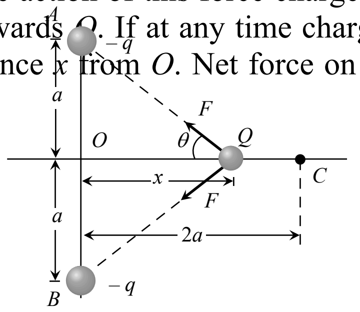
Critical Thinking Questions

1. (d) By symmetry of problem the components of force on Q due to charges at A and B along y-axis will cancel each other while along x-axis will add up and will be along CO. Under the action of this force charge Q will move towards O. If at any time charge Q is at a distance x from O. Net force on charge Q



$F_{net} \Rightarrow 2F\cos\theta = 2\frac{1}{4\pi\varepsilon_0}\frac{-qQ}{(a^2+x^2)}\times\frac{x}{(a^2+x^2)^{1/2}}$

i.e., $F_{net} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQx}{\left(a^2+x^2\right)^{3/2}}$

As the restoring force F_{net} is not linear, motion will be oscillatory (with amplitude $2a$) but not simple harmonic.

2. (c) Charge will move along the circular line of force because $x^2+y^2=1$ is the equation of circle in xy-plane.

3. (a) Because of the presence of positive test charge q_0 in front of positively charged ball, charge on the ball will be redistributed, less charge on the front half surface and more charge on the back half surface. As a result of this net force F between ball and point charge will decrease *i.e.* actual electric field will be greater than F/q_0.

4. (c) Electric field at a distance R is only due to sphere because electric field due to shell inside it is always zero. Hence electric field $=\frac{1}{4\pi\varepsilon_0}\cdot\frac{3Q}{R^2}$

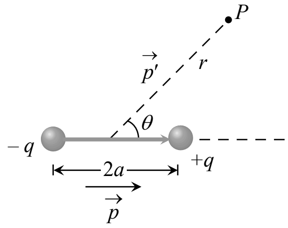
5. (d) $q_1+q_2=Q$ and $\frac{q_1}{4\pi r^2}=\frac{q_2}{4\pi R^2}$ (given)

$q_1=\frac{Qr^2}{R^2+r^2}$ and $q_2=\frac{QR^2}{R^2+r^2}$

Potential at common centre

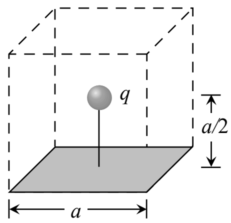
$\frac{1}{4\pi\varepsilon_0}\left[\frac{Qr^2}{(R^2+r^2)r}+\frac{QR^2}{(R^2+r^2)R}\right]=\frac{Q(R+r)}{4\pi\varepsilon_0(R^2+r^2)}$

6. (a) For the given situation, diagram can be drawn as follows



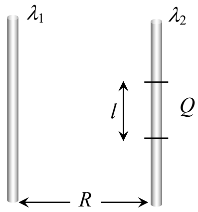
As shown in figure component of dipole moment along the line OP will be $p' = p\cos\theta$. Hence electric potential at point P will be $V=\frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$

7. (d) An imaginary cube can be made by considering charge q at the centre and given square is one of it's face.



So flux from given square (*i.e.* one face) $\phi=\frac{q}{6\varepsilon_0}$

8. (b) Force on l length of the wire 2 is

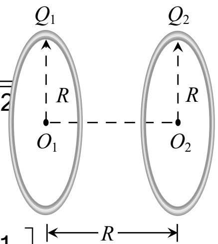


$F_2 = QE_1 = (\lambda_2 l)\frac{2k\lambda_1}{R}$

$\Rightarrow \frac{F_2}{l}=\frac{2k\lambda_1\lambda_2}{R}$

Also $\frac{F_1}{l}=\frac{F_2}{l}=\frac{F}{l}=\frac{2k\lambda_1\lambda_2}{R}$

9. (b) $W=q(V_{O_2}-V_{O_1})$



where $V_{O_1}=\frac{Q_1}{4\pi\varepsilon_0 R}+\frac{Q_2}{4\pi\varepsilon_0 R\sqrt{2}}$

and $V_{O_2}=\frac{Q_2}{4\pi\varepsilon_0 R}+\frac{Q_1}{4\pi\varepsilon_0 R\sqrt{2}}$

$\Rightarrow V_{O_2}-V_{O_1}=\frac{(Q_2-Q_1)}{4\pi\varepsilon_0 R}\left[1-\frac{1}{\sqrt{2}}\right]$

So, $W=\frac{q.(Q_2-Q_1)}{4\pi\varepsilon_0 R}\frac{(\sqrt{2}-1)}{\sqrt{2}}$

10. (c, d) Under electrostatic condition, all points lying on the conductor are in same potential. Therefore, potential at A = potential at B.

From Gauss's theorem, total flux through the surface of the cavity will be q/ε_0.

Note : ❑ Instead of an elliptical cavity, if it would had been a spherical cavity then options (a) and (b) were also correct.

11. (d) $V = \frac{q}{4\pi\varepsilon_0 x_0}\left[1+\frac{1}{3}+\frac{1}{5}+...\right] - \frac{q}{4\pi\varepsilon_0 x_0}\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...\right]$

$= \frac{q}{4\pi\varepsilon_0 x_0}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+....\right] = \frac{q}{4\pi\varepsilon_0 x_0}\log_e 2$

12. (a, c) Here $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qz_0}{(R^2+z_0^2)^{3/2}}$

where Q is the charge on ring and z_0 is the distance of the point from origin.

Then $F = qE = \frac{-Qqz_0}{4\pi\varepsilon_0(R^2+z_0^2)^{3/2}}$

When charge $-q$ crosses origin, force is again towards centre *i.e.*, motion is periodic. Now if $z_0 << R$

$\therefore\ F = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qqz_0}{R^2} \Rightarrow F \propto -z_0$ *i.e.*, motion is S.H.M.

13. (a, c)For non-conducting solid sphere $E_{in} \propto r$

and $E_{out} \propto \frac{1}{r^2}$

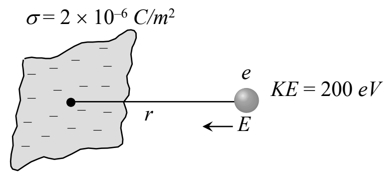
i.e. for $r < R$; E increases as r increases

and for $R < r < \infty$; E decreases as r increases

14. (a) $\int_{-\infty}^{0} -\vec{E}\cdot\vec{dl} =$ potential at centre of non-conducting ring

$= \frac{1}{4\pi\varepsilon_0}\times\frac{q}{r} = \frac{9\times10^9\times1.11\times10^{-10}}{0.5} = 2\ volt$

15. (a) Let an electron is projected towards the plate from the r distance as shown in fig.



