16. (b) $x = a\cos(\omega t + \theta)$ ….(i)

and $v = \frac{dx}{dt} = -a\omega\sin(\omega t + \theta)$ ….(ii)

Given at $t = 0$, $x = 1\,cm$ and $v = \pi$ and $\omega = \pi$

Putting these values in equation (i) and (ii) we will get $\sin\theta = \frac{-1}{a}$ and $\cos\theta = \frac{1}{a}$

$\Rightarrow \sin^2\theta + \cos^2\theta = \left(-\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 \Rightarrow a = \sqrt{2}\,cm$

17. (d) $y = A\sin\omega t = \frac{A\sin 2\pi}{T}t \Rightarrow \frac{A}{2} = A\sin\frac{2\pi t}{T} \Rightarrow t = \frac{T}{12}$.

18. (a) The amplitude is a maximum displacement from the mean position.

19. (c) Equation of motion $y = a\cos\omega t$

$\Rightarrow \frac{a}{2} = a\cos\omega t \Rightarrow \cos\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$

$\Rightarrow \frac{2\pi t}{T} = \frac{\pi}{3} \Rightarrow t = \frac{\frac{\pi}{3} \times T}{2\pi} = \frac{4}{3 \times 2} = \frac{2}{3}\,\text{sec}$

20. (a) Simple harmonic waves are set up in a string fixed at the, two ends.

21. (b)

22. (c) $v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$

$v_2 = \frac{dy_2}{dt} = -0.1\pi\sin\pi t = 0.1\pi\cos\left(\pi t + \frac{\pi}{2}\right)$

Phase difference of velocity of first particle with respect to the velocity of 2nd particle at $t = 0$ is

$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$.

23. (b) $\frac{a_1}{a_2} = \frac{10}{25} = \frac{2}{5}$

24. (c) $y = a\sin\frac{2\pi}{T}t \Rightarrow \frac{a}{2} = a\sin\frac{2\pi t}{3} \Rightarrow \frac{1}{2} = \sin\frac{2\pi t}{3}$

$\Rightarrow \sin\frac{2\pi t}{3} = \sin\frac{\pi}{6} \Rightarrow \frac{2\pi t}{3} = \frac{\pi}{6} \Rightarrow t = \frac{1}{4}\,\text{sec}$

25. (b)

26. (a) $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x' = a\cos\omega t = a\sin\left(\omega t + \frac{\pi}{2}\right)$

$\therefore \Delta\phi = \left(\omega t + \frac{\pi}{2}\right) - \left(\omega t + \frac{\pi}{6}\right) = \frac{\pi}{3}$

Velocity of Simple Harmonic Motion

1. (a) Velocity of a particle executing S.H.M. is given by

$v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{2\pi}{T}\sqrt{\frac{3A^2}{4}} = \frac{\pi A\sqrt{3}}{T}$.

2. (c) $v = \omega\sqrt{(a^2 - y^2)} = 2\sqrt{60^2 - 20^2} = 113\,mm/s$.

3. (c) It is given $v_{max} = 100\,cm/sec$, $a = 10\,cm$.

$\Rightarrow v_{max} = a\omega \Rightarrow \omega = \frac{100}{10} = 10\,rad/sec$

Hence $v = \omega\sqrt{a^2 - y^2} \Rightarrow 50 = 10\sqrt{(10)^2 - y^2}$

$\Rightarrow y = 5\sqrt{3}\,cm$

4. (c) At centre $v_{max} = a\omega = a.\frac{2\pi}{T} = \frac{0.2 \times 2\pi}{0.01} = 40\pi$

5. (b) $v_{max} = a\omega = a\frac{2\pi}{T} = 3 \times \frac{2\pi}{6} = \pi\,cm/s$

6. (c) $v_{max} = \omega a = \frac{2\pi}{T} \times a \Rightarrow v_{max} = \frac{2 \times \pi \times 2}{2} = 2\pi\,m/s$

7. (d) $v_{max} = a\omega = \frac{a.2\pi}{T} = \frac{2\pi a}{T}$

8. (c) $v = \omega\sqrt{a^2 - y^2} \Rightarrow 10 = \omega\sqrt{a^2 - (4)^2}$ and $8 = \omega\sqrt{a^2 - (5)^2}$

On solving $\omega = 2 \Rightarrow \omega = \frac{2\pi}{T} = 2 \Rightarrow T = \pi\,sec$

9. (d) From the given equation, $a = 5$ and $\omega = 4$

$\therefore v = \omega\sqrt{a^2 - y^2} = 4\sqrt{(5)^2 - (3)^2} = 16$

10. (b) $v_{max} = a\omega = a \times \frac{2\pi}{T} = (50 \times 10^{-3}) \times \frac{2\pi}{2} = 0.15\,m/s$

11. (a) $n = \frac{\omega}{2\pi} = \frac{220}{2\pi} = 35\,Hz$

$v_{max} = \omega\,a = 220 \times 0.30\,m/s = 66\,m/s$

12. (d) $v_{max} = a\omega$ and $A_{max} = a\omega^2 \Rightarrow$

$\omega = \frac{A_{max}}{v_{max}} = \frac{4}{2} = 2\,rad/sec$

13. (a) $v_{max} = a\omega = \frac{a \times 2\pi}{T} = \frac{2 \times 10^{-3} \times 2\pi}{0.1} = \frac{\pi}{25}\,m/s$

