4. (a) $U = k|x|^3 \Rightarrow F = -\frac{dU}{dx} = -3k|x|^2$ ...(i)

Also, for SHM $x = a\sin\omega t$ and $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$\Rightarrow$ acceleration $a = \frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow F = ma$

$= m\frac{d^2x}{dt^2} = -m\omega^2 x$ ...(ii)

From equation (i) & (ii) we get $\omega = \sqrt{\frac{3kx}{m}}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3kx}} = 2\pi\sqrt{\frac{m}{3k(a\sin\omega t)}} \Rightarrow T \propto \frac{1}{\sqrt{a}}$.

5. (b, d) Let the velocity acquired by *A* and *B* be *V*, then

$mv = mV + mV \Rightarrow V = \frac{v}{2}$

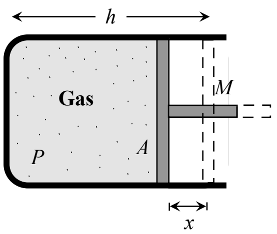
Also $\frac{1}{2}mv^2 = \frac{1}{2}mV^2 + \frac{1}{2}mV^2 + \frac{1}{2}kx^2$

Where *x* is the maximum compression of the spring. On solving the above equations, we get $x = v\left(\frac{m}{2k}\right)^{1/2}$

At maximum compression, kinetic energy of the

$A - B$ system $= \frac{1}{2}mV^2 + \frac{1}{2}mV^2 = mV^2 = \frac{mv^2}{4}$

6. (a) Let the piston be displaced through distance *x* towards left, then volume decreases, pressure increases. If ΔP is increase in pressure and ΔV is decrease in volume, then considering the process to take place gradually (i.e. isothermal)



$P_1V_1 = P_2V_2 \Rightarrow PV = (P + \Delta P)(V - \Delta V)$

$\Rightarrow PV = PV + \Delta PV - P\Delta V - \Delta P\Delta V$

$\Rightarrow \Delta P.V - P.\Delta V = 0$ (neglecting $\Delta P.\Delta V$)

$\Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \frac{P.x}{h}$

This excess pressure is responsible for providing the restoring force (*F*) to the piston of mass *M*.

Hence $F = \Delta P.A = \frac{PAx}{h}$

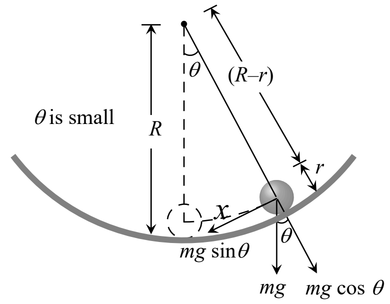
Comparing it with $|F| = kx \Rightarrow k = M\omega^2 = \frac{PA}{h}$

$\Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi\sqrt{\frac{Mh}{PA}}$

Short trick : by checking the options dimensionally. Option (a) is correct.

7. (b) Tangential acceleration, $a_t = -g\sin\theta = -g\theta$

$a_t = -g\frac{x}{(R-r)}$



Motion is S.H.M., with time period

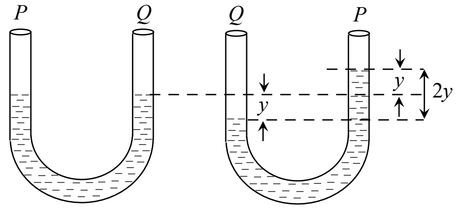
$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{gx}{(R-r)}}} = 2\pi\sqrt{\frac{R-r}{g}}$

8. (a) For resonance amplitude must be maximum which is possible only when the denominator of expression is zero *i.e.*

$a\omega^2 - b\omega + c = 0 \Rightarrow \omega = \frac{+b \pm \sqrt{b^2 - 4ac}}{2a}$

For a single resonant frequency, $b^2 = 4ac$.

9. (d) If the level of liquid is depressed by *y cm* on one side, then the level of liquid in column *P* is *2y cm* higher than *B* as shown.



The weight of extra liquid on the side $P = 2Aydg$.

This becomes the restoring force on mass *M*.

$\therefore$ Restoring acceleration $= \frac{-2Aydg}{M}$

This relation satisfies the condition of SHM i.e. $a \propto -y$.