It will not strike the plate if and only if $KE \le e(E\cdot r)$ (where E = Electric field due to charge plate $= \frac{\sigma}{2\varepsilon_0}$)

$\Rightarrow r \ge \frac{KE}{eE}$. Hence minimum value of r is given by

$r = \frac{KE}{eE} = \frac{200\,eV}{e\times\frac{\sigma}{2\varepsilon_0}} = \frac{400\times8.86\times10^{-12}}{2\times10^{-6}} = 1.77\,mm$

16. (b) $Q = ne$; where n = number of moles × 6.02 × 10^{23} × 10

$\Rightarrow Q = \frac{500}{18}\times6.02\times10^{23}\times10\times1.6\times10^{-19} = 2.67\times10^7\,C$

17. (d) $E_x = -\frac{dV}{dx} = -(6-8y^2),\ E_y = -\frac{dV}{dy} = -(-16xy-8+6z)$

$E_z = -\frac{dV}{dz} = -(6y-8z)$

At origin $x = y = z = 0$ so, $E_x = -6, E_y = 8$ and $E_z = 0$

$\Rightarrow E = \sqrt{E_x^2 + E_y^2} = 10\ N/C$.

Hence force $F = QE = 2\times10 = 20\,N$

18. (b) Flux linked with the given sphere $\phi = \frac{Q}{\varepsilon_o}$;

where Q = Charge enclosed by the sphere.

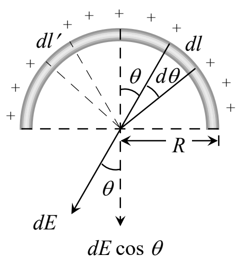
Hence $Q = \phi\varepsilon_o = (EA)\varepsilon_o$

$\Rightarrow Q = 4\pi(\gamma_o)^2 \times A\gamma_o\varepsilon_o = 4\pi\varepsilon_o A\gamma_o^3$.

19. (a) From figure $dl = R\,d\theta$;

Charge on $dl = \lambda R\,d\theta$ $\quad\left\{\lambda = \frac{q}{\pi R}\right\}$

Electric field at centre due to dl is $dE = k.\frac{\lambda R d\theta}{R^2}$.



We need to consider only the component $dE\cos\theta$, as the component $dE\ \sin\theta$ will cancel out because of the field at C due to the symmetrical element dl'.

Total field at centre $= 2\int_0^{\pi/2} dE\cos\theta$

$= \frac{2k\lambda}{R}\int_0^{\pi/2}\cos\theta\,d\theta = \frac{2k\lambda}{R} = \frac{q}{2\pi^2\varepsilon_0 R^2}$

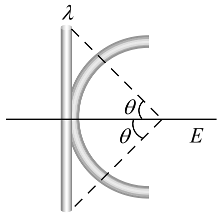
Alternate method : As we know that electric field due to a finite length charged wire on it's perpendicular bisector is given by $E = \frac{2k\lambda}{R}\sin\theta$.

If it is bent in the form of a semicircle then $\theta = 90°$

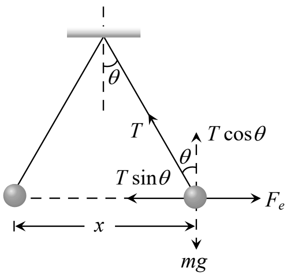
$\Rightarrow E = \frac{2k\lambda}{R}$

$= 2 \times \frac{1}{4\pi\varepsilon_0}\left(\frac{q/\pi R}{R}\right)$

$= \frac{q}{2\pi^2\varepsilon_0 R^2}$



20. (a)



In equilibrium $F_e = T\sin\theta$ .... (i)

$mg = T\cos\theta$ .... (ii)

$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\varepsilon_o x^2 \times mg}$ also $\tan\theta \approx \sin\theta = \frac{x/2}{L}$

Hence $\frac{x}{2L} = \frac{q^2}{4\pi\varepsilon_o x^2 \times mg}$

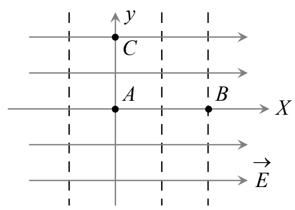
$\Rightarrow x^3 = \frac{2q^2 L}{4\pi\varepsilon_o mg} \Rightarrow x = \left(\frac{q^2 L}{2\pi\varepsilon_o mg}\right)^{1/3}$

21. (d) Outside the charged sphere, (for equal distances from centre) if electric fields at two points are same then both points must be equipotential points.

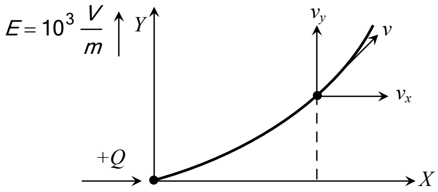
22. (c) Option (a) shows lines of force starting from one positive charge and terminating at another. Option (b) has one line of force making closed loop. Option (d) shows all lines making closed loops. All these are not correct. Only option (c) is correct.

23. (b) Potential decreases in the direction of electric field. Dotted lines are equipotential lines

$\therefore V_A = V_C$ and $V_A > V_B$



24. (c) Body moves along the parabolic path.



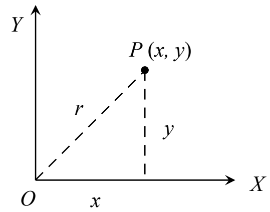
For vertical motion : By using $v = u + at$

$\Rightarrow v_y = 0 + \frac{QE}{m}.t = \frac{10^{-6} \times 10^3}{10^{-3}} \times 10 = 10\, m/sec$

For horizontal motion – It's horizontal velocity remains the same *i.e.* after 10 *sec*, horizontal velocity of body $v_x = 10\ m/sec$.

