a = 2$.

Example: 8 The points of extremum of the function $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)dt$ are

(a) $x = 0$ (b) $x = 1$ (c) $x = \frac{1}{2}$ (d) $x = -1$

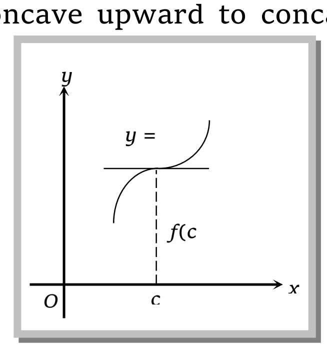
Solution: (b,d) $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)dt \Rightarrow \phi'(x) = e^{-x^2/2}(1-x^2)$

Now $\phi'(x) = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$

Hence, $x = \pm 1$ are points of extremum of $\phi(x)$.

4.3.6 Point of Inflection

A point of inflection is a point at which a curve is changing concave upward to concave downward or vice-versa. A curve $y = f(x)$ has one of its points $x = c$ as an inflection point, if $f''(c) = 0$ or is not defined and if $f''(x)$ changes sign as x increases through $x = c$.



The later condition may be replaced by $f'''(c) \neq 0$, when $f'''(c)$ exists.

Thus, $x = c$ is a point of inflection if $f''(c) = 0$ and $f'''(c) \neq 0$.

Properties of maxima and minima

(i) If $f(x)$ is continuous function in its domain, then at least one maxima and one minima must lie between two equal values of x.

(ii) Maxima and minima occur alternately, that is, between two maxima there is one minimum and vice-versa.

(iii) If $f(x) \to \infty$ as $x \to a$ or b and $f'(x) = 0$ only for one value of x (say c) between a and b, then $f(c)$ is necessarily the minimum and the least value.

If $f(x) \to -\infty$ as $x \to a$ or b, then $f(c)$ is necessarily the maximum and the greatest value.

4.3.7 Greatest and Least Values of a Function in a given Interval

If a function $f(x)$ is defined in an interval [*a, b*], then greatest or least values of this function occurs either at $x = a$ or $x = b$ or at those values of *x* where $f'(x) = 0$.

Remember that a maximum value of the function $f(x)$ in any interval [*a, b*] is not necessarily its greatest value in that interval. Thus greatest value of *f*(*x*) in interval [*a, b*] = max. $[f(a), f(b), f(c)]$

Least value of $f(x)$ interval [*a, b*]= min. $[f(a), f(b), f(c)]$

Where $x = c$ is a point such that $f'(c) = 0$

Example: 9 The maximum and minimum values of $x^3 - 18x^2 + 96$ in interval (0, 9) are **[RPET 1999]**

(a) 160, 0 (b) 60, 0 (c) 160, 128 (d) 120, 28

Solution: (c) Let $y = x^3 - 18x^2 + 96x \Rightarrow \frac{dy}{dx} = 3x^2 - 36x + 96 = 0$

$\therefore$ $x^2 - 12x + 32 = 0 \Rightarrow (x-4)(x-8) = 0, x = 4, 8$

Now, $\frac{d^2y}{dx^2} = 6x - 36$ at $x = 4, \frac{d^2y}{dx^2} = 24 - 36 = -12 < 0$

$\therefore$ at $x = 4$ function will be maximum and $[f(x)]_{\text{max.}} = 64 - 288 + 384 = 160$ at $x = 8\, \frac{d^2y}{dx^2} = 48 - 36 = 12 > 0$

$\therefore$ at $x = 8$ function will be minimum and $[f(x)]_{\text{min.}} = 128$.

Example: 10 The minimum value of the function $2\cos 2x - \cos 4x$ in $0 \le x \le \pi$ is

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) – 3

Solution: (d) $y = 2\cos 2x - \cos 4x = 2\cos 2x(1 - \cos 2x) + 1 = 4\cos 2x \sin^2 x + 1$

Obviously, $\sin^2 x \ge 0$

Therefore, to be least value of *y*, cos 2*x* should be least *i.e.*, – 1. Hence least value of *y* is – 4 + 1 = –3.

Example: 11 On [1, *e*] the greatest value of $x^2 \log x$ **[AMU 2002]**

(a) e^2 (b) $\frac{1}{e}\log\frac{1}{\sqrt{e}}$ (c) $e^2 \log\sqrt{e}$ (d) None of these

Solution: (a) $f(x) = x^2 \log x \Rightarrow f'(x) = (2\log x + 1)x$

Now $f'(x) = 0 \Rightarrow x = e^{-1/2}, 0$

$\because$ $0 < e^{-1/2} < 1$, $\because$ None of these critical points lies in the interval $[1, e]$

$\therefore$ So we only compare the value of $f(x)$ at the end points 1 and *e*. We have $f(1) = 0, f(e) = e^2$

$\therefore$ greatest value = e^2

4.3.8 Maxima and Minima of Functions of Two Variables

If a function is defined in terms of two variables and if these variables are associated with a given relation then by eliminating one variable, we convert function in terms of one variable and then find maxima and minima by known methods.

