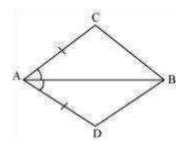
# <u>Class IX Chapter 7 – Triangles</u> <u>Maths</u>

Exercise 7.1 Question

1:

In quadrilateral ACBD, AC = AD and AB bisects ∠A (See the given figure). Show that

 $\cong$ 



Answer:

 $\Delta ABC~\Delta ABD.$  What can you say about BC and BD?

In  $\triangle$ ABC and  $\triangle$ ABD,

AC = AD (Given)

 $\angle CAB = \angle DAB (AB bisects \angle A)$ 

AB = AB (Common)

 $^{∴}$  ∆ABC  $\cong$  ∆ABD (By SAS congruence rule)

BC = BD (By CPCT)

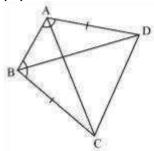
Therefore, BC and BD are of equal lengths.

## Question 2:

ABCD is a quadrilateral in which AD = BC and  $\angle$  DAB =  $\angle$  CBA (See the given figure). Prove that

- (i)  $\triangle ABD \cong \triangle BAC$
- (ii) BD = AC

(iii) ABD = BAC.



Answer:

In  $\triangle ABD$  and  $\triangle BAC$ ,

AD = BC (Given)

*L L* 

DAB = CBA (Given)

AB = BA (Common)

 $^{\perp}$   $\triangle$ ABD  $\cong$   $\triangle$ BAC (By SAS congruence rule)

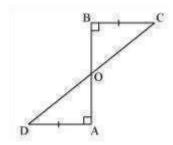
 $\dot{}$  BD = AC (By CPCT) And,  $\angle$  ABD

= BAC (By CPCT)

## Question 3:

AD and BC are equal perpendiculars to a line segment AB (See the given figure).

Show that CD bisects AB.



In  $\triangle$ BOC and  $\triangle$ AOD,

∠ BOC = AOD (Vertically opposite angles)

 $\angle$  CBO = DAO (Each 90°)

BC = AD (Given)

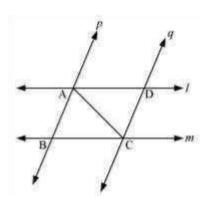
 $^{\circ}$   $\Delta$ BOC  $\cong$   $\Delta$ AOD (AAS congruence rule)

∴ BO = AO (By CPCT)

CD bisects AB.

Question 4: I and m are two parallel lines intersected by another pair of parallel lines p and q (see

the given figure). Show that  $\triangle ABC \stackrel{\cong}{\triangle} CDA$ .



## Answer:

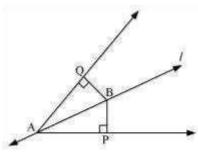
In  $\triangle ABC$  and  $\triangle CDA$ ,

 $\angle BAC = \angle DCA$  (Alternate interior angles, as p || q) AC = CA (Common)

ΔABC ΔCDA (By ASA congruence rule)

#### Question 5:

Line I  ${}_{.}$ A is the bisector of an angle and B is any point on I. BP and BQ are perpendiculars from B to the arms of  ${}_{.}$ A (see the given figure). Show that: i)  $\Delta$ APB  ${}_{.}$  $\Delta$ AQB ( ii) BP = BQ or B is equidistant from the arms of  ${}_{.}$ ( A.



Answer:

In  $\triangle APB$  and  $\triangle AQB$ ,

$$\therefore$$
 APB = AQB (Each 90°)

$$\therefore$$
 PAB = QAB (I is the angle bisector of  $\therefore$ A)

AB = AB (Common)

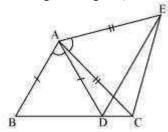
$$\triangle$$
 ΔAPB  $\triangle$  ΔAQB (By AAS congruence rule)  $\triangle$  BP = BQ (By CPCT)

rms of ∴A. Or,

it can be said that B is equidistant from the a

## Question 6:

In the given figure, AC = AE, AB = AD and ABAD = ABAC. Show that BC = DE.