14. (b) $A = \omega^2 y \Rightarrow \omega = \sqrt{A/y} = \sqrt{\frac{8}{2}} = 2\,rad/sec$

Now $v_{max} = a\omega = 6 \times 2 = 12\,cm/sec$

15. (c) $v_{max} = a\omega \Rightarrow \omega = \frac{v_{max}}{a} = \frac{10}{4}$

Now, $v = \omega\sqrt{a^2 - y^2} \Rightarrow v^2 = \omega^2(a^2 - y^2) \Rightarrow y^2 = a^2 - \frac{v^2}{\omega^2}$

$\Rightarrow y = \sqrt{a^2 - \frac{v^2}{\omega^2}} = \sqrt{4^2 - \frac{5^2}{(10/4)^2}} = 2\sqrt{3}\,cm$

16. (b) The particles will meet at the mean position when P completes one oscillation and Q completes half an oscillation

So $\frac{v_P}{v_Q} = \frac{a\omega_P}{a\omega_Q} = \frac{T_Q}{T_P} = \frac{6}{3} = \frac{2}{1}$

17. (b) $\frac{v_{max}}{A_{max}} = \frac{a\omega}{a\omega^2} = \frac{1}{\omega}$

18. (a) Velocity is same. So by using $v = a\omega$

$\Rightarrow A_1\omega_1 = A_2\omega_2 = A_3\omega_3$

19. (d) In S.H.M. at mean position velocity is maximum

So $v = a\omega$ (maximum)

20. (b)

21. (b)

22. (c) Acceleration $A = \omega^2 y \Rightarrow \omega = \sqrt{\frac{A}{y}} = \sqrt{\frac{0.5}{0.02}} = 5$

Maximum velocity $v_{max} = a\omega = 0.1 \times 5 = 0.5$

23. (d) At mean position velocity is maximum

i.e., $v_{max} = \omega a \Rightarrow \omega = \frac{v_{max}}{a} = \frac{16}{4} = 4$

$\therefore\ v = \omega\sqrt{a^2 - y^2} \Rightarrow 8\sqrt{3} = 4\sqrt{4^2 - y^2}$

$\Rightarrow 192 = 16(16 - y^2) \Rightarrow 12 = 16 - y^2 \Rightarrow y = 2\,cm$.

24. (a) $v_{max} = a\omega = 3 \times 100 = 300$

25. (a) $x = 3\sin 2t + 4\cos 2t$. From given equation

$a_1 = 3, a_2 = 4,$ and $\phi = \frac{\pi}{2}$

$\therefore\ a = \sqrt{a_1^2 + a_2^2} = \sqrt{3^2 + 4^2} = 5 \Rightarrow$

$v_{max} = a\omega = 5 \times 2 = 10$

26. (c) Velocity in mean position $v = a\omega$, velocity at a distance of half amplitude.

$v = \omega\sqrt{a^2 - y^2} = \omega\sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{3}{2}}\,a\omega = \sqrt{\frac{3}{2}}\,v$

27. (a) $x = A\cos\left(\omega t + \frac{\pi}{4}\right)$ and $v = \frac{dx}{dt} = -A\omega\sin\left(\omega t + \frac{\pi}{4}\right)$

For maximum speed,

$\sin\left(\omega t + \frac{\pi}{4}\right) = 1 \Rightarrow \omega t + \frac{\pi}{4} = \frac{\pi}{2}$ or $\omega t = \frac{\pi}{2} - \frac{\pi}{4} \Rightarrow$

$t = \frac{\pi}{4\omega}$

Acceleration of Simple Harmonic Motion

1. (d) $F = -kx$

2. (c) The stone execute S.H.M. about centre of earth with time period $T = 2\pi\sqrt{\frac{R}{g}}$; where R = Radius of earth.

3. (c) Acceleration $= \omega^2 a$ at extreme position is maximum.

4. (d) $-a\omega^2$ when it is at one extreme point.

5. (a) Maximum acceleration $= a\omega^2 = a \times 4\pi^2 n^2$

$= 0.01 \times 4 \times (\pi)^2 \times (60)^2 = 144\pi^2\,m/sec$

6. (a) Maximum acceleration

$A_{max} = a\omega^2 = \frac{a \times 4\pi^2}{T^2} = \frac{1 \times 4 \times (3.14)^2}{0.2 \times 0.2}$

$F_{max} = m \times A_{max} = \frac{0.1 \times 4 \times (3.14)^2}{0.2 \times 0.2} = 98.596\,N$

7. (a) Maximum velocity $= a\omega = 16$

Maximum acceleration $= \omega^2 a = 24$

$\Rightarrow a = \frac{(a\omega)^2}{\omega^2 a} = \frac{16 \times 16}{24} = \frac{32}{3}\,m$

8. (d) Acceleration $\propto$ – displacement, and direction of acceleration is always directed towards the equilibrium position.

9. (d) Maximum force $= m(a\omega^2) = ma\left(\frac{4\pi^2}{T^2}\right)$

$= 0.5\left(\frac{4\pi^2}{\pi^2/25}\right) \times 0.01 = 0.5\,N$

10. (d) $a_{max} = \omega^2 a = \left(\frac{\pi}{4}\right)^2 a = 0.62\,cm/sec^2$ $[\because a = 1]$

11. (a) For S.H.M. $F = -kx$.

