Assignment

Continuity

Basic Level

1. If the function $f(x)=\begin{cases}5x-4 & \text{, if } 0<x\le 1\\ 4x^2+3bx, & \text{if } 1<x<2\end{cases}$ is continuous at every point of its domain, then the value of b is **[Rajasthan PET 2000]**

(a) –1 (b) 0 (c) 1 (d) None of these

2. If $f(x)=\begin{cases}\dfrac{\log(1+2ax)-\log(1-bx)}{x}, & x\ne 0\\ k & , x=0\end{cases}$ is continuous at $x=0$, then k equals **[Rajasthan PET 1998]**

(a) $2a+b$ (b) $2a-b$ (c) $b-2a$ (d) $b+a$

3. If $f(x)=\begin{cases}x & \text{, when } 0\le x<1\\ k-2x & \text{, when } 1\le x\le 2\end{cases}$ is continuous at $x=1$, then value of k is **[Rajasthan PET 1993]**

(a) 1 (b) –1 (c) 3 (d) 2

4. If $f(x)=\begin{cases}x & , x<0\\ 1 & , x=0\\ x^2 & , x>0\end{cases}$, then true statement is **[Rajasthan PET 1992; DCE 2001]**

(a) $\lim_{x\to 0} f(x)=1$ (b) $\lim_{x\to 0} f(x)=0$ (c) $f(x)$ is continuous at x = 0 (d) $\lim_{x\to 0} f(x)$ does not exist

5. If $f(x)=\dfrac{x-a}{\sqrt{x}-\sqrt{a}}$ is continuous at x = a, then $f(a)$ equals

(a) $\sqrt{a}$ (b) $2\sqrt{a}$ (c) a (d) $2a$

6. If $f(x)=|x-b|$, then function **[Roorkee 1984]**

(a) Is continuous $\forall x$ (b) Is continuous at $x=\infty$ (c) Is discontinuous at $x=b$ (d) None of these

7. If $f(x)=\begin{cases}\dfrac{x^4-16}{x-2}, & \text{when } x\ne 2\\ 16 & \text{, when } x=2\end{cases}$ then

(a) $f(x)$ is continuous at $x=2$ (b) $f(x)$ is discontinuous at $x=2$

(c) $\lim_{x\to 2} f(x)=16$ (d) None of these

8. In the following discontinuous function is **[Rajasthan PET 1984]**

(a) $\sin x$ (b) x^2 (c) $\dfrac{1}{1-2x}$ (d) $\dfrac{1}{1+x^2}$

9. If $f(x)=\begin{cases}x^2 & \text{, when } x\le 1\\ x+5, & \text{when } x>1\end{cases}$ then **[MP PET 1996]**

(a) $f(x)$ is continuous at $x=1$ (b) $f(x)$ is discontinuous at $x=1$

(c) $\lim_{x\to 1} f(x)=1$ (d) None of these

10. If $f(x)=\begin{cases}1+x, \text{ when } x\le 2\\ 5-x, \text{ when } x>2\end{cases}$ then

(a) $f(x)$ is continuous at x=2 (b) $f(x)$ is discontinuous at x=2 (c) $f(x)$ is discontinuous at $x=0$ (d) None of these

11. The point of discontinuity of the function $f(x)=\frac{1+\cos 5x}{1-\cos 4x}$ is

(a) $x=0$ (b) $x=\pi$ (c) $x=\pi/2$ (d) All of these

12. Function $f(x)=|x|$ is **[Rajasthan PET 1992]**

(a) Discontinuous at $x=0$ (b) Discontinuous at $x=1$ (c) Continuous at all points (d) Discontinuous at all points

13. If $f(x)=\begin{cases}x^2, & \text{when } x\ne 1\\ 2, & \text{when } x=1\end{cases}$ then

(a) $\lim_{x\to 1} f(x)=2$ (b) $f(x)$ is continuous at x = 1 (c) $f(x)$ is discontinuous at x = 1 (d) None of these

14. Let $f(x)=\begin{cases}\frac{\sin \pi x}{5x}, & x\ne 0\\ k, & x=0\end{cases}$. If $f(x)$ is continuous at $x=0$, then k =

(a) $\frac{\pi}{5}$ (b) $\frac{5}{\pi}$ (c) 1 (d) 0

15. Function $f(x)=x-|x|$ is

(a) Discontinuous at $x=0$ (b) Discontinuous at x = 1 (c) Continuous at all points (d) Discontinuous at all points

16. Function $f(x)=x+|x|$ is

(a) Continuous at all points (b) Discontinuous at $x=0$ (c) Discontinuous at $x=1$ (d) Discontinuous at all points

17. If $f(x)$ is continuous function and $g(x)$ is discontinuous function, then correct statement is

(a) $f(x)+g(x)$ is continuous function (b) $f(x)-g(x)$ is continuous function

(c) $f(x)+g(x)$ is discontinuous function (d) $f(x).g(x)$ is discontinuous function