#### Answer:

It is given that ∴BAD = ∴EAC

$$\therefore$$
BAD +  $\therefore$ DAC =  $\therefore$ EAC +  $\therefore$ DAC

∴BAC = ∴DAE

In  $\triangle BAC$  and  $\triangle DAE$ , AB = AD

(Given) ∴BAC =

::DAE (Proved above)

AC = AE (Given)

∴ ΔBAC ∴ ΔDAE (By SAS congruence rule)

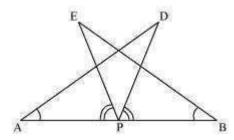
BC = DE (By CPCT)

#### Question 7:

AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that  $\cancel{B}AD = ..ABE$  and ..EPA = ..DPB (See the given figure). Show that i)

.. ΔDAP ΔEBP (

(ii) AD = BE



It is given that EPA = DPB

AP = BP (P is mid-point of AB)

ΔDAP ΔEBP (ASA congruence rule)

 $^{\perp}$  AD = BE (By CPCT)

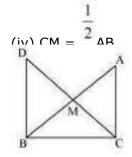
## Question 8:

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point

B (see the given figure). Show that: i)

ii) DBC is a right angle. (iii)

ΔDBC - ΔACB (



#### Answer:

(i) In  $\triangle$ AMC and  $\triangle$ BMD, AM = BM (M is the mid-point of AB)

∴AMC = ∴BMD (Vertically opposite angles)

CM = DM (Given)

∴ ΔAMC ∴ ΔBMD (By SAS congruence rule)

 $^{*}$  AC = BD (By CPCT) And,

ACM = BDM (By CPCT) ii)

ACM = BDM (

However, ACM and BDM are alternate interior angles.

Since alternate angles are equal,

It can be said that DB || AC

 $\therefore$  DBC +  $\therefore$ ACB = 180° (Co-interior angles)

 $^{\circ}$  DBC + 90° = 180°

DBC = 90°

```
(iii) In ΔDBC and ΔACB,
DB = AC (Already proved)

∴DBC = ∴ACB (Each 90°)

BC = CB (Common)

∴ ΔDBC ΔACB (SAS congruence rule) iv)

ΔDBC ΔACB (

∴ AB = DC (By CPCT)

∴ AB = 2 CM

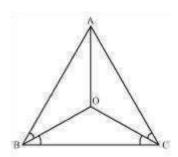
∴ CM = 2 AB
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Exercise 7.2 Question

1:

In an isosceles triangle ABC, with AB = AC, the bisectors of  $\triangle B$  and  $\triangle C$  intersect each other at O. Join A to O. Show that:

i) OB = OC (ii) AO bisects -A ( Answer:



(i) It is given that in triangle ABC, AB = AC

 $\therefore$   $\therefore$  ACB = ABC (Angles opposite to equal sides of a triangle are equal)

$$\therefore \frac{1}{2} \therefore ACB = \frac{1}{2} \therefore ABC$$

Λ.

OB = OC (Sides opposite to equal angles of a triangle are also equal)

(ii) In  $\triangle$ OAB and  $\triangle$ OAC, AO =AO (Common)

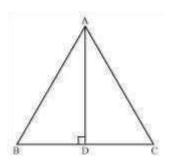
$$AB = AC (Given)$$

Therefore, ∆OAB <sup>\*</sup>∆OAC (By SSS congruence rule)

" AO bisects A.

#### Question 2:

In  $\triangle$ ABC, AD is the perpendicular bisector of BC (see the given figure). Show that  $\triangle$ ABC is an isosceles triangle in which AB = AC.



#### Answer:

In  $\triangle$ ADC and  $\triangle$ ADB,

$$AD = AD (Common)$$

$$ADC = ADB (Each 90^{\circ})$$

CD = BD (AD is the perpendicular bisector of BC)

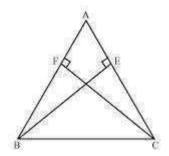
∴ ΔADC ∴ ΔADB (By SAS congruence rule)

AB = AC (By CPCT)

Therefore, ABC is an isosceles triangle in which AB = AC.

## Question 3:

ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see the given figure). Show that these altitudes are equal.



## Answer:

In  $\triangle AEB$  and  $\triangle AFC$ ,

 $^{\circ}$ AEB and AFC (Each 90°) A =

"A (Common angle)

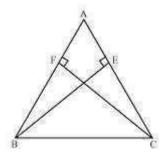
$$AB = AC (Given)$$

∴ 
$$\triangle$$
AEB ∴  $\triangle$ AFC (By AAS congruence rule) ∴ BE = CF (By CPCT)

Question 4:

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see the

given figure). Show that   
 (i) 
$$^{\Delta}\!\mathsf{ABE} : ^{\Delta}\!\mathsf{ACF}$$



Answer:

(ii) AB = AC, i.e., ABC is an isosceles triangle.