$\therefore$ Force = Mass × Acceleration $\propto -x$

$\Rightarrow F = -Akx$; where A and k are positive constants.

12. (a) Velocity $v = a\omega = a \times 2\pi n$

$= 0.06 \times 2\pi \times 15 = 5.65\,m/s$

Acceleration $A = \omega^2 a = 4\pi^2 n^2 a = 5.32 \times 10^2\,m/s^2$

13. (d) $A_{max} = a\omega^2 \Rightarrow a = \frac{A_{max}}{\omega^2} = \frac{7.5}{(3.5)^2} = 0.61\,m$

14. (a) $a = 10 \times 10^{-2}\,m$ and $\omega = 10\,rad/sec$

$A_{max} = \omega^2 a = 10 \times 10^{-2} \times 10^2 = 10\,m/sec^2$

15. (a) $A_{max} = \omega^2 a$

16. (d) $A_{max} = 4\pi^2 n^2 a = 4\pi^2 \times (50)^2 \times 0.02 = 200\pi^2\,m/s$

17. (d) $A = -\omega^2 y$ at mean position $y = 0$

So acceleration is minimum (zero).

18. (d) In S.H.M. $v=\sqrt{a^2-y^2}$ and $a=-\omega^2 y$ when $y=a$

$\Rightarrow v_{\min}=0$ and $a_{\max}=-\omega^2 a$

19. (b) Comparing given equation with standard equation,

$y=a\sin(\omega t+\phi)$, we get, $a=2\,cm,\ \omega=\frac{\pi}{2}$

$\therefore A_{\max}=\omega^2 A=\left(\frac{\pi}{2}\right)^2\times 2=\frac{\pi^2}{2}cm/s^2$.

20. (c) Velocity $v=\omega\sqrt{A^2-x^2}$ and acceleration $=\omega^2 x$

Now given, $\omega^2 x=\omega\sqrt{A^2-x^2}\Rightarrow$ $\omega^2.1=\omega\sqrt{2^2-1^2}$

$\Rightarrow \omega=\sqrt{3}\ \therefore\ T=\frac{2\pi}{\omega}=\frac{2\pi}{\sqrt{3}}$

21. (c) $a=-\omega^2 x\Rightarrow\left|\frac{a}{x}\right|=\omega^2$

Energy of Simple Harmonic Motion

1. (d) $E=\frac{1}{2}m\omega^2a^2\Rightarrow E\propto a^2$

2. (a) P.E. $=\frac{1}{2}m\omega^2x^2$

It is clear P.E. will be maximum when x will be maximum *i.e.*, at $x=\pm A$

3. (d) Let x be the point where K.E. = P.E.

Hence $\frac{1}{2}m\omega^2(a^2-x^2)=\frac{1}{2}m\omega^2x^2$

$\Rightarrow 2x^2=a^2\Rightarrow x=\frac{a}{\sqrt{2}}=\frac{4}{\sqrt{2}}=2\sqrt{2}\ cm$

4. (a) Since maximum value of $\cos^2\omega t$ is 1.

$\therefore K_{\max}=K_o\cos^2\omega t=K_o$

Also $K_{\max}=PE_{\max}=K_o$

5. (a) $F=-kx\Rightarrow dW=Fdx=-kxdx$

So $\int_0^W dW=\int_0^x -kx\,dx\Rightarrow W=U=-\frac{1}{2}kx^2$

6. (c) Suppose at displacement y from mean position potential energy = kinetic energy

$\Rightarrow \frac{1}{2}m(a^2-y^2)\omega^2=\frac{1}{2}m\omega^2y^2$

$\Rightarrow a^2=2y^2\Rightarrow y=\frac{a}{\sqrt{2}}$

7. (c) Total energy in SHM $E=\frac{1}{2}m\omega^2a^2$; (where a = amplitude)

Potential energy

$U=\frac{1}{2}m\omega^2(a^2-y^2)=E-\frac{1}{2}m\omega^2y^2$

When $y=\frac{a}{2}\Rightarrow U=E-\frac{1}{2}m\omega^2\left(\frac{a^2}{4}\right)=E-\frac{E}{4}=\frac{3E}{4}$

8. (b) $\frac{\text{Potential energy}(U)}{\text{Total energy}(E)}=\frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2}=\frac{y^2}{a^2}$

So $\frac{2.5}{E}=\frac{\left(\frac{a}{2}\right)^2}{a^2}\Rightarrow E=10J$

9. (d) Kinetic energy $T=\frac{1}{2}m\omega^2(a^2-x^2)$

and potential energy, $V=\frac{1}{2}m\omega^2x^2$

$\therefore\frac{T}{V}=\frac{a^2-x^2}{x^2}$

10. (c) $\frac{U}{U_{\max}}=\frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2}\Rightarrow\frac{1}{4}=\frac{y^2}{a^2}\Rightarrow y=\frac{a}{2}$

11. (c) Kinetic energy $K=\frac{1}{2}m\omega^2(a^2-y^2)$

$=\frac{1}{2}\times 10\times\left(\frac{2\pi}{2}\right)^2[10^2-5^2]=375\,\pi^2 ergs$

12. (b) $\frac{U}{E}=\frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2}=\frac{y^2}{a^2}=\frac{\left(\frac{a}{2}\right)^2}{a}=\frac{1}{4}$

13. (a)

14. (a) The time period of potential energy and kinetic energy is half that of SHM.