Example: 12 x and y be two variables such that $x > 0$ and $xy = 1$. Then the minimum value of $x + y$ is

[Kurukshetra CEE 1988; MP PET 2002]

(a) 2 (b) 3 (c) 4 (d) 0

Solution: (a) $xy = 1 \Rightarrow y = \frac{1}{x}$ and let $z = x + y$

$z = x + \frac{1}{x} \Rightarrow \frac{dz}{dx} = 1 - \frac{1}{x^2} \Rightarrow \frac{dz}{dx} = 0 \Rightarrow 1 - \frac{1}{x^2} = 0 \Rightarrow x = -1, +1$ and $\frac{d^2z}{dx^2} = \frac{2}{x^3}$

$\left(\frac{d^2z}{dx^2}\right)_{x=1} = \frac{2}{1} = 2 = +ve$, $\therefore$ $x = 1$ is point of minima.

$x = 1, y = 1$, $\therefore$ minimum value = $x + y = 2$.

Example: 13 The sum of two non-zero numbers is 4. The minimum value of the sum of their reciprocals is

(a) $\frac{3}{4}$ (b) $\frac{6}{5}$ (c) 1 (d) None of these

Solution: (c) Let $x + y = 4$ or $y = 4 - x$

$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$ or $f(x) = \frac{4}{xy} = \frac{4}{x(4-x)}$

$f(x) = \frac{4}{4x - x^2}$, $f'(x) = \frac{-4}{(4x - x^2)^2}.(4 - 2x)$

Put $f'(x) = 0 \Rightarrow 4 - 2x = 0 \Rightarrow x = 2$ and $y = 2$

$\therefore$ min. $\left(\frac{1}{x} + \frac{1}{y}\right) = \frac{1}{2} + \frac{1}{2} = 1$.

Example: 14 The real number which most exceeds its cube is **[MP PET 2000]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{2}}$ (d) None of these

Solution: (b) Let number = x, then cube = x^3

Now $f(x) = x - x^3$ (Maximum) $\Rightarrow f'(x) = 1 - 3x^2$

Put $f'(x) = 0 \Rightarrow 1 - 3x^2 = 0 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

Because $f''(x) = -6x = -ve$. when $x = +\frac{1}{\sqrt{3}}$.

4.3.9 Geometrical Results related to Maxima and Minima

The following results can easily be established.

(1) The area of rectangle with given perimeter is greatest when it is a square.

(2) The perimeter of a rectangle with given area is least when it is a square.

(3) The greatest rectangle inscribed in a given circle is a square.

(4) The greatest triangle inscribed in given circle is equilateral.

(5) The semi vertical angle of a cone with given slant height and maximum volume is $\tan^{-1}\sqrt{2}$

(6) The height of a cylinder of maximum volume inscribed in a sphere of radius a is a $2a/\sqrt{3}$.

Important Tips

☞ ***Equilateral triangle:*** *Area* = $(\sqrt{3}/4)x^2$, *where x is its side.*

- ***Square:*** *Area =* a^2*, perimeter =* $4a$*, where a is its side.*
- ***Rectangle:*** *Area = ab, perimeter =* $2(a+b)$*, where a, b are its sides.*
- ***Trapezium:*** *Area =* $\frac{1}{2}(a+b)h$*, where a, b are lengths of parallel sides and h be the distance between them.*
- ***Circle:*** *Area =* πa^2*, perimeter =* $2\pi a$*, where a is its radius.*
- ***Sphere:*** *Volume =* $\frac{4}{3}\pi a^3$*, surface area =* $4\pi a^2$*, where a is its radius.*
- ***Right circular cone:*** *Volume =* $\frac{1}{3}\pi r^2 h$*, curved surface =* $\pi r l$*, where r is the radius of its base, h is its height and l is its slant height.*
- ***Cylinder:*** *Volume =* $\pi r^2 h$*, whole surface =* $2\pi r(r+h)$*, where r is the radius of the base and h is its height.*

Example: 15 The adjacent sides of a rectangle with given perimeter as 100 *cm* and enclosing maximum area are **[MP PET 1**

(a) 10 *cm* and 40 *cm* (b) 20 *cm* and 30 *cm* (c) 25 *cm* and 25 *cm* (d) 15 *cm* and 35 *cm*

Solution: (c) $2x+2y=100 \Rightarrow x+y=50$ .....(i)

Let area of rectangle is *A*, $\therefore A = xy \Rightarrow y = \frac{A}{x}$

From (i), $x + \frac{A}{x} = 50 \Rightarrow A = 50x - x^2 \Rightarrow \frac{dA}{dx} = 50 - 2x$

for maximum area $\frac{dA}{dx} = 0$

$\therefore 50 - 2x = 0 \Rightarrow x = 25$ and $y = 25$

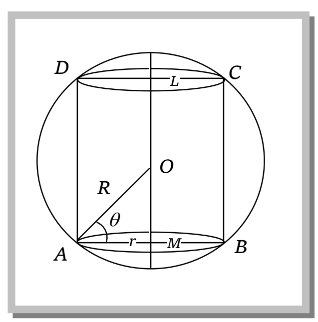
$\therefore$ adjacent sides are 25 *cm* and 25 *cm*.