```
(i) In \triangleABE and \triangleACF,
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ABE and ACF (Each 90°)

 $^{-1}A = A^{-1}(Common angle)$ 

BE = CF (Given)

- .. ΔABE .. ΔACF (By AAS congruence rule)
- (ii) It has already been proved that

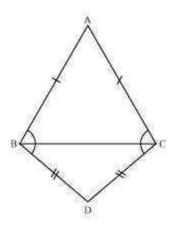
ΔABE ΔACF

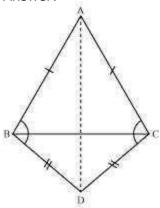
∴ AB = AC (By CPCT)

Question 5:

ABC and DBC are two isosceles triangles on the same base BC (see the given figure).

Show that .ABD = .ACD.





Let us join AD.

In  $\triangle$ ABD and  $\triangle$ ACD,

AB = AC (Given)

BD = CD (Given)

AD = AD (Common side)

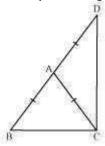
∴ ΔABD ΔACD (By SSS congruence rule)

∴ ∴ ABD = ÆCD (By CPCT)

## Question 6:

 $\Delta ABC$  is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD

= AB (see the given figure). Show that ∴BCD is a right angle.



Answer:

In ΔABC,

$$AB = AC (Given)$$

∴ ∴ ACB = ∴ABC (Angles opposite to equal sides of a triangle are also equal)

In ΔACD,

AC = AD

∴ ∴ ADC = ∴ACD (Angles opposite to equal sides of a triangle are also equal)

In ΔBCD,

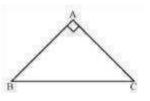
 $^{\circ}ABC + BCD + ADC = 180^{\circ}$  (Angle sum property of a triangle)

$$^{\circ}$$
 ACB + ACB + ACD +  $^{\circ}$  ACD = 180°

Question 7:

ABC is a right angled triangle in which  $\triangle A = 90^{\circ}$  and AB = AC. Find  $\triangle B$  and  $\triangle C$ .

Answer:



is given that

$$AB = AC$$

 $\dot{C}$  = B (Angles opposite to equal sides are also equal)

 $^{\circ}A + B + C = 180^{\circ}$  (Angle sum property of a triangle)

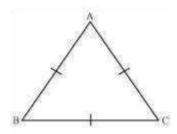
$$^{\circ}$$
 90° + B + C = 180°

$$B = C = 45^{\circ}$$

## Question 8:

Show that the angles of an equilateral triangle are 60° each.

Answer:



Let us consider that ABC is an equilateral triangle.