15. (b) If at any instant displacement is y then it is given that $U=\frac{1}{2}\times E\Rightarrow$

$\frac{1}{2}m\omega^2y^2=\frac{1}{2}\times\left(\frac{1}{2}m\omega^2a^2\right)$

$\Rightarrow y=\frac{a}{\sqrt{2}}=\frac{6}{\sqrt{2}}=4.2\,cm$

16. (b) So $a=6cm,\ \omega=100rad/\sec$

$K_{\max}=\frac{1}{2}m\omega^2a^2=\frac{1}{2}\times 1\times(100)^2\times(6\times 10^{-2})^2=18\ J$

17. (c) In S.H.M., frequency of K.E. and P.E.

= 2 × (Frequency of oscillating particle)

18. (b) Total energy $U=\frac{1}{2}Ka^2$

19. (d) $\frac{U}{E}=\frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2}=\frac{y^2}{a^2}\Rightarrow \frac{U}{80}=\frac{\left(\frac{3}{4}a\right)^2}{a^2}=\frac{9}{16}\Rightarrow$

$U=45\,J$

20. (c)

21. (c) Maximum potential energy position is $y=\pm a$ and maximum kinetic energy position is $y=0$

22. (c) $Mg=Kl \Rightarrow U_{\max}=\frac{1}{2}Kl^2=\frac{1}{2}mgl$

23. (b) $\frac{U}{E}=\frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2}=\frac{y^2}{a^2}=\frac{\left(\frac{a}{2}\right)^2}{a^2}=\frac{1}{4}\Rightarrow U=\frac{E}{4}$

24. (b) In S.H.M., at mean position *i.e.* at $x = 0$ kinetic energy will be maximum and *pE* will be minimum. Total energy is always constant.

25. (a) In SHM for a complete cycle average value of kinetic energy and potential energy are equal *i.e.* $<E> = <U> =\frac{1}{4}m\omega^2a^2$

26. (c) Total energy $=\frac{1}{2}m\omega^2a^2=$ constant

27. (c) Kinetic energy at mean position,

$K_{\max}=\frac{1}{2}mv_{\max}^2.\Rightarrow v_{\max}=\sqrt{\frac{2K_{\max}}{m}}$

$=\sqrt{\frac{2\times 16}{0.32}}=\sqrt{100}=10\,m/s$

28. (a) $E=\frac{1}{2}ma^2\omega^2=\frac{1}{2}ma^2\left(\frac{4\pi^2}{T^2}\right)\Rightarrow E\propto\frac{a^2}{T^2}$

29. (b) $\frac{K}{E}=\frac{\frac{1}{2}m\omega^2(a^2-y^2)}{\frac{1}{2}m\omega^2a^2}=\frac{a^2-y^2}{a^2}=1-\frac{y^2}{a^2}$

So, $\frac{\left(\frac{3E}{4}\right)}{E}=1-\frac{y^2}{a^2}\Rightarrow\frac{y^2}{a^2}=1-\frac{3}{4}=\frac{1}{4}\Rightarrow y=\frac{a}{2}.$

30. (c) Kinetic energy $K=\frac{1}{2}mv^2=\frac{1}{2}ma^2\omega^2\cos^2\omega t$

$=\frac{1}{2}m\omega^2a^2(1+\cos 2\omega t)$ hence kinetic energy varies periodically with double the frequency of S.H.M. *i.e.* 2ω.

31. (c) $E=\frac{1}{2}m\omega^2a^2\Rightarrow\frac{E'}{E}=\frac{a'^2}{a^2}\Rightarrow\frac{E'}{E}=\frac{\left(\frac{3}{4}a\right)^2}{a^2}$

$\left(\because a'=\frac{3}{4}a\right)$

$\Rightarrow E'=\frac{9}{16}E$

32. (d) In simple harmonic motion, energy changes from kinetic to potential and potential to kinetic but the sum of two always remains constant.

33. (b) Body collides elastically with walls of room. So, there will be no loss in its energy and it will remain colliding with walls of room, so it's motion will be periodic.

There is no change in energy of the body, hence there is no acceleration, so it's motion is not SHM.

34. (b) $E_1=\frac{1}{2}Kx^2\Rightarrow x=\sqrt{\frac{2E_1}{K}}$, $E_2=\frac{1}{2}Ky^2\Rightarrow y=\sqrt{\frac{2E_2}{K}}$

and $E=\frac{1}{2}K(x+y)^2\Rightarrow x+y=\sqrt{\frac{2E}{K}}$

$\Rightarrow\sqrt{\frac{2E_1}{K}}+\sqrt{\frac{2E_2}{K}}=\sqrt{\frac{2E}{K}}\Rightarrow\sqrt{E_1}+\sqrt{E_2}=\sqrt{E}$

Time Period and Frequency

1. (b) In the given case, $\frac{\text{Displacement}}{\text{Acceleration}}=\frac{1}{b}$

$\therefore$ Time period $T=2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}=\frac{2\pi}{\sqrt{b}}$

2. (c) On comparing with standard equation $\frac{d^2y}{dt^2}+\omega^2y=0$ we get

$\omega^2=K\Rightarrow\omega=\frac{2\pi}{T}=\sqrt{K}\Rightarrow T=\frac{2\pi}{\sqrt{K}}.$

3. (b) Ball execute S.H.M. inside the tunnel with time period $T=2\pi\sqrt{\frac{R}{g}}=84.63$ min

Hence time to reach the ball from one end to the other end of the tunnel $t=\frac{84.63}{2}=42.3$ min.