Velocity after 10 *sec* $v = \sqrt{v_x^2 + v_y^2} = 10\sqrt{2}\ m/sec$

25. (b) $E_x = -\frac{dV}{dx} = -ky$; $E_y = -\frac{dV}{dy} = -kx$

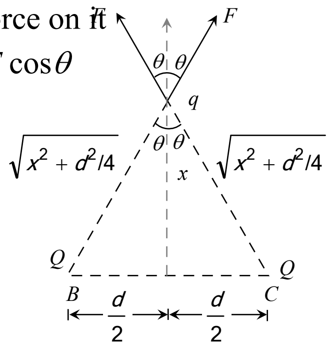


$\Rightarrow E = \sqrt{E_x^2 + E_y^2} = k\sqrt{x^2 + y^2} = kr \Rightarrow E \propto r$

26. (c) Suppose third charge is similar to Q and it is q

So net force on it

$F_{net} = 2F\cos\theta$



Where $F = \frac{1}{4\pi\varepsilon_0}.\frac{Qq}{\left(x^2 + \frac{d^2}{4}\right)}$ and

$\cos\theta = \frac{x}{\sqrt{x^2 + \frac{d^2}{4}}}$

$$\therefore F_{net} = 2\times\frac{1}{4\pi\varepsilon_0}.\frac{Qq}{\left(x^2+\frac{d^2}{4}\right)}\times\frac{x}{\left(x^2+\frac{d^2}{4}\right)^{1/2}}$$

$$= \frac{2Qqx}{4\pi\varepsilon_0\left(x^2+\frac{d^2}{4}\right)^{3/2}}$$

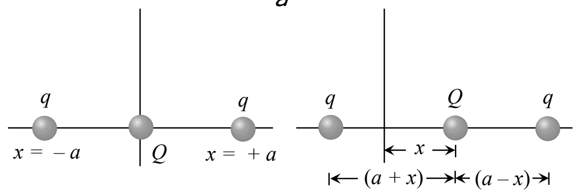
For F_{net} to be maximum $\frac{dF_{net}}{dx} = 0$

i.e. $\frac{d}{dx}\left[\frac{2Qqx}{4\pi\varepsilon_0\left(x^2+\frac{d^2}{4}\right)^{3/2}}\right] = 0$

or $\left[\left(x^2+\frac{d^2}{4}\right)^{-3/2} - 3x^2\left(x^2+\frac{d^2}{4}\right)^{-5/2}\right] = 0$

i.e. $x = \pm\frac{d}{2\sqrt{2}}$

27. (b) Initially according to figure (i) potential energy of Q is $U_i = \frac{2kqQ}{a}$ ......(i)



(i) (ii)

According to figure (ii) when charge Q is displaced by small distance x then it's potential energy now

$U_f = kqQ\left[\frac{1}{(a+x)}+\frac{1}{(a-x)}\right] = \frac{2kqQa}{(a^2-x^2)}$ ....(ii)

Hence change in potential energy

$\Delta U = U_f - U_i = 2kqQ\left[\frac{a}{a^2-x^2}-\frac{1}{a}\right] = \frac{2kqQx^2}{(a^2-x^2)}$

Since $x << a$ so $\Delta U = \frac{2kqQx^2}{a^2} \Rightarrow \Delta U \propto x^2$

28. (a) Suppose distance of closest approach is r, and according to energy conservation applied for elementary charge.

Energy at the time of projection = Energy at the distance of closest approach

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}.\frac{(Ze).e}{r} \Rightarrow r = \frac{Ze^2}{2\pi\varepsilon_0 mv^2}$

29. (a) When dipole is given a small angular displacement θ about it's equilibrium position, the restoring torque will be

$\tau = -pE\sin\theta = -pE\theta$ (as $\sin\theta = \theta$)

or $I\frac{d^2\theta}{dt^2} = -pE\theta$ (as $\tau = I\alpha = I\frac{d^2\theta}{dt^2}$)

or $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ with $\omega^2 = \frac{pE}{I} \Rightarrow \omega = \sqrt{\frac{pE}{I}}$

30. (c) Electric field is perpendicular to the equipotential surface and is zero every where inside the metal.

31. (c) $E = \frac{1}{4\pi\varepsilon_0}.\left[\frac{5\times10^{-9}}{(1\times10^{-2})^2}-\frac{5\times10^{-9}}{(2\times10^{-2})^2}+\frac{5\times10^{-9}}{(4\times10^{-2})^2} - \frac{(5\times10^{-9})}{(8\times10^{-2})^2}+.....\right]$

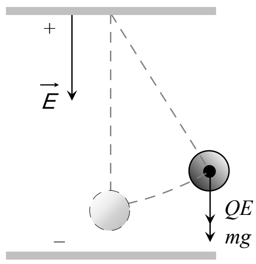
$\Rightarrow E = \frac{9\times10^9\times5\times10^{-9}}{10^{-4}}\left[1-\frac{1}{(2)^2}+\frac{1}{(4)^2}-\frac{1}{(8)^2}+...\right]$

$\Rightarrow E = 45\times10^4\left[1+\frac{1}{(4)^2}+\frac{1}{(16)^2}+...\right] - 45\times10^4\left[\frac{1}{(2)^2}+\frac{1}{(8)^2}+\frac{1}{(32)^2}+...\right]$

$\Rightarrow E = 45\times10^4\left[\frac{1}{1-\frac{1}{16}}\right] - \frac{45\times10^4}{(2)^2}\left[1+\frac{1}{4^2}+\frac{1}{(16)^2}+..\right]$

$E = 48\times10^4 - 12\times10^4 = 36\times10^4\ N/C$

32. (c)

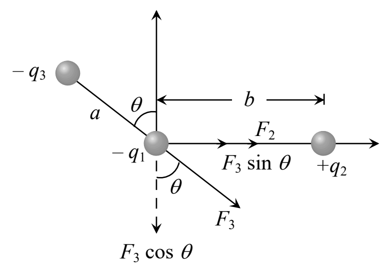


Net downward force $mg' = mg + QE$

$\Rightarrow$ Effect acceleration $g' = \left(g+\frac{QE}{m}\right)$

Hence time period $T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\left(g+\frac{QE}{m}\right)}}$

33. (c)



F_2 = Force applied by q_2 on $-q_1$

F_3 = Force applied by $(-q_3)$ on $-q_1$

x-component of Net force on $-q_1$ is

$F_x = F_2 + F_3 \sin\theta = k\frac{q_1q_2}{b^2} + k.\frac{q_1q_3}{a^2}\sin\theta$

$\Rightarrow F_x = k\left[\frac{q_1q_2}{b^2} + \frac{q_1q_3}{a^2}\sin\theta\right]$