Hence time period $T = 2\pi\sqrt{\frac{\text{Displacement}}{|\text{Acceleration}|}}$

$= 2\pi\sqrt{\frac{y}{\frac{2Aydg}{M}}} \Rightarrow T = 2\pi\sqrt{\frac{M}{2Adg}}$

10. (b) Time taken by particle to move from $x=0$ (mean position) to $x = 4$ (extreme position) $= \frac{T}{4} = \frac{1.2}{4} = 0.3\ s$

Let t be the time taken by the particle to move from $x=0$ to $x=2\ cm$

$y = a\sin\omega t \Rightarrow 2 = 4\sin\frac{2\pi}{T}t \Rightarrow \frac{1}{2} = \sin\frac{2\pi}{1.2}t$

$\Rightarrow \frac{\pi}{6} = \frac{2\pi}{1.2}t \Rightarrow t = 0.1\ s$. Hence time to move from $x = 2$ to $x = 4$ will be equal to $0.3 - 0.1 = 0.2$ s

Hence total time to move from $x = 2$ to $x = 4$ and back again $= 2 \times 0.2 = 0.4$ sec

11. (c) For body to remain in contact $a_{max} = g$

$\therefore \omega^2 A = g \Rightarrow 4\pi^2 n^2 A = g$

$\Rightarrow n^2 = \frac{g}{4\pi^2 A} = \frac{10}{4(3.14)^2 0.01} = 25 \Rightarrow n = 5\ Hz$

12. (c) Under the influence of one force $F_1 = m\omega_1^2 y$ and under the action of another force, $F_2 = m\omega_2^2 y$.

Under the action of both the forces $F = F_1 + F_2$

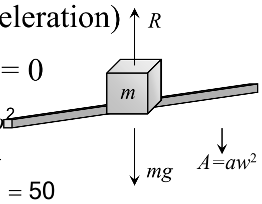
$\Rightarrow m\omega^2 y = m\omega_1^2 y + m\omega^2 y$

$\Rightarrow \omega^2 = \omega_1^2 + \omega_2^2 \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$

$\Rightarrow T = \sqrt{\frac{T_1^2 T_2^2}{T_1^2 + T_2^2}} = \sqrt{\frac{\left(\frac{4}{5}\right)^2\left(\frac{3}{5}\right)^2}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2}} = 0.48$ sec

13. (a) By drawing free body diagram of object during the downward motion at extreme position, for equilibrium of mass

$mg - R = mA$ (A = Acceleration)



For critical condition $R = 0$

so $mg = mA \Rightarrow mg = ma\omega^2$

$\Rightarrow \omega = \sqrt{g/a} = \sqrt{\frac{9.8}{3.92 \times 10^{-3}}} = 50$

$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1256\,sec$

14. (a) Using $x = A\sin\omega t$

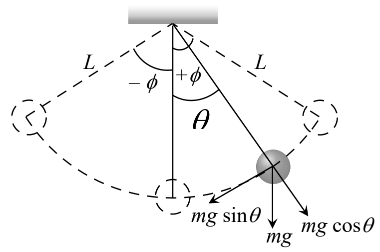
For $x = A/2$, $\sin\omega T_1 = 1/2 \Rightarrow T_1 = \frac{\pi}{6\omega}$

For $x = A$, $\sin\omega(T_1 + T_2) = 1 \Rightarrow T_1 + T_2 = \frac{\pi}{2\omega}$

$\Rightarrow T_2 = \frac{\pi}{2\omega} - T_1 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$ *i.e.* $T_1 < T_2$

Alternate method : In S.H.M., velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme position. Therefore the time taken for the particle to go from 0 to $\frac{A}{2}$ will be less than the time taken to go from $\frac{A}{2}$ to A. Hence $T_1 < T_2$.

15. (b, c) From following figure it is clear that



$T - Mg\cos\theta =$ Centripetal force

$\Rightarrow T - Mg\cos\theta = \frac{Mv^2}{L}$

Also tangential acceleration $|a_T| = g\sin\theta$.

16. (c) If t is the time taken by pendulums to come in same phase again first time after $t = 0$.

and $N_S =$ Number of oscillations made by shorter length pendulum with time period T_S.

$N_L =$ Number of oscillations made by longer length pendulum with time period T_L.