4. (d) Given max velocity $\omega a=1$ and maximum acceleration $\omega^2a=1.57$

$\therefore\frac{\omega^2a}{\omega a}=1.57\Rightarrow\omega=1.57\Rightarrow\frac{2\pi}{T}=1.57\Rightarrow T=4$

5. (b) $\omega=\frac{2\pi}{T}=100\pi\Rightarrow T=0.02$ sec

6. (a) At mean position, the kinetic energy is maximum.

Hence $\frac{1}{2}ma^2\omega^2 = 16$

On putting the values we get $\omega = 10 \Rightarrow T = \frac{2\pi}{\omega} = \frac{\pi}{5}$ sec

7. (d) $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{3}{12}} = \pi = 3.14$ sec

8. (d) $\omega = \sqrt{\frac{k}{m}} \Rightarrow \frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} \Rightarrow 2 = \sqrt{\frac{m_1}{m_2}} \Rightarrow m_2 = \frac{m_1}{4}$

9. (d)

10. (a) $\omega = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}} = \sqrt{\frac{2.0}{0.02}} = 10\, rad\, s^{-1}$

11. (b) From given equation $\omega = 3000$, $\Rightarrow$ $n = \frac{\omega}{2\pi} = \frac{3000}{2\pi}$

12. (b)

13. (b) Given, $v = \pi\, cm/\text{sec}$, $x = 1\, cm$ and $\omega = \pi s^{-1}$

using $v = \omega\sqrt{a^2 - x^2} \Rightarrow \pi = \pi\sqrt{a^2 - 1}$

$\Rightarrow 1 = a^2 - 1 \Rightarrow a = \sqrt{2}\, cm$

14. (b) Length of the line = Distance between extreme positions of oscillation = 4 *cm*

So, Amplitude $a = 2\, cm$

also $v_{max} = 12\, cm/s$

$\because v_{max} = \omega a = \frac{2\pi}{T}a$

$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 2}{12} = 1.047\,\text{sec}$

15. (a) Comparing given equation with standard equation,

$x = a\cos(\omega t + \phi)$ we get, $a = 0.01$ and $\omega = \pi$

$\Rightarrow 2\pi n = \pi \Rightarrow n = 0.5\, Hz$

16. (d) $y = 5\sin(\pi t + 4\pi)$, comparing it with standard equation

$y = a\sin(\omega t + \phi) = a\sin\left(\frac{2\pi t}{T} + \phi\right)$

$a = 5\, m$ and $\frac{2\pi t}{T} = \pi t \Rightarrow T = 2$ *sec.*

17. (d) $v_{max} = a\omega = a \times 2\pi n \Rightarrow n = \frac{v_{max}}{2\pi a} = \frac{31.4}{2 \times 3.14 \times 5} = 1\, Hz$

18. (d) From the given equitation $\omega = 2\pi n = 4\pi \Rightarrow$ $n = 2\, Hz$

Simple Pendulum

1. (c) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$

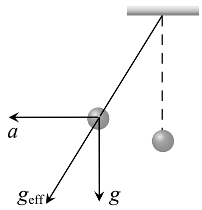
2. (a) Inside the mine *g* decreases

hence from $T = 2\pi\sqrt{\frac{l}{g}}$; *T* increase

3. (b) When a little mercury is drained off, the position of *c.g.* of ball falls (*w.r.t.* fixed and) so that effective length of pendulum increases hence *T* increase.

4. (b) Initially time period was $T = 2\pi\sqrt{\frac{l}{g}}$.

When train accelerates, the effective value of *g* becomes $\sqrt{(g^2 + a^2)}$ which is greater than *g*



Hence, new time period, becomes less than the initial time period.

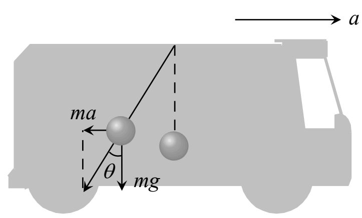
5. (b) As we know $g = \frac{GM}{R^2}$

$\Rightarrow \frac{g_{earth}}{g_{planet}} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$

Also $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} \Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}}$

$\Rightarrow T_p = 2\sqrt{2}$ sec.

6. (b) In accelerated frame of reference, a fictitious force (pseudo force) *ma* acts on the bob of pendulum as shown in figure.



Hence, $\tan\theta = \frac{ma}{mg} = \frac{a}{g}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$ in the backward direction.