$\Rightarrow F_x = k\cdot q_1\left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right] \Rightarrow$

$F_x \propto \left(\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right)$

34. (a) In case of a charged conducting sphere

$V_{\text{inside}} = V_{\text{centre}} = V_{\text{surface}} = \frac{1}{4\pi\varepsilon_o}.\frac{q}{R}, \quad V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$

If a and b are the radii of sphere and spherical shell respectively, then potential at their surface will be

$V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{a}$ and $V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{b}$

$\therefore V = V_{\text{sphere}} - V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}.\left[\frac{Q}{a} - \frac{Q}{b}\right]$

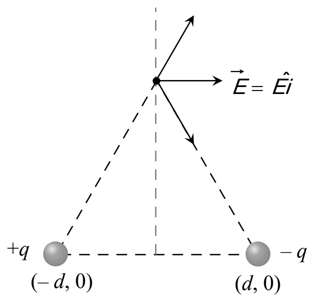
Now when the shell is given charge $(-3Q)$, then the potential will be

$V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} + \frac{(-3Q)}{b}\right], \quad V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b} + \frac{(-3Q)}{b}\right]$

$\therefore V_{\text{sphere}} - V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} - \frac{Q}{b}\right] = V$

35. (a) From figure, it is clear that $\vec{E}$ at all points on the y-axis is along $\hat{i}$. Here $\vec{E}$ of all points on x-axis cannot have the same direction.

Here electric potential at origin is zero so no work is done in bringing a test charge from infinity to origin.



Here dipole moment is in $-x$ direction ($-q$ to $+q$).

Hence only option (a) is correct.

36. (a) By the concept of electrical image, it is considered that an equal but opposite charge present on the other side of the plate at equal distance. Hence force

$F = \frac{40\times 40}{4^2} = 100\ dynes$

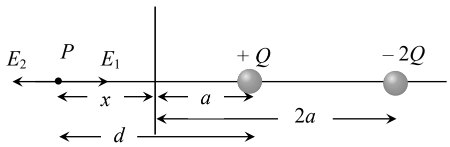
37. (d) Energy $= \frac{1}{2}\varepsilon_0 E^2 \times (A\times d) = \frac{1}{2}\varepsilon_0\left(\frac{V^2}{d^2}\right)Ad$

$= \frac{1}{2}\times\frac{8.85\times 10^{-12}\times(10^5)^2\times 25\times 10^6}{0.75\times 10^3} = 1475\,J$

38. (b) Suppose electric field is zero at a point P lies at a distance d from the charge $+Q$.

At P $\frac{kQ}{d^2} = \frac{k(2Q)}{(a+d)^2}$

$\Rightarrow \frac{1}{d^2} = \frac{2}{(a+d)^2} \Rightarrow d = \frac{a}{(\sqrt{2}-1)}$



Since $d > a$ *i.e.* point P must lies on negative x-axis as shown at a distance x from origin hence $x = d - a = \frac{a}{(\sqrt{2}-1)} - a = \sqrt{2}\,a$. Actually P lies on negative x-axis so $x = -\sqrt{2}\,a$

39. (d) If the charges are arranged according to the option (d), the electric fields due to P and S and due to Q and T add to zero, while due to U and R will be added up.

40. (d) Charge q will momentarily come to rest at a distance r from charge Q when all it's kinetic energy converted to potential energy *i.e.* $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}.\frac{qQ}{r}$

Therefore the distance of closest approach is given by

$r = \frac{qQ}{4\pi\varepsilon_0}.\frac{2}{mv^2} \Rightarrow r \propto \frac{1}{v^2}$

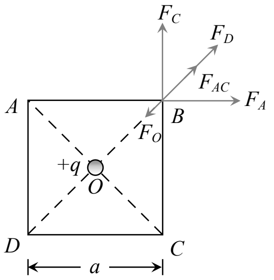
Hence if v is doubled, r becomes one fourth.

41. (b) If all charges are in equilibrium , system is also in equilibrium.

Charge at centre : charge q is in equilibrium because no net force acting on it corner charge :

If we consider the charge at corner B. This charge will experience following forces

$$F_A = k\frac{Q^2}{a^2},\ F_C = \frac{kQ^2}{a^2},\ F_D = \frac{kQ^2}{(a\sqrt{2})^2} \text{ and } F_O = \frac{KQq}{(a\sqrt{2})^2}$$



Force at B away from the centre = $F_{AC} + F_D$

$$= \sqrt{F_A^2 + F_C^2} + F_D = \sqrt{2}\frac{kQ^2}{a^2} + \frac{kQ^2}{2a^2} = \frac{kQ^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

Force at B towards the centre $= F_O = \frac{2kQq}{a^2}$

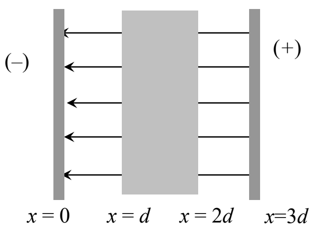
For equilibrium of charge at B, $F_{AC} + F_D = F_O$

$$\Rightarrow \frac{KQ^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right) = \frac{2KQq}{a^2} \Rightarrow q = \frac{Q}{4}\left(1 + 2\sqrt{2}\right)$$

42. (c) Capacitance will increase but not 5 times (because dielectric is not filled completely). Hence new capacitance may be 200 $\mu\mu F$.

43. (b, c) Even after introduction of dielectric slab, direction of electric field will be perpendicular to the plates and directed from positive plate to negative plate.

Further, magnitude of electric field in air $= \frac{\sigma}{\varepsilon_0}$



Magnitude of electric field in dielectric $= \frac{\sigma}{K\varepsilon_0}$

Similarly electric lines always flows from higher to lower potential, therefore, electric potential increases continuously as we move from $x = 0$ to $x = 3d$.