Example: 16 The radius of the cylinder of maximum volume, which can be inscribed a sphere of radius *R* is **[AMU 1999]**

(a) $\frac{2}{3}R$ (b) $\sqrt{\frac{2}{3}}R$ (c) $\frac{3}{4}R$ (d) $\sqrt{\frac{3}{4}}R$

Solution: (b) If *r* be the radius and *h* the height, the from the figure, $r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow h^2 = 4(R^2 - r^2)$

Now, $V = \pi r^2 h = 2\pi r^2\sqrt{R^2 - r^2}$



$\therefore \frac{dV}{dr} = 4\pi r\sqrt{R^2 - r^2} + 2\pi r^2 . \frac{1}{2}\frac{(-2r)}{\sqrt{R^2 - r^2}}$

For max. or min., $\frac{dV}{dr} = 0$

$\Rightarrow 4\pi r\sqrt{R^2 - r^2} = \frac{2\pi r^3}{\sqrt{R^2 - r^2}} \Rightarrow 2(R^2 - r^2) = r^2$

$\Rightarrow 2R^2 = 3r^2 \Rightarrow r = \sqrt{\frac{2}{3}}R \Rightarrow \frac{d^2V}{dr^2} = -ve$. Hence *V* is max. when $r = \sqrt{\frac{2}{3}}R$.

Example: 17 The ratio of height of a cone having maximum volume which can be inscribed in a sphere with the diameter of sphere is

[MNR 1985]

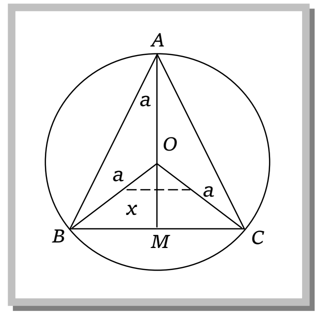
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

Solution: (a) Let $OM = x$

Then height of cone *i.e.,* $h = x + a$ (where a is radius of sphere)

Radius of base of cone = $\sqrt{a^2 - x^2}$

Therefore, volume $V = \frac{1}{3}\pi(a^2 - x^2)(x + a) \Rightarrow \frac{dV}{dx} = \frac{\pi}{3}(a + x)(a - 3x)$



Now, $\frac{dV}{dx} = 0 \Rightarrow x = -a, \frac{a}{3}$

But $x \neq -a$, So, $x = \frac{a}{3}$

The volume is maximum at $x = \frac{a}{3}$

Height of a cone $h = a + \frac{a}{3} = \frac{4}{3}a$

Therefore ratio of height and diameter = $\frac{\frac{4}{3}a}{2a} = \frac{2}{3}$.

Assignment

Maxima and Minima

Basic Level

1. The maximum value of $f(x) = \frac{x}{4 + x + x^2}$ on [– 1, 1] is **[MP PET 2000]**

(a) $\frac{-1}{4}$ (b) $\frac{-1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{5}$

2. Maximum value of $x(1 - x)^2$ when $0 \le x \le 2$, is **[MP PET 1997]**

(a) 2 (b) $\frac{4}{27}$ (c) 5 (d) 0

3. The maximum value of $2x^3 - 24x + 107$ in the interval [– 3, 3] is

(a) 75 (b) 89 (c) 125 (d) 139

4. The maximum value of the function $f(x) = 3\sin x + 4\cos x$ is

(a) 3 (b) 4 (c) 5 (d) 7

5. If the function $f(x) = x^4 - 62x^2 + ax + 9$ is maximum at $x = 1$, then the value of *a* is

(a) 120 (b) – 120 (c) 52 (d) 128

6. The maximum value of $f(\theta) = a\sin\theta + b\cos\theta$ is **[MP PET 1999; UPSEAT 2000]**

(a) $\frac{a}{b}$ (b) $\frac{a}{\sqrt{a^2+b^2}}$ (c) $\sqrt{ab}$ (d) $\sqrt{a^2+b^2}$

7. The minimum value of the function $y = 2x^3 - 21x^2 + 36x - 20$ is

(a) – 128 (b) – 126 (c) – 120 (d) None of these

8. $\frac{x}{1 + x\tan x}$ is maximum at **[UPSEAT 1999]**

(a) $x = \sin x$ (b) $x = \cos x$ (c) $x = \frac{\pi}{3}$ (d) $x = \tan x$

9. The minimum value of the expression $7 - 20x + 11x^2$ is

(a) $\frac{177}{11}$ (b) $-\frac{177}{11}$ (c) $-\frac{23}{11}$ (d) $\frac{23}{11}$

10. The minimum value of $2x^2 + x - 1$ is **[EAMCET 2003]**

(a) $\frac{-1}{4}$ (b) $\frac{3}{2}$ (c) $\frac{-9}{8}$ (d) $\frac{9}{4}$

11. The maximum value of *xy* subject to $x + y = 8$, is **[MNR 1995]**

(a) 8 (b) 16 (c) 20 (d) 24

12. If $A + B = \frac{\pi}{2}$, the maximum value of $\cos A \cos B$ is **[AMU 1999]**

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{4}{3}$

13. If $xy = c^2$, then minimum value of $ax + by$ is **[Rajasthaan PET 2001]**

(a) $c\sqrt{ab}$ (b) $2c\sqrt{ab}$ (c) $-c\sqrt{ab}$ (d) $-2c\sqrt{ab}$

14. If $a^2x^4 + b^2y^4 = c^6$, then maximum value of xy is **[Rajasthan PET 2001]**

(a) $\frac{c^2}{\sqrt{ab}}$ (b) $\frac{c^3}{ab}$ (c) $\frac{c^3}{\sqrt{2ab}}$ (d) $\frac{c^3}{2ab}$