18. Function $f(x)=\begin{cases}-1, \text{ when } x<-1\\ -x, \text{ when } -1\le x\le 1\\ 1, \text{ when } x>1\end{cases}$ is continuous **[Rajasthan PET 1986]**

(a) Only at $x=1$ (b) Only at $x=-1$ (c) At both $x=1$ and $x=-1$ (d) Neither at x=1 nor at x=–1

Advance Level

19. Let $f(x)=\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$ the value which should be assigned to f at $x=0$ so that it is continuous everywhere is

[MP PET 1992]

(a) $\frac{1}{2}$ (b) –2 (c) 2 (d) 1

20. The value of $f(0)$ so that the function $f(x)=\frac{\sqrt{1+x}-(1+x)^{1/3}}{x}$ becomes continuous is equal to

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) 2 (d) $\frac{1}{3}$

21. If $f(x)=\begin{cases}\frac{|x-a|}{x-a} & \text{when } x\neq a\\ 1 & \text{when } x=a\end{cases}$ then **[AI CBSE 1983]**

(a) $f(x)$ is continuous at $x=a$ (b) $f(x)$ is discontinuous at $x=a$ (c) $\lim_{x\to a} f(x)=1$ (d) None of these

22. If $f(x)=\begin{cases}\frac{x}{e^{1/x}+1}, & \text{when } x\neq 0\\ 0, & \text{when } x=0\end{cases}$ then **[BIT Rnchi 1999]**

(a) $\lim_{x\to 0^+} f(x)=1$ (b) $\lim_{x\to 0^-} f(x)=1$ (c) $f(x)$ is continuous at $x = 0$ (d) None of these

23. If the function $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & \text{when } x\neq\frac{\pi}{2}\\ 3, & \text{when } x=\frac{\pi}{2}\end{cases}$ be continuous at $x=\frac{\pi}{2}$, then k=

(a) 3 (b) 6 (c) 12 (d) None of these

24. A function $f(x)$ is defined in $[0,1]$ as follows $f(x)=\begin{cases}x, \text{ if } x \text{ is rational}\\ 1-x, \text{ if } x \text{ is irrational}\end{cases}$, then correct statement is

(a) $f(x)$ is continuous at $x=0$ (b) $f(x)$ is continuous at $x=1$

(c) $f(x)$ is continuous at $x=\frac{1}{2}$ (d) $f(x)$ is everywhere discontinuous

25. If $f(x)=\begin{cases}\frac{e^{1/x}-1}{e^{1/x}+1}, & x\neq 0\\ 1, & x=0\end{cases}$, then at $x=0, f(x)$ is **[BITS (Mesra) 1998]**

(a) Continuous (b) Left continuous (c) Right continuous (d) None of these

26. The function $f(x)=\begin{cases}x+2, & 1\le x\le 2\\ 4, & x=2\\ 3x-2, & x>2\end{cases}$ is continuous **[DCE 1999]**

(a) $x=2$ only (b) $x\le 2$ (c) $1\le x$ (d) None of these

27. If $f(x)=\begin{cases}1, & \text{when } 0<x\le\frac{3\pi}{4}\\ 2\sin\frac{2x}{9}, & \text{when } \frac{3\pi}{4}<x<\pi\end{cases}$ then **[IIT 1991]**

(a) $f(x)$ is continuous at $x=0$ (b) $f(x)$ is continuous at $x=\pi$

(c) $f(x)$ is continuous at $x=\frac{3\pi}{4}$ (d) $f(x)$ is discontinuous at $x=\frac{3\pi}{4}$

28. If $f(x)=\begin{cases}1/2-x, & 0<x<1/2\\ 0, & x=0\\ 1/2, & x=1/2\\ 3/2-x, & 1/2<x<1\\ 1, & x=1\end{cases}$, then false statement is **[Rajasthan PET 1984 (Similar to MP PET 1996)]**

(a) $f(x)$ is discontinuous at $x = 0$ (b) $f(x)$ is continuous at $x=\frac{1}{2}$

(c) $f(x)$ is discontinuous at $x=1$ (d) $f(x)$ is continuous at $x=\frac{1}{4}$

29. $f(x)=\frac{\sqrt{1+px}-\sqrt{1-px}}{x}, -1 \le x < 0 = \frac{2x+1}{x-2}, 0 \le x \le 1$ is continuous in the interval $[-1,1]$ then p equals

(a) –1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1

30. The function $f(x)=\begin{cases} x^2/a & , 0 \le x < 1 \\ a & , 1 \le x < \sqrt{2} \\ (2b^2-4b)/x^2, & \sqrt{2} \le x < \infty \end{cases}$ is continuous for $0 \le x < \infty$, then the most suitable values of a and b are