44. (d) If length of the foil is l then $C = \frac{k\varepsilon_0(l \times b)}{d}$

$$\Rightarrow 2\times10^{-6} = \frac{2.5\times8.85\times10^{-12}(l\times400\times10^{-3})}{0.15\times10^{-3}}$$

$\Rightarrow l = 33.9\ m$

45. (a, b) By using

$V = V_0 e^{-t/CR} \Rightarrow 40 = 50\, e^{-1/CR} \Rightarrow e^{-1/CR} = 4/5$

Potential difference after 2 *sec*

$$V = V_0 e^{-2/CR} = 50(e^{-1/CR})^2 = 50\left(\frac{4}{5}\right)^2 = 32\ V$$

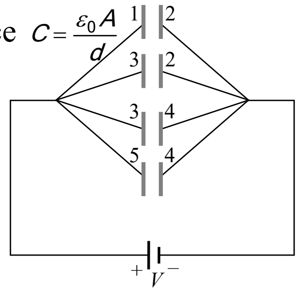
Fraction of energy after 1 *sec*

$$= \frac{\frac{1}{2}C(V_f)^2}{\frac{1}{2}C(V_i)^2} = \left(\frac{40}{50}\right)^2 = \frac{16}{25}$$

46. (a) $U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{x}\right)V^2$

$$\therefore \frac{dU}{dt} = \frac{1}{2}\varepsilon_0 AV^2\left(-\frac{1}{x^2}\frac{dx}{dt}\right) \Rightarrow \frac{dU}{dt} \propto x^{-2}$$

47. (c) The given circuit can be redrawn as follows. All capacitors are identical and each having capacitance $C = \frac{\varepsilon_0 A}{d}$



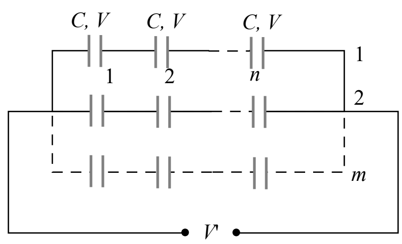
|Charge on each capacitor| = |Charge on each plate|

$= \frac{\varepsilon_0 A}{d}V$

Plate 1 is connected with positive terminal of battery so charge on it will be $+\frac{\varepsilon_0 A}{d}.V$

Plate 4 comes twice and it is connected with negative terminal of battery, so charge on plate 4 will be $-\frac{2\varepsilon_0 A}{d}V$

48. (b) Suppose $C = 8\ \mu F$, $C' = 16\mu F$ and $V = 250\ V$, $V' = 1000V$



Suppose m rows of given capacitors are connected in parallel and each row contains n capacitors then potential difference across each capacitor $V = \frac{V}{n}$ and equivalent capacitance of network $C = \frac{mC}{n}$ on putting the values we get $n = 4$ and $m = 8$

$\therefore$ Total capacitors $= n \times m = 4 \times 8 = 32$

Short Trick : For such type of problems number of capacitors $= \frac{C}{C} \times \left(\frac{V}{V}\right)^2 = \frac{16}{8}\left(\frac{1000}{250}\right)^2 = 32$

49. (b) This combination forms a G.P. $S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} ......$

Sum of infinite G.P. $S = \frac{a}{1-r}$

Here a = first term = 1 and r = common ratio $= \frac{1}{2}$

$\Rightarrow S = \frac{1}{1 - \frac{1}{2}} = 2 \Rightarrow C_{eq} = 2\mu F$

50. (a) $q_1 = 2CV,\ q_2 = CV$

Now condenser of capacity C is filled with dielectric K, therefore $C_2 = KC$

As charge is conserved

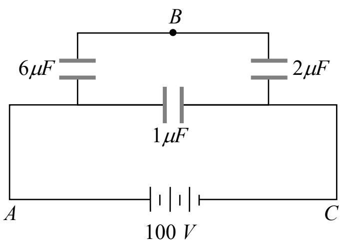
$\therefore\ q_1 + q_2 = (C_2 + 2C)V' \Rightarrow V' = \frac{3CV}{(K+2)C} = \frac{3V}{K+2}$

51. (c) $C_{eq} = \frac{(3+3)\times(1+1)}{(3+3)+(1+1)} + 1 = \left(\frac{6\times 2}{6+2}\right) + 1 = \frac{5}{2}\mu F$

$\therefore\ Q = C \times V = \frac{5}{2} \times 100 = 250\mu C$

Charge in $6\mu F$ branch $= VC = \left(\frac{6\times 2}{6+2}\right)100$

$= 150\,\mu C$



$V_{AB} = \frac{150}{6} = 25\ V$ and $V_{BC} = 100 - V_{AB} = 75\ V$

52. (c) Initially potential difference across both the capacitor is same hence energy of the system is

$U_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$ ......(i)

In the second case when key K is opened and dielectric medium is filled between the plates, capacitance of both the capacitors becomes $3C$, while potential difference across A is V and potential difference across B is $\frac{V}{3}$ hence energy of the system now is

$U_2 = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{10}{6}CV^2$ ......(ii)

So, $\frac{U_1}{U_2} = \frac{3}{5}$

53. (c) Total charge $= (2C)(2V) + (C)(-V) = 3CV$

$\therefore$ Common potential $= \frac{3CV}{3C} = V$

$\therefore$ Energy $= \frac{1}{2}(3C)(V)^2 = \frac{3}{2}CV^2$

54. (b) Charge on capacitor A is given by

$Q_1 = 15 \times 10^{-6} \times 100 = 15 \times 10^{-4}\ C$

Charge on capacitor B is given by

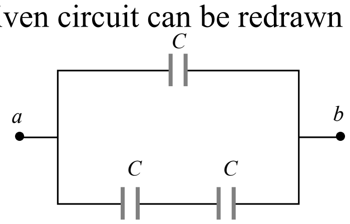
$Q_2 = 1 \times 10^{-6} \times 100 = 10^{-4}\ C$

Capacity of capacitor A after removing dielectric $= \frac{15\times 10^{-6}}{15} = 1\mu F$

Now when both capacitors are connected in parallel their equivalent capacitance will be $C_{eq} = 1 + 1 = 2\mu F$

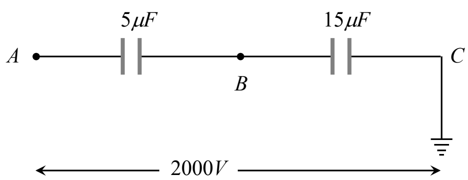
So common potential $= \frac{(15\times 10^{-4}) + (1\times 10^{-4})}{2\times 10^{-6}} = 800\,V.$

55. (d) The given circuit can be redrawn as follows



$\Rightarrow C_{eq} = \frac{3C}{2} = \frac{3\varepsilon_0 A}{2d}$

56. (c) The given circuit can be redrawn as follows



$(V_A - V_B) = \left(\frac{15}{5+15}\right) \times 2000 \Rightarrow V_A - V_B = 1500\,V$

$\Rightarrow 2000 - V_B = 1500\,V \Rightarrow V_B = 500\,V$

57. (a) If the value of C is chosen as $4\mu F$, the equivalent capacity across every part of the section will be $4\mu F$.