Then $t = N_S T_S = N_L T_L$

$\Rightarrow N_S 2\pi\sqrt{\frac{5}{g}} = N_L \times 2\pi\sqrt{\frac{20}{g}}$ $\left(\because T = 2\pi\sqrt{\frac{l}{g}}\right)$

$\Rightarrow N_S = 2N_L$ i.e. if $N_L = 1 \Rightarrow N_S = 2$

17. (a) Tension in the string when bob passes through lowest point

$T = mg + \frac{mv^2}{r} = mg + mv\omega$ $(\because v = r\omega)$

putting $v = \sqrt{2gh}$ and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

we get $T = m(g + \pi\sqrt{2gh})$

18. (d) When the bob is immersed in water its effective weight $= \left(mg - \frac{m}{\rho}g\right) = mg\left(\frac{\rho - 1}{\rho}\right)$

$\therefore g_{eff} = g\left(\frac{\rho - 1}{\rho}\right)$

$\frac{T}{T} = \sqrt{\frac{g}{g_{eff}}} \Rightarrow T = T\sqrt{\frac{\rho}{(\rho - 1)}}$

19. (a) Time period $T \propto \sqrt{l} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\Delta\theta$

Also according to thermal expansion $l' = (1 + \alpha\Delta\theta)$

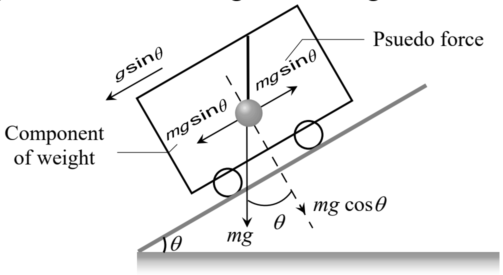
$\frac{\Delta l}{l} = \alpha + \theta$. Hence $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\Delta\theta$

$= \frac{1}{2} \times 12 \times 10^{-6} \times (40 - 20) = 12 \times 10^{-5}$

$\Rightarrow \Delta T = 12 \times 10^{-5} \times 86400$ *seconds / day*

$\therefore \Delta T \approx 10.3$ *seconds / day*

20. (a) See the following force diagram.



Vehicle is moving down the frictionless inclined surface so, it's acceleration is $g\sin\theta$. Since vehicle is accelerating, a pseudo force $m(g\sin\theta)$ will act on bob of pendulum which cancel the $\sin\theta$ component of weight of the bob.

Hence net force on the bob is $F_{net} = mg\cos\theta$ or net acceleration of the bob is $g_{eff} = g\cos\theta$

$\therefore$ Time period $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{g\cos\theta}}$

21. (c) $\because t_o = 2\pi\sqrt{\frac{l}{g}}$

Effective weight of bob inside water,

$W = mg - \text{thrust} = V\rho g - V\rho' g$

$\Rightarrow V\rho g_{eff} = V(\rho - \rho')g$, where, $\rho =$ Density of bob

$\Rightarrow g_{eff} = \left(1 - \frac{\rho'}{\rho}\right)g$ and $\rho' =$ Density of water

$\therefore t = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(1 - \rho'/\rho)g}}$

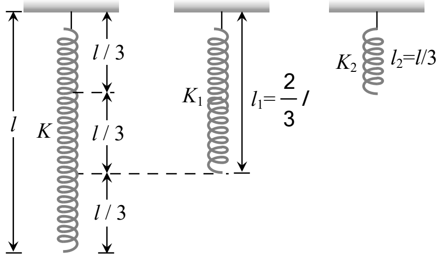
$\therefore \frac{t}{t_0} = \sqrt{\frac{1}{1 - \rho'/\rho}} = \sqrt{\frac{1}{1 - \frac{3}{4}}}$

$(\because \rho' = 10^3\ kg/m^3$

$\rho = \frac{4}{3} \times 10^3\ kg/m^3$

$\Rightarrow t = 2t_0$.

22. (b)



Force constant $(k) \propto \frac{1}{\text{Length of spring}}$

$\Rightarrow \frac{K}{K_1} = \frac{l_1}{l} = \frac{\frac{2}{3}l}{l} \Rightarrow K_1 = \frac{3}{2}K$.