7. (c)

8. (c) $T = 2\pi\sqrt{\frac{l}{g}}$ (Independent of mass)

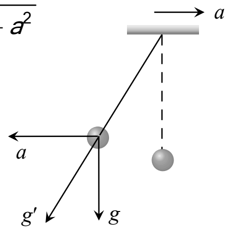
9. (c) In stationary lift $T = 2\pi\sqrt{\frac{l}{g}}$

In upward moving lift $T' = 2\pi\sqrt{\frac{l}{(g+a)}}$

($a =$ Acceleration of lift)

$\Rightarrow \frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{g}{\left(g+\frac{g}{4}\right)}} = \sqrt{\frac{4}{5}} \Rightarrow T' = \frac{2T}{\sqrt{5}}$

10. (d) $g' = \sqrt{g^2 + a^2}$



11. (d) In the given case effective acceleration $g_{eff.} = 0 \Rightarrow T = \infty$

12. (b) $p_{max} = \sqrt{2mE_{max}}$

13. (a) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \sqrt{\frac{l}{g}}$ = constant

$\Rightarrow l \propto g, \Rightarrow \frac{l_m}{1} = \frac{1}{6}\frac{g}{g} \Rightarrow l_m = \frac{1}{6}m$

14. (d) $T \propto \sqrt{l} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{0.02}{2} = 0.01 \Rightarrow \Delta T = 0.01\,T$

Loss of time per day $= 0.01 \times 24 \times 60 \times 60$ $= 864$ sec

15. (d) $\frac{T'}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{g}{g+5g}} = \sqrt{\frac{1}{6}} \Rightarrow T' = \frac{T}{\sqrt{6}}$

16. (b) $T \propto \sqrt{l} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2} \times 1\% = 0.5\%$

17. (b) At B, the velocity is maximum using conservation of mechanical energy

$\Delta PE = \Delta KE \Rightarrow mgH = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gH}$

18. (c)

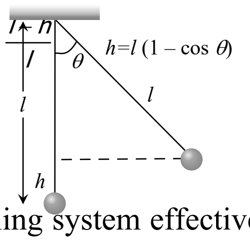
19. (c) If suppose bob rises up to a height h as shown then after releasing potential energy at extreme position becomes kinetic energy of mean position

$\Rightarrow mgh = \frac{1}{2}mv_{max}^2 \Rightarrow v_{max} = \sqrt{2gh}$

Also, from figure $\cos\theta = \frac{l-h}{l}$

$\Rightarrow h = l(1 - \cos\theta)$

So, $v_{max} = \sqrt{2gl(1-\cos\theta)}$

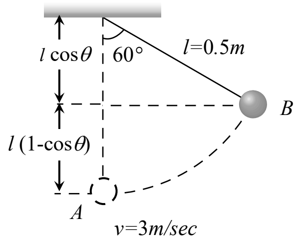


20. (c) $T = 2\pi\sqrt{\frac{l}{g}}$; for freely falling system effective $g = 0$

so $T = \infty$ or $n = 0$

It means that pendulum does not oscillate at all.

21. (d) Let bob velocity be v at point B where it makes an angle of 60° with the vertical, then using conservation of mechanical energy



$KE_A + PE_A = KE_B + PE_B$

$\Rightarrow \frac{1}{2}m \times 3^2 = \frac{1}{2}mv^2 + mgl(1-\cos\theta)$

$\Rightarrow 9 = v^2 + 2 \times 10 \times 0.5 \times \frac{1}{2} \Rightarrow v = 2\ m/s$

22. (d) $T \propto \sqrt{l} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} \Rightarrow \frac{2}{T_2} = \sqrt{\frac{l}{4l}} \Rightarrow T_2 = 4\,sec$

23. (c) Remains the same because time period of simple pendulum T is independent of mass of the bob

24. (c) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{l}{T^2} = \frac{g}{4\pi^2}$ = constant

25. (d) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_P}{T_e} = \sqrt{\frac{g_e}{g_P}} = \sqrt{\frac{2}{1}} \Rightarrow T' = \sqrt{2}T$

26. (a) If initial length $l_1 = 100$ then $l_2 = 121$

By using $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$

Hence, $\frac{T_1}{T_2} = \sqrt{\frac{100}{121}} \Rightarrow T_2 = 1.1\,T_1$

% increase $= \frac{T_2 - T_1}{T_1} \times 100 = 10\%$

27. (a) $\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{100}{400}}$ (If $l_1 = 100$ then $l_2 = 400$)

$\Rightarrow T_2 = 2T_1$

Hence % increase $= \frac{T_2 - T_1}{T_1} \times 100 = 100\%$

28. (b) $T = 2\pi\sqrt{l/g} \Rightarrow l = \frac{gT^2}{4\pi^2} = \frac{9.8 \times 4}{4 \times \pi^2} = 99\ cm$

29. (d) This is the case of freely falling lift and in free fall of lift effective g for pendulum will be zero. So $T = 2\pi\sqrt{\frac{l}{0}} = \infty$

30. (d) After standing centre of mass of the oscillating body will shift upward therefore effective length will decrease and by $T \propto \sqrt{l}$, time period will decrease.

31. (c) $T = 2\pi\sqrt{l/g} = 2\pi\sqrt{\frac{1}{\pi^2}} = 2\ sec$

32. (c) Time period is independent of mass of pendulum.

33. (b) $T \propto \sqrt{l}$ Time period depends only on effective length. Density has no effect on time period. If length made 4 times then time period becomes 2 times.