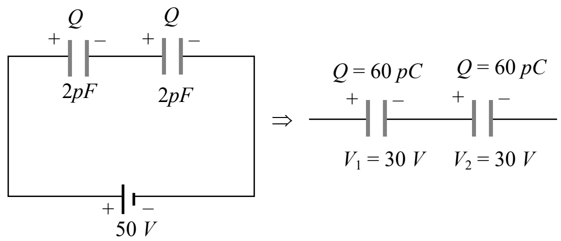
58. (c) Plane conducting surfaces facing each other must have equal and opposite charge densities. Here as the plate areas are equal, $Q_2 = -Q_3$.

The charge on a capacitor means the charge on the inner surface of the positive plate (here it is Q_2)

Potential difference between the plates $= \frac{\text{charge}}{\text{capacitance}} = \frac{Q_2}{C} = \frac{2Q_2}{2C}$
$= \frac{Q_2 - (-Q_2)}{2C} = \frac{Q_2 - Q_3}{2C}$.

59. (d) Charges on capacitors are $Q_1 = 30 \times 2 = 6\,pC$ and $Q_2 = 20 \times 3 = 60\,pC$ or $Q_1 = Q_2 = Q(\text{say})$

The situation is situation is similar as the two capacitors in series are first charged with a battery of emf 50 V and then disconnected



$\therefore$ when S_3 is closed $V_1 = 30\,V$ and $V_2 = 20\,V$

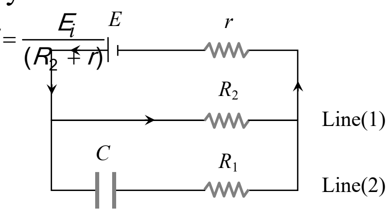
60. (a) The $\pm q$ charges appearing on the inner surfaces of A, are bound charges. As B is uncharged initially, as it is isolated, the charges on A will not be affected on closing the switch S. No charge will flow in to B.

61. (c) As $Q = CV$, $(Q_1)_{max} = 10^{-6} \times 6 \times 10^3 = 6mC$

While $(Q_2)_{max} = 3 \times 10^{-6} \times 4 \times 10^3 = 12mC$

However in series charge is same so maximum charge on C_2 will also be 6 mC (and not 12 mC) and potential difference across it $V_2 = 6mC/3\ \mu F = 2KV$ and as in series $V = V_1 + V_2$ so $V_{max} = 6KV + 2KV = 8KV$

62. (c) In steady state current drawn from the battery $i = \frac{E}{(R_2 + r)}$



In steady state capacitor is fully charged hence No current will flow through line (2) Hence potential difference across line (1) is $V = \frac{E}{(R_2 + r)} \times R_2$, the same potential difference appears across the capacitor, so charge on capacitor $Q = C \times \frac{ER_2}{(R_2 + r)}$

63. (b) $V = V_0(e^{-\lambda t})$

After 1 *seconds*

$V_1 = 320(e^{-\lambda}) \Rightarrow 240 = 320(e^{-\lambda}) \Rightarrow e^{-\lambda} = \frac{3}{4}$

After 2 *seconds*

$V_2 = 320(e^{-\lambda})^2 = 320 \times \left(\frac{3}{4}\right)^2 = 180\ volt$

After 3 *seconds*

$V_3 = 320(e^{-\lambda})^3 = 320 \times \left(\frac{3}{4}\right)^3 = 135\ volt$

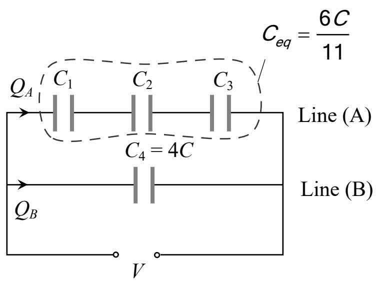
64. (b) $C = \frac{\varepsilon_0 A}{x}$; $\therefore$ $\frac{dC}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{1}{x}\right)$

$= \frac{-\varepsilon_0 A}{x^2}\left(\frac{dx}{dt}\right) = \frac{-\varepsilon_0 A}{d^2}\left(\frac{dx}{dt}\right)$

$\Rightarrow \left|\frac{dC}{dt}\right| = \frac{\varepsilon_0 A}{d^2} v$ *i.e.* $\left|\frac{dC}{dt}\right| \propto \frac{1}{d^2}$

65. (b) $\frac{1}{2}CV^2 = m.s\Delta T \Rightarrow V = \sqrt{\frac{2ms\Delta T}{C}}$

66. (b) The given circuit can be redrawn as follows



$$C_{eq} = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1} = \frac{6C}{11}$$

$$\frac{Q_A}{Q_B} = \frac{C_A}{C_B} = \frac{6C/11}{4C} = \frac{3}{22}$$

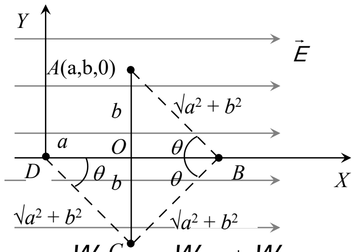
67. (a) $V_R = \frac{V_0}{4} = V_0 e^{-\frac{t}{RC}} \Rightarrow \frac{1}{4} = e^{-\frac{t}{10}}$

$\Rightarrow 4 = e^{\frac{t}{10}} \Rightarrow \log_e 4 = \frac{t}{10} \Rightarrow t = 10\log 4 =$ 13.86 *s*

$(RC = 2.5 \times 10^6 \times 4 \times 10^{-6} = 10)$

Graphical Questions

1. (b) As electric field is a conservative field Hence the work done does not depend on path

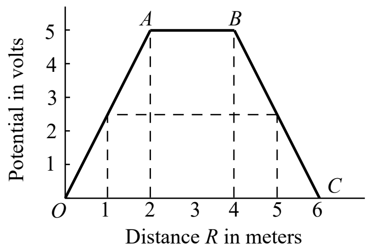


$\therefore W_{ABCD} = W_{AOD} = W_{AO} + W_{OD}$

$= Fb\cos 90^o + Fa\cos 180^o = 0 + qEa(-1) = -qEa$

2. (a) Intensity at 5*m* is same as at any point between *B* and *C* because the slope of *BC* is same throughout (*i.e.*, electric field between *B* and *C* is uniform). Therefore electric field at *R* = 5*m* is equal to the slope of line *BC* hence by $E = \frac{-dV}{dr}$;

$$E = -\frac{(0-5)}{6-4} = 2.5\frac{V}{m}$$



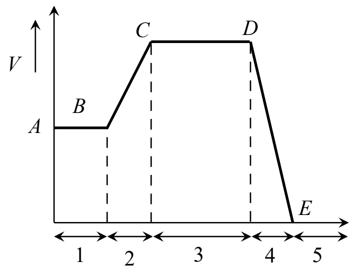
At *R* = 1 *m*, $E = -\frac{(5-0)}{(2-0)} = -2.5\frac{V}{m}$

and at *R* = 3*m* potential is constant so *E* = 0.