23. (b) The wire may be treated as a string for which force constant $k_1 = \frac{\text{Force}}{\text{Extension}} = \frac{YA}{L}$ $\left(\because Y = \frac{F}{A} \times \frac{L}{\Delta L}\right)$

Spring constant of the spring $k_2 = K$

Hence spring constant of the combination (series)

$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{(YA/L)K}{(YA/L) + K} = \frac{YAK}{YA + KL}$

$\because$ Time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\left[\frac{(YA + KL)m}{YAK}\right]^{1/2}$

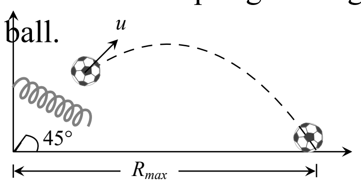
24. (a) Slope is irrelevant hence $T = 2\pi\left(\frac{M}{2K}\right)^{1/2}$

25. (b) For forced oscillation,

$x = x_0 \sin(\omega t + \phi)$ and $F = F_0 \cos\omega t$

where, $x_0 = \frac{F_o}{m(\omega_o^2 - \omega^2)} \propto \frac{1}{m(\omega_o^2 - \omega^2)}$.

26. (b) For getting horizontal range, there must be some inclination of spring with ground to project ball.



$\Rightarrow R_{max} = \frac{u^2}{g}$ …..(i)

But K.E. acquired by ball = P.E. of spring gun

$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}kx^2 \Rightarrow u^2 = \frac{kx^2}{m}$ …..(ii)

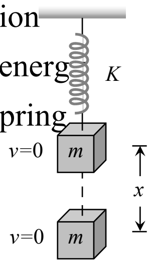
From equation (i) and (ii)

$R_{max} = \frac{kx^2}{mg} = \frac{600\times(5\times10^{-2})^2}{15\times10^{-3}\times10} = 10m.$

27. (b) Let x be the maximum extension of the spring. From energy conservation

Loss in gravitational potential energy

= Gain in potential energy of spring



$Mgx = \frac{1}{2}Kx^2$

$\Rightarrow x = \frac{2Mg}{K}$

28. (b) $y = 4\cos^2\left(\frac{t}{2}\right)\sin 1000\, t$

$\Rightarrow y = 2(1+\cos t)\sin 1000\, t$

$\Rightarrow y = 2\sin 1000\, t + 2\cos t \sin 1000 t$

$\Rightarrow y = 2\sin 1000\, t + \sin 999\, t + \sin 1001\, t$

It is a sum of three S.H.M.

29. (a, c) Let simple harmonic motions be represented by

$y_1 = a\sin\left(\omega t - \frac{\pi}{4}\right)$; $y_2 = a\sin\omega t$ and

$y_3 = a\sin\left(\omega t + \frac{\pi}{4}\right)$. On superimposing, resultant SHM will be

$y = a\left[\sin\left(\omega t - \frac{\pi}{4}\right) + \sin\omega t + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$

$= a\left[2\sin\omega t\cos\frac{\pi}{4} + \sin\omega t\right]$

$= a[\sqrt{2}\sin\omega t + \sin\omega t] = a(1+\sqrt{2})\sin\omega t$

Resultant amplitude $= (1+\sqrt{2})a$

Energy is S.H.M. ∝ (Amplitude)^2

$\therefore \frac{E_{Resultant}}{E_{Single}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$

$\Rightarrow E_{Resultant} = (3+2\sqrt{2})E_{Single}$

30. (d) $y = \sin^2\omega t = \frac{1-\cos 2\omega t}{2} \Rightarrow$ Period, $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

The given function is not satisfying the standard differential equation of S.H.M. $\frac{d^2y}{dx^2} = -\omega^2 y$. Hence it represents periodic motion but not S.H.M.

31. (c) $y = Kt^2 \Rightarrow \frac{d^2y}{dt^2} = a_y = 2K = 2 \times 1 = 2 m/s^2$ ($\because K = 1 m/s^2$)

Now, $T_1 = 2\pi\sqrt{\frac{l}{g}}$ and $T_2 = 2\pi\sqrt{\frac{l}{(g+a_y)}}$

Dividing, $\frac{T_1}{T_2} = \sqrt{\frac{g+a_y}{g}} \Rightarrow \sqrt{\frac{6}{5}} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{6}{5}$

32. (b) From the relation of restitution $\frac{h_n}{h_0} = e^{2n}$ and

$h_n = h_0(1-\cos 60°) \Rightarrow \frac{h_n}{h_0} = 1-\cos 60° = \left(\frac{2}{\sqrt{5}}\right)^{2n}$

$\Rightarrow 1 - \frac{1}{2} = \left(\frac{4}{5}\right)^n \Rightarrow \frac{1}{2} = \left(\frac{4}{5}\right)^n$

Taking log of both sides we get

$\log 1 - \log 2 = n(\log 4 - \log 5)$

$0 - 0.3010 = n(0.6020 - 0.6990)$

$-0.3010 = -n\times 0.097 \Rightarrow n = \frac{0.3010}{0.097} = 3.1 \approx 3$

33. (a) As a is the side of cube σ is its density.