15. The function $f(x) = 2x^3 - 15x^2 + 36x + 4$ is maximum at **[Karnataka CET 2001]**

(a) $x = 2$ (b) $x = 4$ (c) $x = 0$ (d) $x = 3$

16. The function $f(x) = x^{-x}, (x \in R)$ attains a maximum value at $x =$

(a) 2 (b) 3 (c) $\frac{1}{e}$ (d) 1

17. The function $y = a(1 - \cos x)$ is maximum when $x =$ **[Kerala (Engg.) 2002]**

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{-\pi}{2}$ (d) $\frac{-\pi}{6}$

18. The minimum value of $\left(x^2 + \frac{250}{x}\right)$ is **[Haryana CEE 2002]**

(a) 75 (b) 50 (c) 25 (d) 55

19. In the graph of the function $\sqrt{3}\sin x + \cos x$ the maximum distance of a point from *x*-axis is

(a) 4 (b) 2 (c) 1 (d) $\sqrt{3}$

20. The function $f(x) = x + \sin x$ has **[AMU 2000]**

(a) A minimum but no maximum (b) A maximum but no minimum

(c) Neither maximum nor minimum (d) Both maximum and minimum

21. The point for the curve $y = xe^x$

(a) $x = -1$ is minimum (b) $x = 0$ is minimum (c) $x = -1$ is maximum (d) $x = 0$ is maximum

22. 36 is factorized into two factors in such a way that sum of factors is minimum, then the factors are

(a) 2, 18 (b) 9, 4 (c) 3, 12 (d) None of these

23. The necessary condition to be maximum or minimum for the function is

(a) $f'(x) = 0$ and it is sufficient (b) $f''(x) = 0$ and it is sufficient

(c) $f'(x) = 0$ but it is not sufficient (d) $f'(x) = 0$ and $f''(x) = -ve$

24. The maximum and minimum value of the function $3x^4 - 8x^3 + 12x^2 - 48x + 25$ in the interval [1, 3]

(a) 16, – 39 (b) – 16, 39 (c) 6, – 9 (d) None of these

25. If $f(x) = 2x^3 - 3x^2 - 12x + 5$ and $x \in [-2, 4]$, then the maximum value of function is at the following value of *x* **[MP PET 1987**

(a) 2 (b) – 1 (c) – 2 (d) 4

26. The minimum value of $|x| + |x + \frac{1}{2}| + |x - 3| + |x - \frac{5}{2}|$ is

(a) 0 (b) 2 (c) 4 (d) 6

27. The maximum value of the function $x^3 + x^2 + x - 4$ is

(a) 127 (b) 4

(c) Does not have a maximum value (d) None of these

28. The function $x^5 - 5x^4 + 5x^3 - 10$ has a maximum when *x* =

(a) 3 (b) 2 (c) 1 (d) 0

29. If $x - 2y = 4$, the minimum value of *xy* is **[UPSEAT 2003]**

(a) – 2 (b) 2 (c) 0 (d) – 3

30. The minimum value of $x^2 + \frac{1}{1 + x^2}$ is at **[UPSEAT 2003]**

(a) $x = 0$ (b) $x = 1$ (c) $x = 4$ (d) $x = 3$

31. The maximum and minimum value of the function $|\sin 4x + 3|$ are

(a) 1, 2 (b) 4, 2 (c) 2, 4 (d) – 1, 1

32. The maximum value of function $x^3 - 12x^2 + 36x + 17$ in the interval [1, 10] is

(a) 17 (b) 177 (c) 77 (d) None of these

33. Let $f(x) = (x - p)^2 + (x - q)^2 + (x - r)^2$. Then $f(x)$ has a minimum at $x = \lambda$, where λ is equal to

(a) $\frac{p + q + r}{3}$ (b) $3\sqrt{pqr}$ (c) $\frac{3}{\frac{1}{p} + \frac{1}{q} + \frac{1}{r}}$ (d) None of these

34. The function $x^2 \log x$ in the interval (1, *e*) has

(a) A point of maximum (b) A point of minimum

(c) Points of maximum as well as of minimum (d) Neither a point of maximum nor minimum

35. The two parts of 100 for which the sum of double of first and square of second part is minimum, are

(a) 50, 50 (b) 99, 1 (c) 98, 2 (d) None of these

36. Of the given perimeter, the triangle having maximum area is

(a) Isosceles triangle (b) Right angled triangle (c) Equilateral (d) None of these