3. (b) Electric field in the region 1, 3 and 5 is zero *i.e.* $E_1 = E_3 = E_5$

Slope of the line *BC* < Slope of the line *DE*

i.e. $E_2 < E_4$



4. (a) Electric field due to a hollow spherical conductor is governed by following equation $E = 0$, for $r < R$ ...(i)

and $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ for $r \geq R$ ....(ii)

i.e. inside the conductor field will be zero and outside the conductor will vary according to $E \propto \frac{1}{r^2}$

5. (b) $V_{inside} = \frac{Q}{4\pi\varepsilon_0 R}$ for $r \leq R$ ....(i)

and $V_{out} = \frac{Q}{4\pi\varepsilon_0 r}$ for $r \geq R$ ....(ii)

i.e. potential inside the hollow spherical shell is constant and outside varies according to $V \propto \frac{1}{r}$.

6. (b) $E_{inside} = \frac{\rho}{3\varepsilon_0}r$ $(r < R)$

$E_{outside} = \frac{\rho R^3}{3\varepsilon_0 r^2}$ $(r \geq R)$

i.e. inside the uniformly charged sphere field varies linearly $(E \propto r)$ with distance and outside varies according to $E \propto \frac{1}{r^2}$

7. (d) From $V = \frac{Q}{C}$. For constant *Q*, $V \propto \frac{1}{C}$ *i.e.* '*V*' varies hyperbolically with *C*.

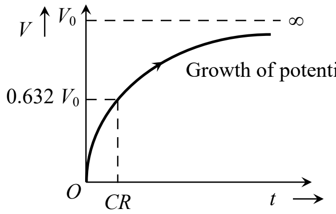
8. (b) While drawing the dielectric plate outside, the capacitance decreases till the entire plate comes out and then becomes constant. So, *V* increases and then becomes constant.

9. (c) According to graph we can say that potential difference across the capacitor C_1 is more than that across C_2. Since charge *Q* is same *i.e.*, $Q = C_1V_1 = C_2V_2$

$\Rightarrow \frac{C_1}{C_2} = \frac{V_2}{V_1} \Rightarrow C_1 < C_2 \quad (V_1 > V_2)$.

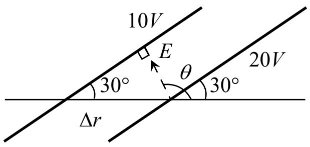
10. (a) For charging of capacitor $q = q_0\left(1 - e^{-\frac{t}{CR}}\right)$

and potential difference $V = V_0\left(1 - e^{-\frac{t}{CR}}\right)$



11. (d) $U = \frac{1}{2}QV =$ Area of triangle OAB

12. (c) Using $dV = -\vec{E}.\vec{dr}$



$\Rightarrow \Delta V = -E\Delta r\cos\theta$

$\Rightarrow E = \frac{-\Delta V}{\Delta r\cos\theta}$

$\Rightarrow E = \frac{-(20-10)}{10\times10^{-2}\cos120°}$

$= \frac{-10}{10\times10^{-2}(-\sin30°)} = \frac{-10^2}{-1/2} = 200\ V/m$

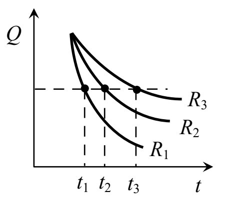
Direction of E be perpendicular to the equipotential surface *i.e.* at 120° with x-axis.

13. (c) During the discharge of a capacitor through a resistance charge at any instant $Q = Q_0e^{-t/CR}$

$\Rightarrow \frac{Q_0}{Q} = e^{t/CR} \Rightarrow t = CR\log_e\frac{Q_0}{Q}$

If $Q \to$ constant, then $t \propto R$

Now, draw a line parallel to the time axis as shown. Suppose this line cut the graphs at points 1, 2 and 3. Corresponding time are t_1, t_2 and t_3 respectively. Hence from graph $t_1 < t_2 < t_3$



$\Rightarrow R_1 < R_2 < R_3$

14. (a) As we know $Q = CV$.

15. (d) At mid point, $E = 0$

Before mid point, E is positive. This is maximum near the charge and decreases towards mid point.

After mid point, E is negative, The curve crosses x-axes at x-axis at $x = d/2$. From centre to end, E decreases.

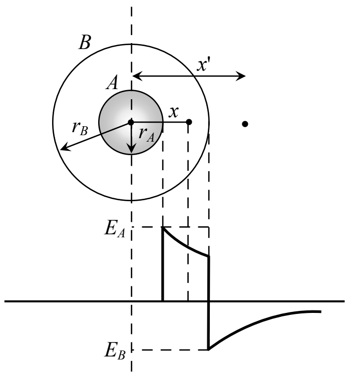
The variation is shown by curve.

16. (b) In case of RC circuit $i = \frac{E}{R}e^{-t/RC}$

$\therefore \log_e i = -\frac{t}{RC} + \log_e\frac{E}{R}$

When R is doubled, the slope of the curve increases. Further at $t = 0$, the current will be less for an increased value of resistance.

17. (a) Inside the shell A, electric field $E_{in} = 0$



At the surface of shell A,

$E_A = \frac{kQ_A}{r_A^2} \longrightarrow$ (a fixed positive value)

Between the shell A and B, at a distance x from the common centre

$E = \frac{k.Q_A}{x^2} \longrightarrow$ (as x increases E decreases)

At the surface of shell B,

$E_B = \frac{k.(Q_A - Q_B)}{r_B^2} \longrightarrow$ (a fixed negative value because $|Q_A| < |Q_B|$)

Outside the both shell, at a distance x from the common centre

$E_{out} = \frac{k(Q_A - Q_B)}{x'^2} \longrightarrow$ (as x' increases negative value of E_{out} decreases and it becomes zero at $x = \infty$)

Assertion and Reason

1. (d) Gravitational force is the dominating force in nature and not coulomb's force. Gravitational force is the weakest force. Also, Coulomb's force>> gravitational force.