Mass of cube is $a^3\sigma$, its weight $= a^3\sigma g$

Let h be the height of cube immersed in liquid of density ρ in equilibrium then, $F = a^2 h\rho g =$

If it is pushed down by y then the buoyant force $F' = a^2(h+y)\rho g$

Restoring force is

$\Delta F = F' - F = a^2(h+y)\sigma g - a^2 h\sigma g$

$= a^2 y\rho g$

Restoring acceleration

$= \frac{\Delta F}{M} = -\frac{a^2 y\rho g}{M} = -\frac{a^2\rho g}{a^3\sigma}y$

Motion is S.H.M.

$\Rightarrow T = 2\pi\sqrt{\frac{a^3\sigma}{a^2\rho g}} = 2\pi\sqrt{\frac{a\sigma}{\rho g}}$

34. (b) As here two masses are connected by two springs, this problem is equivalent to the oscillation of a reduced mass m_r of a spring of effective spring constant.

$T = 2\pi\sqrt{\frac{m_r}{K_{eff.}}}$

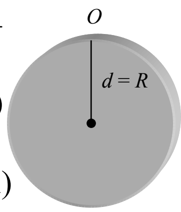
Here $m_r = \frac{m_1 m_2}{m_1+m_2} = \frac{m}{2} \Rightarrow K_{eff.} = K_1 + K_2 = 2K$

$\therefore\ n = \frac{1}{2\pi}\sqrt{\frac{K_{eff.}}{m_r}} = \frac{1}{2\pi}\sqrt{\frac{2K}{m}\times 2} = \frac{1}{\pi}\sqrt{\frac{K}{m}} = \frac{1}{\pi}\sqrt{\frac{0.1}{0.1}} = \frac{1}{\pi}Hz$

35. (d) Time period of a physical pendulum

$T = 2\pi\sqrt{\frac{I_0}{mgd}} = 2\pi\sqrt{\frac{\left(\frac{1}{2}mR^2 + mR^2\right)}{mgR}}$

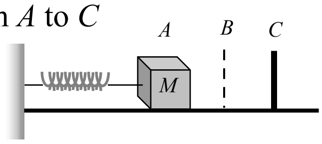
$= 2\pi\sqrt{\frac{3R}{2g}}$ …..(i)



$T_{simple\,pendulum} = 2\pi\sqrt{\frac{l}{g}}$ …..(ii)

Equating (i) and (ii), $l = \frac{3}{2}R$.

36. (c) The total time from A to C



$t_{Ac} = t_{AB} + t_{BC}$

$= (T/4) + t_{BC}$

where T = time period of oscillation of spring mass system

t_{BC} can be obtained from, $BC = AB\sin(2\pi/T)t_{BC}$

Putting $\frac{BC}{AB} = \frac{1}{2}$ we obtain $t_{BC} = \frac{T}{12}$

$\Rightarrow t_{AC} = \frac{T}{4} + \frac{T}{12} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$.

37. (b) When mass 700 *gm* is removed, the left out mass (500 + 400) gm oscillates with a period of 3 sec

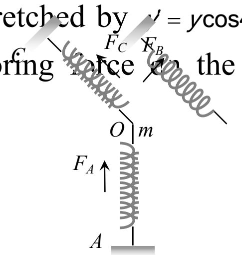
$\therefore\ 3 = t = 2\pi\sqrt{\frac{(500+400)}{k}}$ ……(i)

When 500 *gm* mass is also removed, the left out mass is 400 *gm*.

$\therefore\ t' = 2\pi\sqrt{\frac{400}{k}}$ …..(ii)

$\Rightarrow \frac{3}{t'} = \sqrt{\frac{900}{400}} \Rightarrow t' = 2\,\text{sec}$

38. (b) When the particle of mass m at O is pushed by y in the direction of A The spring A will be compressed by y while spring B and C will be stretched by $y' = y\cos 45°$. So that the total restoring force on the mass m along OA.