37. The function $x^5 - 5x^4 + 5x^3 - 1$ is

(a) Maximum at $x = 3$ and minimum at $x = 1$ (b) Minimum at $x = 1$

(c) Neither maximum nor minimum at $x = 0$ (d) Maximum at $x = 0$

38. If $u = \sqrt{a^2 \cos^2\theta + b^2 \sin^2\theta} + \sqrt{a^2 \sin^2\theta + b^2 \cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by

[AIEEE 2004]

(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$ (c) $(a+b)^2$ (d) $2(a^2+b^2)$

39. The minimum value of $2x + 3y$, when $xy = 6$, is **[MP PET 2003]**

(a) 12 (b) 9 (c) 8 (d) 6

40. The real number x when added to its inverse gives the minimum value of the sum at x equal to **[Rajasthan PET 2000; AIEEE 2003]**

(a) – 2 (b) 2 (c) 1 (d) – 1

41. $x + \frac{1}{x}$ is maximum at **[Rajasthan PET 1991]**

(a) $x = 1$ (b) $x = -1$ (c) $x = 2$ (d) $x = -2$

42. $f(x) = (1-x)^2 e^x$ is minimum at

(a) $x = 1$ (b) $x = -1$ (c) $x = 0$ (d) $x = 2$

43. The maximum value of the function $x^3 - 12x^2 + 45x$ is **[Rajasthan PET 1994]**

(a) 0 (b) 50 (c) 54 (d) 70

Advance Level

44. Let $f(x) = \begin{cases} |x| & , \quad 0 < |x| \le 2 \\ 1 & , \quad x = 0 \end{cases}$, then at $x = 0$ f has **[IIT Screening 2000]**

(a) A local maximum (b) No local maximum (c) A local minimum (d) No extremum

45. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x, then the minimum value of f

(a) Does not exist because f is unbounded (b) Is not attained even though f is bounded

(c) Is equal to 1 (d) Is equal to –1

46. The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is

(a) 0 (b) 1 (c) 2 (d) Infinite

47. On the interval [0, 1] the function $x^{25}(1-x)^{75}$ takes its maximum value at the point **[IIT 1995]**

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

48. x^x has a stationary point at

(a) $x = e$ (b) $x = \frac{1}{e}$ (c) $x = 1$ (d) $x = \sqrt{e}$

49. A minimum value of $\int_0^x te^{-t^2} dt$ is

(a) 1 (b) 2 (c) 3 (d) 0

50. The sum of two numbers is fixed. Then its multiplication is maximum, when

(a) Each number is half of the sum (b) Each number is $\frac{1}{3}$ and $\frac{2}{3}$ respectively of the sum

(c) Each number is $\frac{1}{4}$ and $\frac{3}{4}$ respectively of the sum (d) None of these

51. The value of a so that the sum of the squares of the roots of the equation $x^2 - (a-2)x - a + 1 = 0$ assume the least value, is

(a) 2 (b) 1 (c) 3 (d) 0

52. If from a wire of length 36 *metre* a rectangle of greatest area is made, then its two adjacent sides in *metre* are **[MP PET**

(a) 6, 12 (b) 9, 9 (c) 10, 8 (d) 13, 5

53. The maximum value of $x^4 e^{-x^2}$ is

(a) e^2 (b) e^{-2} (c) $12e^{-2}$ (d) $4e^{-2}$

54. One maximum point of $\sin^p x \cos^q x$ is **[Rajasthan PET 1997; AMU 2000]**

(a) $x = \tan^{-1}\sqrt{(p/q)}$ (b) $x = \tan^{-1}\sqrt{(q/p)}$ (c) $x = \tan^{-1}(p/q)$ (d) $x = \tan^{-1}(q/p)$

55. 20 is divided into two parts so that product of cube of one quantity and square of the other quantity is maximum. The parts are

[Rajasthan PET 1997]

(a) 10, 10 (b) 16, 4 (c) 8, 12 (d) 12, 8

56. The minimum value of $e^{(2x^2-2x+1)\sin^2 x}$ is **[Rorkee 1998]**

(a) e (b) $\frac{1}{e}$ (c) 1 (d) 0

57. Divide 20 into two parts such that the product of one part and the cube of the other is maximum. The two parts are **[DCE 1999]**

(a) (10, 10) (b) (5, 15) (c) (13, 7) (d) None of these

58. The minimum value of $\exp(2 + \sqrt{3}\cos x + \sin x)$ is **[AMU 1999]**

(a) exp(2) (b) $\exp(2 - \sqrt{3})$ (c) exp(4) (d) 1

59. The minimum value of $\frac{\log x}{x}$ in the interval $[2, \infty)$ **[Roorkee 1999]**

(a) Is $\frac{\log 2}{2}$ (b) Is zero (c) Is $\frac{1}{e}$ (d) Does not exist

60. The function $f(x) = ax + \frac{b}{x}, a, b, x > 0$ takes on the least value at x equal to **[AMU 2000]**

(a) b (b) $\sqrt{a}$ (c) $\sqrt{b}$ (d) $\sqrt{b/a}$

61. The area of a rectangle of given perimeter is maximum, when ratio of its length and breadth is

(a) 2 : 1 (b) 3 : 2 (c) 4 : 3 (d) 1 : 1

62. The denominator of a fraction number is greater than 16 of the square of numerator, then least value of the number is

[Rajasthan PET 2000]