Therefore, AB = BC = AC

$$AB = AC$$

-: C:= B (Angles opposite to equal sides of a triangle are equal)

Also,

AC = BC

 $\dot{B} = A$  (Angles opposite to equal sides of a triangle are equal)

Therefore, we obtain ∴A

=  $B = C \cdot \Delta$ In  $\Delta ABC$ ,

 $\dot{A} + B + C = 180^{\circ}$ 

· A + A + A = 180°

. 3 Å = 180°

... Å = 60°

 $\overset{\cdot \cdot}{A} = \overset{\cdot \cdot}{B} = \overset{\cdot \cdot}{C} = 60^{\circ}$  Hence, in an equilateral triangle, all interior angles are of measure 60°.

#### Exercise 7.3

#### Question 1:

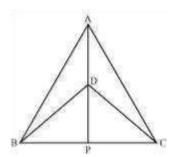
 $\Delta$ ABC and  $\Delta$ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see the given figure). If AD is extended to intersect

BC at P, show that

i) ΔABD ... ΔACD ( ii) ΔABP ΔACP

( iii) AP bisects ∴A as well as D. (

(iv) AP is the perpendicular bisector of BC.



Answer:

(i) In  $\triangle$ ABD and  $\triangle$ ACD,

AB = AC (Given)

BD = CD (Given)

AD = AD (Common)

∴ ΔABD ΔACD (By SSS congruence rule)

" BAD = CAD (By CPCT)

... ... BAP = CAP .... (1)

(ii) In  $\triangle$ ABP and  $\triangle$ ACP,

AB = AC (Given)

BAP = CAP [From equation (1)]

AP = AP (Common)

∴ ΔABP ∴ ΔACP (By SAS congruence rule)

BP = CP (By CPCT) ... (2)

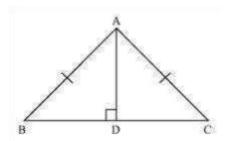
(iii) From equation (1),

∴BAP = ∴CAP

Hence, AP bisects A.

In  $\triangle BDP$  and  $\triangle CDP$ ,

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BD = CD (Given)
DP = DP (Common)
BP = CP [From equation (2)]
∴ ΔBDP ΔCDP (By S.S.S. Congruence rule)
" BDP = CDP (By CPCT) ... (3) Hence,
AP bisects D. iv) ΔBDP ...
ΔCDP (
∴ BPD = CPĐ (By CPCT) .... (4)
\therefore BPD + \therefore CPD = 180 (Linear pair angles)
  BPD +
           BPD = 180
         = 180 [From equation (4)]
 BPD 2
 BPD = 90 ... (5)
From equations (2) and (5), it can be said that AP is the perpendicular bisector of BC.
Question 2:
AD is an altitude of an isosceles triangles ABC in which AB = AC. Show that
i) AD bisects BC (ii) AD bisects .A. (
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(i) In  $\triangle BAD$  and  $\triangle CAD$ ,

ADB = ADC (Each 90° as AD is an altitude)

AB = AC (Given)

AD = AD (Common)

<sup>∴</sup> ΔBAD ∴ ΔCAD (By RHS Congruence rule)

.. BD = CD (By CPCT)

Hence, AD bisects BC.

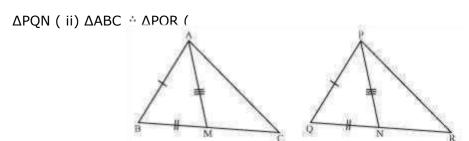
(ii) Also, by CPCT,

BAD = CAD Hence, AD

bisects A. ...

## Question 3:

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of  $\Delta$ PQR (see the given figure). Show that: i)  $\Delta$ ABM



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(i) In  $\triangle$ ABC, AM is the median to BC.

$$\therefore BM = \frac{1}{2}_{BC}$$

$$\therefore QN = \frac{1}{2}QR$$

However, BC = QR

$$\frac{1}{2}_{BC} = \frac{1}{2}_{QR}$$

In  $\Delta ABM$  and  $\Delta PQN_{\mbox{\scriptsize ,}} In$   $\Delta PQR_{\mbox{\scriptsize ,}}$  PN is the median to QR.