$F_{net} = F_A + F_B\cos 45° + F_C\cos 45°$

$= ky + 2ky'\cos 45° = ky + 2k(y\cos 45°)\cos 45° = 2ky$

Also $F_{net} = k'y \Rightarrow k'y = 2ky \Rightarrow k' = 2k$

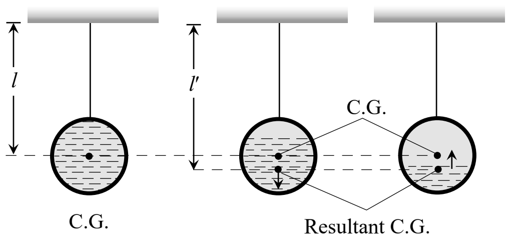
$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$

39. (d) The given system is like a simple pendulum, whose effective length (l) is equal to the distance between point of suspension and C.G. (Centre of Gravity) of the hanging body.

When water slowly flows out the sphere, the C.G. of the system is lowered, and hence l increases, which in turn increases time period (as $T \propto \sqrt{l}$).

After some time weight of water left in sphere become less than the weight of sphere itself, so the resultant C.G. gets clear the C.G. of sphere itself i.e. l decreases and hence T increases.

Finally when the sphere becomes empty, the resulting C.G. is the C.G. of sphere i.e. length becomes equal to the original length and hence the time period becomes equal to the same value as when it was full of water.



40. (b) Let T_1 and T_2 are the time period of the two pendulums $T_1 = 2\pi\sqrt{\frac{100}{g}}$ and $T_2 = 2\pi\sqrt{\frac{121}{g}}$

($T_1 < T_2$ because $l_1 < l_2$).

Let at $t = 0$, they start swinging together. Since their time periods are different, the swinging will not be in unision always. Only when number of completed oscillation differs by an integer, the two pendulum will again begin to swing together.

Let longer length pendulum complete n oscillation and shorter length pendulum complete (n+1) oscillation, for the unision swinging, then $(n+1)T_1 = nT_2$

$$(n+1)\times 2\pi\sqrt{\frac{100}{g}} = n\times 2\pi\sqrt{\frac{121}{g}} \Rightarrow n = 10$$

41. (b) Amplitude of damped oscillator

$A = A_0 e^{-\lambda t}$; $\lambda =$ constant, t = time

For t =1 min. $\frac{A_0}{2} = A_0 e^{-\lambda t} \Rightarrow e^{\lambda} = 2$

For t = 3 min. $A = A_0 e^{-\lambda \times 3} = \frac{A_0}{(e^{\lambda})^3} = \frac{A_0}{2^3}$

$\Rightarrow X = 2^3$

42. (a) The standard differential equation is satisfied by only the function $\sin\omega t - \cos\omega t$. Hence it represents S.H.M.

43. (d) This is the special case of physical pendulum and in this case $T = 2\pi\sqrt{\frac{2l}{3g}}$

$$\Rightarrow \quad T = 2\times 3.14\sqrt{\frac{2\times 2}{3\times 9.8}} = 2.31\,\text{sec} \approx 2.4\,\text{sec}$$

Graphical Questions

1. (a) Because acceleration $\propto$ displacement.

2. (d) Using acceleration $A = -\omega^2 x$

$At - x_{max}$ A will be maximum and positive.

3. (d) Acceleration $= -\omega^2 y$. So $F = -m\omega^2 y$

y is sinusoidal function.

So F will be also sinusoidal function with phase difference π

4. (d) At time $\frac{T}{2}; v = 0$ $\therefore$ Total energy = Potential energy.

5. (b) PE varies from zero to maximum. It is always positive sinusoidal function.

6. (d) Potential energy of particle performing SHM is given by: $PE = \frac{1}{2}m\omega^2 y^2$ *i.e.* it varies parabolically such that at mean position it becomes zero and maximum at extreme position.

7. (a) Potential energy is minimum (in this case zero) at mean position (x = 0) and maximum at extreme position $(x = \pm A)$.

At time t = 0, x = A, hence potential should be maximum. Therefore graph I is correct. Further in graph III. Potential energy is minimum at x = 0, hence this is also correct.