(a) $\frac{-1}{4}$ (b) $\frac{-1}{8}$ (c) $\frac{1}{12}$ (d) $\frac{1}{16}$

63. If for a function $f(x)$, $f'(a) = 0$, $f''(a) = 0, f'''(a) > 0$, then at $x = a$, $f(x)$ is **[MP PET 1994]**

(a) Minimum (b) Maximum (c) Not an extreme points (d) Extreme point

64. The least value of the sum of any positive real number and its reciprocal is **[MP PET 1994]**

(a) 1 (b) 2 (c) 3 (d) 4

65. If x is real, then greatest and least values of $\frac{x^2-x+1}{x^2+x+1}$ are **[Rajasthan PET 1999; AMU 1999; UPSEAT 2002]**

(a) $3, -\frac{1}{2}$ (b) $3, \frac{1}{3}$ (c) $-3, -\frac{1}{3}$ (d) None of these

66. A wire of constant length is given. In which shape it should be bent to surround maximum area

(a) Circle (b) Square (c) Both (a) and (b) (d) Neither (a) nor (b)

67. The function $x\sqrt{1-x^2}, (x>0)$ has

(a) A local maxima (b) A local minima

(c) Neither a local maxima nor a local minima (d) None of these

68. If $x+y=16$ and x^2+y^2 is minimum, the value of x and y are

(a) 3, 13 (b) 4, 12 (c) 6, 10 (d) 8, 8

69. The area of a rectangle will be maximum for the given perimeter. When rectangle is a

(a) Parallelogram (b) Trapezium (c) Square (d) None of these

70. Local maximum value of the function $\frac{\log x}{x}$ is

[MNR 1984; Rajasthan PET 1997, 2002; DCE 2002; Karnataka CET 2000; UPSEAT 2001; MP PET 2002]

(a) e (b) 1 (c) $\frac{1}{e}$ (d) $2e$

71. Local maximum and local minimum values of the function $(x-1)(x+2)^2$ are

(a) – 4, 0 (b) 0, – 4 (c) 4,0 (d) None of these

72. If $f(x)=2x^3-21x^2+36-30$, then which one of the following is correct

(a) $f(x)$ has minimum at $x=1$ (b) $f(x)$ has maximum at $x=6$

(c) $f(x)$ has maximum at $x=1$ (d) $f(x)$ has no maxima or minima

73. If sum of two numbers is 3, then maximum value of the product of first and the square of second is

(a) 4 (b) 3 (c) 2 (d) 1

74. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$ such that min $f(x)>$ max $g(x)$, then the relation between b and c is **[IIT Screening 2003]**

(a) No real value of b and c (b) $0<c<b\sqrt{2}$ (c) $|c|<|b|\sqrt{2}$ (d) $|c|>|b|\sqrt{2}$

75. The minimum value of $[(5+x)(2+x)]/[1+x]$ for non-negative real x is

(a) 12 (b) 1 (c) 9 (d) 8

76. Let $f(x)=\int_0^x \frac{\cos t}{t}dt, x>0$ then $f(x)$ has **[Haryana CEE 2002]**

(a) Maxima when $n=-2,-4,-6.....$ (b) Maxima $n=-1,-3,-5,......$

(c) Minima when $n=0,2,4,......$ (d) Minima when $n=1,3,5,......$

77. The function $f(x)=2x^3-3x^2-12x+4$ has **[DCE 2002]**

(a) No maxima and minima (b) One maximum and one minimum

(c) Two maxima (d) Two minima

78. If $f(x)=\frac{1}{4x^2+2x+1}$, then its maximum value is **[Rajasthan PET 2002]**

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{3}{4}$

79. If PQ and PR are the two sides of a triangle, then the angle between them which gives maximum area of the triangle is

[Kerala (Engg.) 2002]

(a) π (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

80. If $ab = 2a + 3b, a > 0, b > 0$ then the minimum value of *ab* is **[Orissa JEE 2002]**

(a) 12 (b) 24 (c) $\frac{1}{4}$ (d) None of these

81. The perimeter of a sector is *p*. The area of the sector is maximum when its radius is **[Karnataka CET 2002]**

(a) $\sqrt{p}$ (b) $\frac{1}{\sqrt{p}}$ (c) $\frac{p}{2}$ (d) $\frac{p}{4}$

82. The maximum area of the rectangle that can be inscribed in a circle of radius *r* is **[EAMCET 1994]**

(a) πr^2 (b) r^2 (c) $\frac{\pi r^2}{4}$ (d) $2r^2$

83. If $f(x) = \begin{cases} 3x^2 + 12x - 1 & , \ -1 \le x \le 2 \\ 37 - x & , \ 2 < x \le 3 \end{cases}$, then **[IIT 1993]**

(a) $f(x)$ is increasing $[-1, 2]$ (b) $f(x)$ is continuous in [– 1, 3] (c) $f(x)$ is maximum at $x = 2$ (d)