AB = PQ (Given)

BM = QN [From equation (1)]

AM = PN (Given)

∴ ΔABM ΔPQN (SSS congruence rule)

(ii) In  $\triangle$ ABC and  $\triangle$ PQR,

$$AB = PQ (Given)$$

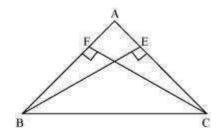
$$BC = QR (Given)$$

∴ ΔABC ∴ ΔPQR (By SAS congruence rule)

Question 4:

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

## Answer:



In  $\triangle$ BEC and  $\triangle$ CFB,

∴BEC = ∴CFB (Each 90°)

BC = CB (Common) BE = CF (Given)

- ΔBEC ΔCFB (By RHS congruency)

BCE = CBF (By CPCT)

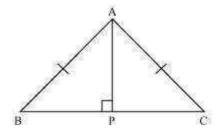
 $^{\circ}$  AB = AC (Sides opposite to equal angles of a triangle are equal)

Hence,  $\triangle ABC$  is isosceles.

Question 5:

A A

ABC is an isosceles triangle with AB = AC. Drawn AP  $\div$  BC to show that B = C.



In  $\triangle APB$  and  $\triangle APC$ ,

 $\therefore APB = \therefore APC (Each 90^{\circ})$ 

AB = AC (Given)

AP = AP (Common)

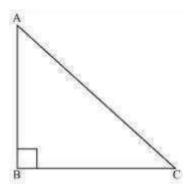
∴ ΔAPB ΔAPC (Using RHS congruence rule)

 $\ddot{B} = C$  (By using CPCT)

Exercise 7.4 Question 1:

Show that in a right angled triangle, the hypotenuse is the longest side.

Answer:



Let us consider a right-angled triangle ABC, right-angled at B.

# In ΔABC,

 $\dot{A} + B + C = 180^{\circ}$  (Angle sum property of a triangle)

$$^{..}A + 90^{\circ} + C^{.} = 180^{\circ}$$

Hence, the other two angles have to be acute (i.e., less than 90°).

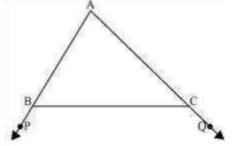
 $\dot{}$   $\dot{}$  B is the largest angle in  $\Delta ABC$ .

[In any triangle, the side opposite to the larger (greater) angle is longer.] Therefore, AC is the largest side in  $\triangle$ ABC.

However, AC is the hypotenuse of  $\Delta$ ABC. Therefore, hypotenuse is the longest side in a right-angled triangle.

#### Question 2:

In the given figure sides AB and AC of  $\triangle$ ABC are extended to points P and Q respectively. Also,  $\triangle$ PBC <  $\triangle$ OCB. Show that AC > AB.



#### Answer:

In the given figure,

$$\dot{ABC} + PBC = 180^{\circ}$$
 (Linear pair)

$$^{\circ}$$
 ABC = 180° - ...PBC ... (1)

Also,

$$^{\circ}$$
ACB +  $^{\circ}$ QCB = 180°

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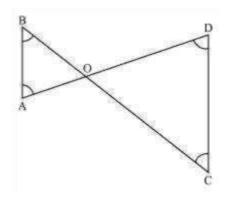
$$ACB = 180^{\circ} - QCB ... (2)$$

As PBC < QCB,

... ABC > ACB [From equations (1) and (2)] .. AC >

AB (Side opposite to the larger angle is larger.) Question 3:

In the given figure,  $\dot{B} < \dot{A}$  and  $\dot{C} < \dot{D}$ . Show that AD < BC.