34. (b) Time period is independent of mass of bob of pendulum.

35. (a) At the surface of moon, g decreases hence time period increases $\left(\text{as}\ T \propto \frac{1}{\sqrt{g}}\right)$

36. (a) When lift falls freely effective acceleration and frequency of oscillations be zero

$g_{eff} = 0 \Rightarrow T = \infty$, hence a frequency = 0.

37. (d) Effective value of 'g' remains unchanged.

38. (b) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{144}{100}} = \frac{12}{10}$

$\Rightarrow T_2 = 1.2\ T_1$

Hence % increase $= \frac{T_2 - T_1}{T_1} \times 100 = 20\%$

39. (c) If amplitude is large motion will not remain simple harmonic.

40. (d) Minimum velocity is zero at the extreme positions.

41. (a) At the time $t = \frac{T}{4} = \frac{4}{4} = 1\sec$ after passing from mean position, the body reaches at it's extreme position. At extreme, position velocity of body becomes zero.

42. (a) No momentum will be transferred because, at extreme position the velocity of bob is zero.

43. (a) In this case frequency of oscillation is given by $n = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + a^2}}{l}}$ where a is the acceleration of car. If a increases then n also increases.

44. (b) As periodic time is independent of amplitude.

45. (d) Frequency $n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} \Rightarrow$

$\frac{l_1}{l_2} = \frac{n_2^2}{n_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$

46. (d) Suppose at $t = 0$, pendulums begins to swing simultaneously.

Hence, they will again swing simultaneously

if $n_1 T_1 = n_2 T_2$

$\Rightarrow \frac{n_1}{n_2} = \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_1}{l_2} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{8}{7}\right)^2 = \frac{64}{49}$

47. (b) $T \propto \frac{1}{\sqrt{g}}$ and g is same in both cases so time period remain same.

48. (a) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$, hence if l made 9 times T becomes 3 times.

Also time period of simple pendulum does not depends on the mass of the bob.

49. (a) As we go from equator to pole the value of g increases. Therefore time period of simple pendulum $\left(T \propto \frac{1}{\sqrt{g}}\right)$ decreases. $\left(\because T \propto \frac{1}{\sqrt{g}}\right)$

50. (a) If v is velocity of pendulum at Q

and 10% energy is lost while moving from P to Q

Hence, by applying conservation of between P and Q

$\frac{1}{2}mv^2 = 0.9(mgh) \Rightarrow v^2 = 2 \times 0.9 \times 10 \times 2 \Rightarrow$

$v = 6\ m/sec$

51. (c) $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}} = \sqrt{\left(\frac{g}{g/4}\right)} \Rightarrow T_2 = 2T_1 = 2T$

52. (c) For stationary lift $T_1 = 2\pi\sqrt{\frac{l}{g}}$

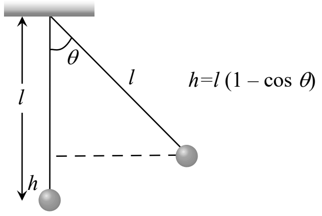
For ascending lift with acceleration a, $T_2 = 2\pi\sqrt{\frac{l}{g+a}}$

$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{g+a}{g}} \Rightarrow \frac{T}{T_2} = \sqrt{\frac{g + \frac{g}{3}}{g}} = \sqrt{\frac{4}{3}} \Rightarrow T_2 = \frac{\sqrt{3}}{2}T$

53. (c) $T \propto \sqrt{l}$

54. (d) Kinetic energy will be maximum at mean position.

From law of conservation of energy maximum kinetic energy at mean position = Potential energy at displaced position



$\Rightarrow K_{max} = mgh = mgl(1-\cos\theta)$

55. (c)

56. (b) As it is clear that in vacuum, the bob will not experience any frictional force. Hence, there shall be no dissipation therefore, it will oscillate with constant amplitude.

57. (c) The effective acceleration in a lift descending with acceleration $\frac{g}{3}$ is

$g_{eff} = g - \frac{g}{3} = \frac{2g}{3}$

$\therefore\ T = 2\pi\sqrt{\left(\frac{L}{g_{eff}}\right)} = 2\pi\sqrt{\left(\frac{L}{2g/3}\right)} = 2\pi\sqrt{\left(\frac{3L}{2g}\right)}$

58. (a)

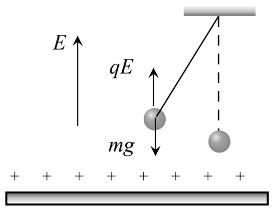
59. (c) According to the principle of conservation of energy, $\frac{1}{2}mv^2 = mgh$ or

$v = \sqrt{2gh} = \sqrt{2\times 9.8\times 0.1} = 1.4\ m/s$

60. (a) In this case time period of pendulum becomes

$T' = 2\pi\sqrt{\frac{l}{\left(g+\frac{qE}{m}\right)}}$

$\Rightarrow T' < T$



61. (b) In deep mine $g' = g\left(1-\frac{d}{R}\right)$; *i.e.*, g decreases so according to $n \propto \sqrt{g}$, frequency also decreases.