84. If $f'(x) = (x - a)^{2n}(x - b)^{2p+1}$ when *n* and *p* are positive integers, then

(a) $x = a$ is a point of minimum (b) $x = a$ is a point of maximum

(c) $x = a$ is not a point of maximum or minimum (d) None of these

85. *N* characters of information are held on magnetic tape, in batches of *x* characters each, the batch processing time is $\alpha + \beta x^2$ seconds, α and β are constants. The optical value of *x* for fast processing is

(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta}{\alpha}$ (c) $\sqrt{\frac{\alpha}{\beta}}$ (d) $\sqrt{\frac{\beta}{\alpha}}$

86. If $f(x) = \sin^6 x + \cos^6 x$, then

(a) $f(x) \le 1$ (b) $f(x) \le 2$ (c) $f(x) > \frac{1}{4}$ (d) $f(x) > \frac{1}{8}$

87. The maximum and minimum values of $y = \frac{ax^2 + 2bx + c}{Ax^2 + 2Bx + C}$ are those for which

(a) $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$ is equal to zero (b) $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$ is a perfect square

(c) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} \ne 0$ (d) None of these

88. A differentiable function $f(x)$ has a relative minimum at $x = 0$, then the function $y = f(x) + ax + b$ has a relative minimum at $x = 0$ for

(a) All *a* and all *b* (b) All *b* if $a = 0$ (c) All $b > 0$ (d) All $a > 0$

89. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Then area of the triangle is maximum when θ

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

90. The greatest value of the function $f(x) = \frac{\sin 2x}{\sin\left(x + \frac{\pi}{4}\right)}$ on the interval $\left[0, \frac{\pi}{2}\right]$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 1 (d) $-\sqrt{2}$

91. The longest distance of the point (*a*, 0) from the curve $2x^2 + y^2 - 2x = 0$, is given by

(a) $\sqrt{1 - 2a + a^2}$ (b) $\sqrt{1 + 2a + 2a^2}$ (c) $\sqrt{1 + 2a - a^2}$ (d) $\sqrt{1 - 2a + 2a^2}$

92. The function $f(x)=\int_1^x \{2(t-1)(t-2)^3+3(t-1)^2(t-2)^2 dt$ attains its maximum at $x=$

(a) 1 (b) 2 (c) 3 (d) 4

93. If the function $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$ has a positive point of maximum, then

(a) $a\in(3,\infty)\cup(-\infty,-3)$ (b) $a\in(-\infty,-3)\cup\left(3,\frac{29}{7}\right)$ (c) $(-\infty,7)$ (d) $\left(-\infty,\frac{29}{7}\right)$

94. The minimum value of $\left(1+\frac{1}{\sin^n\alpha}\right)\left(1+\frac{1}{\cos^n\alpha}\right)$ is

(a) 1 (b) 2 (c) $(1+2^{n/2})^2$ (d) None of these

95. A cubic $f(x)$ vanishes at $x=-2$ and has relative minimum/maximum at $x=-1$ and $x=\frac{1}{3}$ such that $\int_{-1}^{1} f(x)dx=\frac{14}{3}$. Then $f(x)$ is

(a) x^3+x^2-x (b) x^3+x^2-x+1 (c) x^3+x^2-x+2 (d) x^3+x^2-x-2

96. If $A>0, B>0$ and $A+B=\frac{\pi}{3}$, then the maximum value of $\tan A\tan B$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) 3 (d) $\sqrt{3}$

97. Total number of parallel tangents of f $f_1(x)=x^2-x+1$ and x^3-x^2-2x+1 is equal to

(a) 2 (b) 3 (c) 4 (d) None of these

98. Function $f(x)=|px-q|+r|x|\ (p>0,q>0,r>0)$ attains its minimum value only at one point, if

(a) $p\neq q$ (b) $q\neq r$ (c) $r\neq p$ (d) $p=q=r$

99. The height of right circular cylinder of maximum volume inscribed in a sphere of radius a is

(a) $\frac{a}{\sqrt{3}}$ (b) $\sqrt{3}a$ (c) $\frac{2a}{\sqrt{3}}$ (d) $2\sqrt{3}a$

100. A line is drawn through a fixed point (a, b), $(a>0,b>0)$ to meet the positive direction of the coordinate axes in P,Q respectively. The minimum value of $OP+OQ$ is

(a) $\sqrt{a}+\sqrt{b}$ (b) $(\sqrt{a}+\sqrt{b})^2$ (c) $(\sqrt{a}+\sqrt{b})^3$ (d) None of these

101. For the curve $\frac{C^4}{r^2}=\frac{a^2}{\sin^2\theta}+\frac{b^2}{\cos^2\theta}$, the maximum value of r is

(a) $\frac{c^2}{a+b}$ (b) $\frac{a+b}{c^2}$ (c) $\frac{c^2}{a-b}$ (d) $c^2(a+b)$

102. The coordinates of a point situated on the curve $4x^2+a^2y^2=4a^2(4<a^2<8)$, which are at maximum distance from the point (0, – 2) is

(a) $(a, 0)$ (b) $(2a,-4)$ (c) (0, 2) (d) None of these

103. For what value of k, the function: $f(x)=kx^2+\frac{2k^2-81}{2}x-12$, is maximum at $x=\frac{9}{4}$

(a) $\frac{9}{2}$ (b) -9 (c) $\frac{-9}{2}$ (d) 9

104. If $\alpha<\beta$, then correct statement is

(a) $\alpha-\sin\alpha>\beta-\sin\beta$ (b) $\alpha-\sin\alpha<\beta-\sin\beta$ (c) $\sin\alpha-\alpha<-\sin\beta+\beta$ (d) None of these