Answer:

In ΔAOB,

 $\dot{\cdot}$  B  $\dot{\cdot}$  AO < BO (Side opposite to smaller angle is smaller) ... (1)

In ΔCOD,

∴ C ∹< D

 $^{\circ}$  OD < OC (Side opposite to smaller angle is smaller) ... (2)

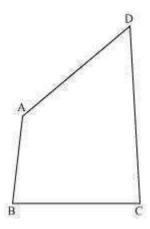
On adding equations (1) and (2), we obtain

AO + OD < BO + OC

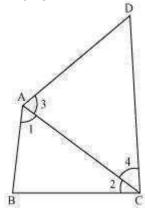
AD < BC

Question 4:

AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD see the given figure). Show that A > C and B > C.



#### Answer:



Let us join AC. In  $\triangle$ ABC,

AB < BC (AB is the smallest side of quadrilateral ABCD)

 $\div$   $\div$  2.4< 1 (Angle opposite to the smaller side is smaller) ... (1)

In ΔADC,

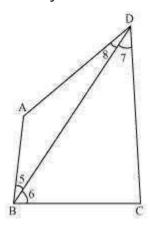
AD < CD (CD is the largest side of quadrilateral ABCD)

 $\div$  4 < 3 (Angle opposite to the smaller side is smaller) ... (2)

On adding equations (1) and (2), we obtain

$$\cdot \cdot C < \cdot \cdot A$$

∴ ∴A > ∴C Let us join BD.



In ΔABD,

AB < AD (AB is the smallest side of quadrilateral ABCD)

 $\dot{}$  8 < 5 (Angle opposite to the smaller side is smaller) ... (3)

In ΔBDC,

BC < CD (CD is the largest side of quadrilateral ABCD)

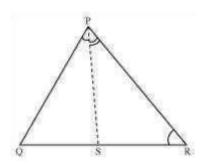
...7. < 6 (Angle opposite to the smaller side is smaller) ... (4)

On adding equations (3) and (4), we obtain

 $\ddot{B} > D \dot{Q}$ uestion

5.

In the given figure, PR > PQ and PS bisects ::QPR. Prove that ::PSR > ::PSQ.



As PR > PQ,

 $\therefore$  PQR > PRQ (Angle opposite to larger side is larger) ... (1) PS is the bisector of QPR.

 $^{\circ}$  PSR is the exterior angle of  $\Delta$ PQS.

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 $_{..}$  ... PSQ is the exterior angle of  $\Delta PRS.$ 

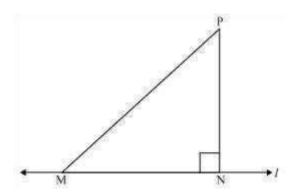
Adding equations (1) and (2), we obtain

$$\dot{P}$$
PQR +  $\dot{Q}$ PS > PR $\dot{Q}$  +  $\dot{R}$ PS

PSR > PSQ [Using the values of equations (3) and (4)]

## Question 6:

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.



Let us take a line I and from point P (i.e., not on line I), draw two line segments PN and PM. Let PN be perpendicular to line I and PM is drawn at some other angle.

In ΔPNM,

∴N = 90°

 $^{.}$ P + N + M  $\stackrel{.}{=}$  180° (Angle sum property of a triangle)

 $\dot{P} + \dot{M} = 90^{\circ}$ 

Clearly,  $\dot{M}$  is an acute angle.

 $\overset{\cdot \cdot }{\stackrel{\cdot \cdot }{M}}< N \overset{\cdot \cdot }{\stackrel{\cdot \cdot }{}}$ 

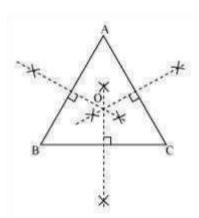
PN < PM (Side opposite to the smaller angle is smaller)

Similarly, by drawing different line segments from P to I, it can be proved that PN is smaller in comparison to them.