[BIT Ranchi 1984]

(a) $a=1, b=-1$ (b) $a=-1, b=1+\sqrt{2}$ (c) $a=-1, b=1$ (d) None of these

31. Let $f(x)=\begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2}, & \text{if } x \ne 2 \\ k & ,\text{If } x=2 \end{cases}$ if $f(x)$ be continuous for all x, then $k=$ **[IIT 1981]**

(a) 7 (b) –7 (c) ± 7 (d) None of these

32. If $f(x)=\begin{cases} \frac{x^2+3x-10}{x^2+2x-15}, \text{when } x \ne -5 \\ a \quad , \text{ when } x=-5 \end{cases}$ is continuous at $x=-5$, then the value of 'a' will be

(a) $\frac{3}{2}$ (b) $\frac{7}{8}$ (c) $\frac{8}{7}$ (d) $\frac{2}{3}$

33. The function $f(x)=[x]^2-[x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at **[IIT 199**

(a) All integers (b) All integers except 0 and 1 (c) All integers except 0
(d) All integers except 1

34. If $f(x)=\frac{1}{2}x-1$, then on the interval $[0, \pi]$

(a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous (b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous

(c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous (d) $\tan[f(x)]$ is continuous but $\frac{1}{f(x)}$ is not continuous

35. Let $f(x)=[x]+\sqrt{x-[x]}$, where $[x]$ denotes the greatest integer function. Then,

(a) $f(x)$ is continuous on R^+ (b) $f(x)$ is continuous on R
(c) $f(x)$ is continuous on $R-Z$ (d) None of these

36. Let $f(x)=[2x^3-5]$, [.] denotes the greatest integer function. Then number of points in (1, 2) where the function is discontinuous, is

(a) 0 (b) 13 (c) 10 (d) 3

37. The number of points at which the function $f(x)=\frac{1}{x-[x]}$ [.] denotes, the greatest integer function) is not continuous is

(a) 1 (b) 2 (c) 3 (d) None of these

38. If $f(x)=\begin{cases} \frac{\sin x}{x}+\cos x, \text{ when } x \ne 0 \\ 2 \quad , \text{ when } x=0 \end{cases}$, then

(a) $\lim_{x \to 0^+} f(x) \ne 2$ (b) $\lim_{x \to 0^-} f(x)=0$ (c) $f(x)$ is continuous at $x=0$ (d) None of these

39. The number of points at which the function $f(x)=\frac{1}{\log|x|}$ is discontinuous is

(a) 1 (b) 2 (c) 3 (d) 4

40. The function $f(x)=p[x+1]+q[x-1]$, where $[x]$ is the greatest integer function is continuous at $x=1$ if

(a) $p-q=0$ (b) $p+q=0$ (c) $p=0$ (d) $q=0$

41. Function $f(x)=\left(1+\frac{x}{a}\right)^{1/x}$ is continuous at $x=0$, if $f(0)$ equals

(a) e^a (b) e^{-a} (c) 0 (d) $e^{1/a}$

42. Let [.] denote the greatest integer function and $f(x)=[\tan^2 x]$. Then **[IIT 1993]**

(a) $\lim_{x\to 0} f(x)$ does not exist (b) $f(x)$ is continuous at $x=0$

(c) $f(x)$ is not differentiable at $x=0$ (d) $f'(0)=1$

43. The function $f(x)=\begin{cases} x+a\sqrt{2}\sin x & 0\le x<\pi/4 \\ 2x\cot x+b & \pi/4\le x\le \pi/2 \\ a\cos 2x-b\sin x & \pi/2<x\le\pi \end{cases}$ is continuous for $0\le x\le\pi$ then *a*, *b* are

(a) $\frac{\pi}{6},\frac{\pi}{12}$ (b) $\frac{\pi}{3},\frac{\pi}{6}$ (c) $\frac{\pi}{6},-\frac{\pi}{12}$ (d) None of these

44. Let $f:R\to R$ be any function. Define $g:R\to R$ by $g(x)=|f(x)|$ for all *x*, Then *g* is **[IIT 2000]**

(a) Onto if *f* is onto (b) One-one if *f* is one-one

(c) Continuous if *f* is continuous (d) Differentiable if *f* is differentiable

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Answer Sheet

Assignment (Basic & Advance Level)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
a	**a**	**c**	**b**	**b**	**a,b**	**b**	**c**	**b**	**a**	**d**	**c**	**c**	**a**	**c**	**a**	**c**	**d**	**d**	**a**
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
b	**c**	**b**	**c**	**c**	**c**	**c**	**b**	**b**	**c**	**a**	**b**	**d**	**b**	**b**	**b**	**d**	**c**	**c**	**b**
41	**42**	**43**	**44**																
d	**b**	**c**	**c**																