105. The difference between two numbers is a if their product is minimum, then number are

(a) $\frac{-a}{2},\frac{a}{2}$ (b) $-a,2a$ (c) $\frac{-a}{3},\frac{2a}{3}$ (d) $\frac{-a}{3},\frac{4a}{3}$

106. If λ, μ be real numbers such that $x^3 - \lambda x^2 + \mu x - 6 = 0$ has its roots real and positive then the minimum value of μ is

(a) $3 \times \sqrt[3]{36}$ (b) 11 (c) 0 (d) None of these

107. Let the tangent to the graph of $y = f(x)$ at the point $x = a$ be parallel to the *x*-axis, let $f'(a-h) > 0$ and $f'(a+h) < 0$, where *h* is a very small positive number. Then the ordinate of the point is

(a) A maximum (b) A minimum

(c) Both a maximum and a minimum (d) Neither a maximum nor a minimum

108. If $a > b > 0$, the minimum value of $a \sec\theta - b \tan\theta$ is

(a) $b - a$ (b) $\sqrt{a^2 + b^2}$ (c) $\sqrt{a^2 - b^2}$ (d) $2\sqrt{a^2 - b^2}$

109. Let the function $f(x)$ be defined as below:

$$f(x) = \sin^{-1}\lambda + x^2, 0 < x < 1; \quad 2x, x \ge 1$$

$f(x)$ can have a minimum at $x = 1$ if the value of λ is

(a) 1 (b) – 1 (c) 0 (d) None of these

110. Let $f(x) = ax^3 + bx^2 + cx + 1$ have extreme at $x = \alpha, \beta$ such that $\alpha\beta < 0$ and $f(\alpha).f(\beta) < 0$. Then the equation $f(x) = 0$ has

(a) Three equal real roots (b) Three distinct real roots

(c) One positive root if $f(\alpha) < 0$ and $f(\beta) > 0$ (d) One negative root if $f(\alpha) > 0$ and $f(\beta) < 0$

111. Let $f(x) = 1 + 2x^2 + 2^2 x^4 + ... + 2^{10} x^{20}$; Then $f(x)$ has

(a) More than one minimum (b) Exactly one minimum (c) At least one maximum(d)

112. Let the function $f(x)$ be defined as follows:

$$f(x) = x^3 + x^2 - 10x, \quad -1 \le x < 0$$

$$\cos x, \quad 0 \le x < \frac{\pi}{2}; \quad 1 + \sin x, \quad \frac{\pi}{2} \le x \le \pi$$

Then $f(x)$ has

(a) A local minimum at $x = \frac{\pi}{2}$ (b) A local maximum at $x = \frac{\pi}{2}$

(c) An absolute minimum at $x = -1$ (d) An absolute maximum at $x = \pi$

113. Two part of 64 such that the sum of their cubes is minimum will be

(a) 44, 20 (b) 16, 48 (c) 32, 32 (d) 50, 14

114. If *x* be real then the minimum value of $f(x) = 3^{x+1} + 3^{-(x+1)}$ is

(a) 2 (b) 6 (c) $\frac{2}{3}$ (d) $\frac{7}{9}$

115. The minimum value of $e^{(2x^2 - 2x - 1)\sin^2 x}$ is **[Roorkee 1998]**

(a) e (b) $\frac{1}{e}$ (c) 1 (d) 0

116. The semi-vertical angle of a right circular cone of given slant height and maximum volume is

(a) $\tan^{-1} 2$ (b) $\tan^{-1}\sqrt{2}$ (c) $\tan^{-1} 1/2$ (d) $\tan^{-1} 1/\sqrt{2}$

117. If $0 < a < x$, then the minimum value of $\log_a x + \log_a a$ is **[IIT 1984]**

(a) 2 (b) – 2 (c) 2*a* (d) Does not exist

118. Which point of the parabola $y = x^2$ is nearest to the point (3, 0)

(a) (– 1, 1) (b) (1, 1) (c) (2, 4) (d) (– 2, 4)

119. The point of inflexion for the curve $y = x^{5/2}$ is **[Rajasthan PET 1989, 1992]**

(a) (1, 1) (b) (0, 0) (c) (1, 0) (d) (0, 1)

***

Answer Sheet

Assignment (Basic & Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	a	d	c	a	d	a	b	c	c	b	a	b	c	a	c	a	a	b	c
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	d	c	a	d	d	c	c	a	a	b	b	a	d	b	c	c	a	a	c
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	a	c	a	d	b	d	b	d	a	b	b	d	a	d	c	b	d	d	d
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
d	b	c	b	b	a	a	d	c	c	b	c	a	d	c	a,d	b	a	d	b
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
d	d	d	c	c	a,c	b,c	b	a	c	d	a	b	c	c	b	d	d	c	b
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	
a	c	b	b	a	a	a	c	d	b,c	b	b	c	a	c	b	d	b	b	