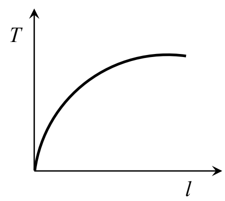
8. (a) $f = \frac{1}{T} = \frac{1}{0.04} = 25\ Hz$

9. (d) From graph, slope $K = \frac{F}{x} = \frac{8}{2} = 4$

$$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow T = 2\pi\sqrt{\frac{0.01}{4}} = 0.3\,\text{sec}$$

10. (b) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l \propto T^2$ (Equation of parabola)

11. (b) $T \propto \sqrt{l} \Rightarrow T^2 \propto l$



12. (d) In simple harmonic motion

$y = a\sin\omega t$ and $v = a\omega\cos\omega t$ from this we have

$\frac{y^2}{a^2} + \frac{v^2}{a^2\omega^2} = 1$, which is a equation of ellipse.

13. (c)

14. (a) In S.H.M. when acceleration is negative maximum or positive maximum, the velocity is zero so kinetic energy is also zero. Similarly for zero acceleration, velocity is maximum so kinetic energy is also maximum.

15. (b) Total potential energy = 0.04 J

Resting potential energy =0.01 J

Maximum kinetic energy =(0.04–0.01)

$= 0.03J = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}ka^2$

$$0.03 = \frac{1}{2} \times k \times \left(\frac{20}{1000}\right)^2$$

$$k = 0.06 \times 2500\ N/m = 150\ N/m.$$

16. (a) Kinetic energy varies with time but is never negative.

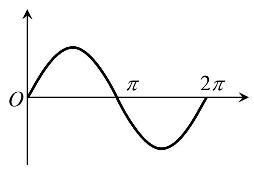
Assertion and Reason

1. (b) Both assertion and reason are correct but reason is not the correct explanation of assertion.

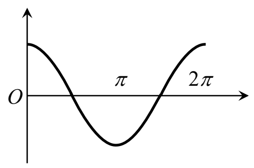
2. (e) In simple harmonic motion, $v = \omega\sqrt{a^2 - y^2}$ as y changes, velocity v will also change. So simple harmonic motion is not uniform motion. But simple harmonic motion may be defined as the projection of uniform circular motion along one of the diameter of the circle.

3. (a) In SHM, the acceleration is always in a direction opposite to that of the displacement *i.e.,* proportional to (–*y*).

4. (a) A periodic function is one whose value repeats after a definite interval of time. $\sin\theta$ and $\cos\theta$ are periodic functions because they repeat itself after 2π interval of time.



sin curve



cos curve

It is also true that moon is smaller than the earth, but this statement is not explaining the assertion.

5. (e) In SHM, $v = \omega\sqrt{a^2 - y^2}$ or $v^2 = \omega^2 a^2 - \omega^2 y^2$.

Dividing both sides by $\omega^2 a^2$, $\frac{v^2}{\omega^2 a^2} + \frac{y^2}{a^2} = 1$.

This is the equation of an ellipse. Hence the graph between v and y is an ellipse not a parabola.

6. (b) $T = 2\pi\sqrt{\frac{l}{g}}$. On moon, g is much smaller compared to g on earth. Therefore, T increases.

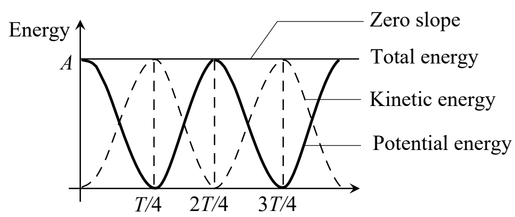
7. (c) Amplitude of oscillation for a forced, damped oscillator is $A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2) + (b\omega/m)^2}}$, where b is constant related to the strength of the resistive force, $\omega_0 = \sqrt{k/m}$ is natural frequency of undamped oscillator (b = 0).

When the frequency of driving force (ω) ≈ ω_0, then amplitude A is very larger.

For $\omega < \omega_0$ or $\omega > \omega_0$, the amplitude decrease.

(a) The total energy of S.H.M. = Kinetic energy of particle + potential energy of particle.

The variation of total energy of the particle in SHM with time is shown in a graph.



(c) Time period of simple pendulum of length l is,

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l} \Rightarrow \sqrt{\frac{\Delta T}{T}} = \frac{1}{2}\frac{\Delta l}{l}$$

$$\therefore \frac{\Delta T}{T} = \frac{1}{2} \times 3 = 1.5\%$$

(c) Frequency of second pendulum $n = (1/2)s^{-1}$. When elevator is moving upwards with acceleration *g/2*, the effective acceleration due to gravity is

$$g = g + a = g + g/2 = 3g/2.$$

As $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ so $n^2 \propto g$.