2. (c) Equivalent capacitance of parallel combination is $C_p = C_1 + C_2 + C_3$.

3. (a) In a hollow spherical shield, the charge is present only on its surface but charge is zero at every point inside the hollow sphere. Hence, the metallic shield in form of hollow shell may be built to block an electric field.

4. (a) Electron has negative charge, in electric field negative charge moves from lower potential to higher potential.

5. (b) By the formula capacitance of a capacitor

$C_1 = \varepsilon_0 \times \frac{KA}{d} \propto \frac{K}{d}$

Hence, $\frac{C_1}{C_2} = \frac{K_1}{d_1} \times \frac{d_2}{K_2} = \frac{K_1}{K_2} \times \frac{d/2}{3K} = \frac{1}{6}$ or $C_2 = 6C_1$

Again for capacity of a capacitor $C = \frac{Q}{V}$

Therefore, capacity of a capacitor does not depend upon the nature of the material of the capacitor.

6. (c) In the given case $V = V_0$ (constant)

Energy stored in the capacitor $= \frac{1}{2}CV^2$

$C \to KC$, so energy stored will become A times

$Q = CV$, so Q will become K times

$\therefore$ Surface charge density $\sigma' = \frac{Kq}{A} = K\sigma_0$.

7. (e) If electric lines of forces cross each other, then the electric field at the point of intersection will have two direction simultaneously which is not possible physically.

8. (b) Electron and proton have same amount of charge so they have same coulomb force. They have different accelerations because they have different masses $(a = \frac{F}{m})$

Therefore, both assertion and reason are true and reason is the correct explanation of the assertion.

9. (b)

10. (b) Charge is always conserved but energy is lost in the term of heat.

11. (e) It is an example of conversation of charges.

12. (a) Potential is constant on the surface of a sphere so it behaves as an equipotential surface.

13. (a) Capacitance is basically a geometrical quantity.

14. (e) Battery is disconnected from the capacitor.

So Q = constant. Energy $= \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A}$

$\Rightarrow$ Energy $\propto d$

15. (a)

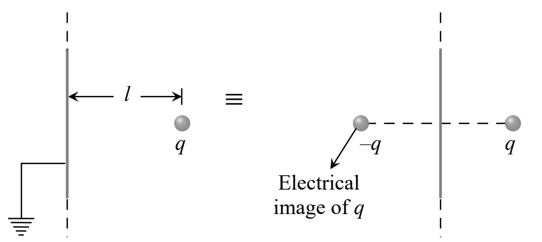
16. (a)

17. (c) $Q = \pm ne$ and charge lesser than 1 C is possible.

18. (a) This is the concept of electric image.

If we are asked to find the force between an infinite earthed conductor and a point charge q placed at perpendicular distance l from the earthed conductor (see figure), than we proceed as follows.

Firstly, the conductor being earthed implies $V = 0$.



So, we redraw the situation in which we replace the conductor and introduce an **IMAGE** charge -q as shown.

The force between the two charges (object charge q and image charge $-q$) is the electrostatic force between the infinite grounded conductor and q.

So, $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2l)^2} \Rightarrow F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4l^2}$ (attractive in nature)

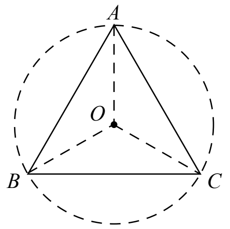
19. (b) As $\sigma_1 = \sigma_2$ (Given)

$\therefore \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}$, or $\frac{q_1}{q_2} = \frac{r_1^2}{r_2^2}$ [Let r_1 and r_2 be two different radii]

Then the ratio of electric field intensities near the surface of spherical conductor,

$$\frac{E_1}{E_2} = \frac{q_1}{4\pi\varepsilon_0 r_1^2} \times \frac{4\pi\varepsilon_0 r_2^2}{q_2} = \frac{q_1}{q_2} \times \frac{r_2^2}{r_1^2} = 1 \quad i.e. \ E_1 = E_2$$

20. (a) Resultant of electric intensity at *O* due to *B* and *C* is equal and opposite to that due to *A*.



21. (d) The rate of decrease of electric field is different in the two cases. In case of a point charge, it decreases as $1/r^2$ but in the case of electric dipole it decreases more rapidly, as $E \propto 1/r^3$.

22. (d) The whole charge of a conductor can be transferred to another isolated conductor, if it is placed inside the hollow insulated conductor and connected with it.

23. (d) Electric potential of a charged conductor depends not only on the amount of charge and volume but also on the shape of the conductor. Hence if their shapes are different, they may have different electric potential.

24. (b) Since the electric field is directed from south to north hence rate of change of potential will be along this direction, but it is zero along east and west.

25. (d) Electric field at the nearby point will be resultant of existing field and field due to the charge brought. It may increase or decrease if the charge is positive or negative depending on the position of the point with respect to the charge brought.

26. (d) The electric field due to one charged plate at the location of the other is $E = \sigma/2\varepsilon_0$ and the force per unit area is $F = \sigma E = \sigma^2/2\varepsilon_0$.

27. (a) A charged cloud induces opposite charge on pointed conductors. At sharp points of the conductor surface density of charge is very high and charge begins to leak from the pointed ends by setting up oppositely charged electric wind. This wind, when comes in contact with the charged cloud, neutralizes some of its charge lowering the potential difference between the cloud and the building. This reduces the chances of lightening striking the building [if the lightening strikes the building, the charge is conducted to the earth and the building remains safe].

28. (c) A charged capacitor, after removing the battery, does not discharge itself. If this capacitor is touched by someone, he may feel shock due to large charge still present on the capacitor. Hence it should be handled cautiously otherwise this may cause a severe shock.

29. (b) During take off and landing, the friction between tyres and the run way may cause electrification of tyres. Due to conducting nature of tyre, the charge so collected is conducted to a ground and electrical sparking is avoided.

30. (c) When the bird perches on a single high power line, no current passes through its body because its body is at equipotential surface *i.e.*, there is no potential difference. While when man touches the same line, standing bare foot on ground the electrical circuit is completed through the ground. The hands of man are at high potential and his feet's are at low potential. Hence large amount of current flows through the body of the man and person therefore gets a fatal shock.