Therefore, it can be observed that of all line segments drawn from a given point not
on it, the perpendicular line segment is the shortest.
Exercise 7.5 Question
1:
ADC is a twistered. I seeks a maint in the interior of AADC which is acreditatent from all
ABC is a triangle. Locate a point in the interior of $\triangle$ ABC which is equidistant from all
the vertices of $\triangle ABC$ .
Answer:
Circumcentre of a triangle is always equidistant from all the vertices of that triangle.

Circumcentre is the point where perpendicular bisectors of all the sides of the triangle

meet together.



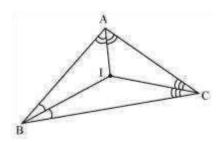
In  $\triangle$ ABC, we can find the circumcentre by drawing the perpendicular bisectors of sides AB, BC, and CA of this triangle. O is the point where these bisectors are meeting together. Therefore, O is the point which is equidistant from all the vertices of  $\triangle$ ABC.

## Question 2:

In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

#### Answer:

The point which is equidistant from all the sides of a triangle is called the incentre of the triangle. Incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.



Here, in  $\triangle$ ABC, we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. I is the point where these angle bisectors are intersecting each other. Therefore, I is the point equidistant from all the sides of  $\triangle$ ABC.

## Question 3:

In a huge park people are concentrated at three points (see the given figure)

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A: where there are different slides and swings for children,

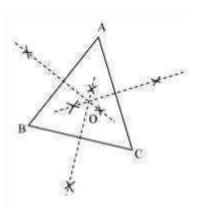
B: near which a man-made lake is situated,

C: which is near to a large parking and exit.

Where should an ice-cream parlour be set up so that maximum number of persons can approach it?

(Hint: The parlor should be equidistant from A, B and C) Answer:

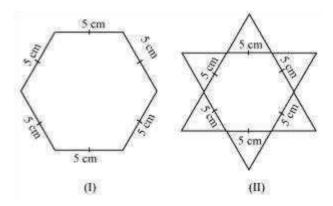
Maximum number of persons can approach the ice-cream parlour if it is equidistant from A, B and C. Now, A, B and C form a triangle. In a triangle, the circumcentre is the only point that is equidistant from its vertices. So, the ice-cream parlour should be set up at the circumcentre O of  $\Delta ABC$ .



In this situation, maximum number of persons can approach it. We can find circumcentre O of this triangle by drawing perpendicular bisectors of the sides of this triangle.

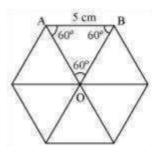
#### Question 4:

Complete the hexagonal and star shaped rangolies (see the given figures) by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



### Answer:

It can be observed that hexagonal-shaped rangoli has 6 equilateral triangles in it.



Area of 
$$\triangle OAB$$
 
$$= \frac{\sqrt{3}}{4} (side)^2 = \frac{\sqrt{3}}{4} (5)^2$$

$$= \frac{\sqrt{3}}{4} (25) = \frac{25\sqrt{3}}{4} \text{ cm}^2$$

$$=6 \times \frac{25\sqrt{3}}{4} = \frac{75\sqrt{3}}{2} \text{ cm}^2$$

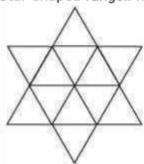
Area of hexagonal-shaped rangoli

Area of equilateral triangle having its side as 1 cm =  $\frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}$  cm<sup>2</sup>

Number of equilateral triangles of 1 cm side that can be filled

in this hexagonal-shaped 
$$rangoli = \frac{\frac{75\sqrt{3}}{2}}{\frac{\sqrt{3}}{4}} = 150$$

Star-shaped rangoli has 12 equilateral triangles of side 5 cm in it.



Area of star-shaped rangoli = 
$$12 \times \frac{\sqrt{3}}{4} \times (5)^2 = 75\sqrt{3}$$

Number of equilateral triangles of 1 cm side that can be filled

in this star-shaped rangoli 
$$=\frac{75\sqrt{3}}{\frac{\sqrt{3}}{4}} = 300$$

Therefore, star-shaped rangoli has more equilateral triangles in it.