$\therefore \frac{n_1^2}{n_2^2} = \frac{g_1}{g} = \frac{3g/2}{g} = \frac{3}{2}$ or, $\frac{n_1}{n} = \sqrt{\frac{3}{2}}$ =1.225

or, $n_1 = 1.225n = 1.225 \times (1/2) = 0.612\ s^{-1}$.

(b) Energy of damped oscillator at an any instant t is given by

$E = E_0 e^{-bt/m}$ [where $E_0 = \frac{1}{2}kx^2 =$ maximum energy]

Due to damping forces the amplitude of oscillator will go on decreasing with time whose energy is expressed by above equation.

(b) In SHM. $K.E. = \frac{1}{2}m\omega^2(a^2 - y^2)$ and $P.E. = \frac{1}{2}m\omega^2 y^2$.

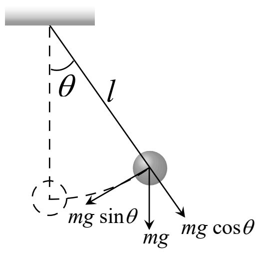
For $K.E. = P.E. \Rightarrow 2y^2 = a^2 \Rightarrow y = a/\sqrt{2}$. Since total energy remains constant through out the motion, which is $E = K.E. + P.E.$ So, when $P.E.$ is maximum then $K.E.$ is zero and viceversa.

(a) Total energy of the harmonic oscillator,

$E = \frac{1}{2}m\omega^2 a^2$ *i.e.,* $E \propto a^2$.

Therefore $\frac{E'}{E} = \left(\frac{2a}{a}\right)^2$ or, $E' = 4E$.

(b) In simple pendulum, when bob is in deflection position, the tension in the string is $T = mg\cos\theta + \frac{mv^2}{l}$. Since the value of θ is different at different positions, hence tension in the string is not constant throughout the oscillation. At end points θ is maximum; the value of $\cos\theta$ is least, hence the value of tension in the string is least. At the mean position, the value of $\theta = 0°$ and $\cos 0° = 1$, so the value of tension is largest.



Also velocity is given by $v = \omega\sqrt{a^2 - y^2}$ which is maximum when y = 0, at mean position.

(e) Spring constant $\propto \frac{1}{\text{Length of spring}}$

$\Rightarrow k' = \frac{k}{n}$

Also, spring constant depends on material properties of the spring.

Hence assertion is false, but reason is true.

(a) The time period of a oscillating spring is given by,

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T \propto \frac{1}{\sqrt{k}}$. Since the spring constant is large for hard spring, therefore hard spring has a less periodic time as compared to soft spring.

(e) In simple harmonic motion the velocity is given by,

$v = \omega\sqrt{a^2 - y^2}$ at extreme position, $y = a$.

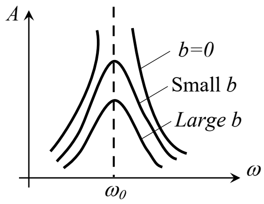
$\therefore v = 0$. But acceleration $A = -\omega^2 a$, which is maximum at extreme position.

(a) If the soldiers while crossing a suspended bridge march in steps, the frequency of marching steps of soldiers may match with the natural frequency of oscillations of the suspended bridge. In that situation resonance will take place, then the amplitude of oscillation of the suspended bridge will increase enormously, which may cause the collapsing of the bridge. To avoid situations the soldiers are advised to go out steps on suspended bridge.

(a) From equation, amplitude of oscillation

$$A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega / m)^2}}$$

In absence of damping force ($b = 0$), that the steady state amplitude approaches infinity as $\omega \to \omega_0$. That is, if there is no resistive force in the system and then it is possible to drive an oscillator with sinusoidal force at the resonance frequency, the amplitude of motion will build up without limit. This does not occur in practice because some damping is always present in real oscillation.



20. (b)

21. (c) The amplitude of an oscillating pendulum decreases with time because of friction due to air. Frequency of pendulum is independent $\left(T = \frac{1}{2\pi}\sqrt{\frac{g}{l}}\right)$ of amplitude.

22. (b) $x = a\sin\omega t$ and $v = \frac{dx}{dt} = a\omega\cos\omega t$

It is clear phase difference between 'x' and 'a' is $\pi/2$.

23. (e)