Spring Pendulum

1. (d) Maximum velocity $= a\omega = a\sqrt{\frac{k}{m}}$

Given that $a_1\sqrt{\frac{K_1}{m}} = a_2\sqrt{\frac{K_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{K_2}{K_1}}$

2. (d) Given spring system has parallel combination, so

$k_{eq} = k_1 + k_2$ and time period $T = 2\pi\sqrt{\frac{m}{(k_1+k_2)}}$

3. (b) $T = 2\pi\sqrt{\frac{m}{k}}$. Also spring constant ($k$) $\propto \frac{1}{\text{Length}(l)}$, when the spring is half in length, then k becomes twice.

$\therefore\ T' = 2\pi\sqrt{\frac{m}{2k}} \Rightarrow \frac{T'}{T} = \frac{1}{\sqrt{2}} \Rightarrow T' = \frac{T}{\sqrt{2}}$

4. (b) $\omega = \sqrt{\frac{k}{m}}$

5. (b) With respect to the block the springs are connected in parallel combination.

$\therefore$ Combined stiffness $k = k_1 + k_2$ and

$n = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$

6. (c) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \frac{n_S}{n_P} = \sqrt{\frac{k_S}{k_P}} \Rightarrow \frac{n_s}{n_p} = \sqrt{\frac{\left(\frac{k}{2}\right)}{2k}} = \frac{1}{2}$

7. (c) In series $k_{eq} = \frac{k_1k_2}{k_1+k_2}$ so time period

$T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

8. (b) Force constant $k = \frac{F}{x} = \frac{0.5\times 10}{0.2} = 25\ N/m$

Now $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25}{25}} = 0.628\ sec$

9. (a) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m \propto T^2 \Rightarrow \frac{m_2}{m_1} = \frac{T_2^2}{T_1^2}$

$\Rightarrow \frac{M+m}{M} = \left(\frac{\frac{5}{4}T}{T}\right)^2 \Rightarrow \frac{m}{M} = \frac{9}{16}$

10. (c) Spring constant $(k) \propto \frac{1}{\text{Length of the spirng}(l)}$

as length becomes half, k becomes twice is $2k$.

11. (a) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \frac{n}{n'} = \sqrt{\frac{k}{m}\times\frac{m'}{K'}} = \sqrt{\frac{k}{m}\times\frac{2m}{2K}} = 1 \Rightarrow$

$n' = n$

12. (b) As mg produces extension x, hence $k = \frac{mg}{x}$

$\therefore\ T = 2\pi\sqrt{\frac{(M+m)}{k}} = 2\pi\sqrt{\frac{(M+m)x}{mg}}$

13. (d) For the given figure $f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ …..(i)

If one spring is removed, then $k_{eq} = k$ and

$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ ….(ii)

From equation (i) and (ii), $\frac{f}{f'} = \sqrt{2} \Rightarrow$ $f' = \frac{f}{\sqrt{2}}$

14. (c) $\because\ mg = kx \Rightarrow \frac{m}{k} = \frac{x}{g} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{g}}$

$= 2\pi\sqrt{\frac{9.8\times10^{-2}}{9.8}} = \frac{2\pi}{10} sec$

15. (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2}{80}} = 0.31 sec$

16. (d) Spring is cut into two equal halves so spring constant of each part = $2k$

These parts are in parallel so $K_{eq} = 2K + 2K = 4K$

Extension force (*i.e.* W) is same hence by using $F = kx \Rightarrow 4k \times x' = kx \Rightarrow x' = \frac{x}{4}$.

17. (a) In this case springs are in parallel, so $k_{eq} = k_1 + k_2$

and $\omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$

18. (d) Force constant $(k) \propto \frac{1}{\text{Length of the spring}(l)}$

$\Rightarrow \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{2}{1}$

19. (b) Standard equation for given condition

$x = a\cos\frac{2\pi}{T}t \Rightarrow x = -0.16\cos(\pi t)$

[As $a = -0.16$ *meter*, $T = 2$ *sec*]

20. (c) By using conservation of mechanical energy

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow x = v\sqrt{m/k}$

21. (c) Given elastic energies are equal *i.e.*, $\frac{1}{2}k_1x_1^2 = \frac{1}{2}k_2x_2^2$

$\Rightarrow \frac{k_1}{k_2} = \left(\frac{x_2}{x_1}\right)^2$ and using $F = kx$

$\Rightarrow \frac{F_1}{F_2} = \frac{k_1x_1}{k_2x_2} = \frac{k_1}{k_2}\times\sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{k_1}{k_2}}$

22. (c) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow n \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$

$\Rightarrow \frac{n}{n_2} = \sqrt{\frac{4m}{m}} \Rightarrow n_2 = \frac{n}{2}$

23. (d) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{4m}{m}} = 2$

$\Rightarrow T_2 = 2\times2 = 4s$

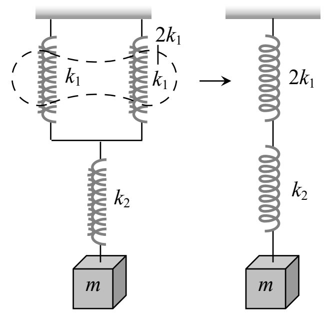
24. (a) $T \propto \frac{1}{\sqrt{k}} \Rightarrow$

$T_1 : T_2 : T_3 = \frac{1}{\sqrt{k}} : \frac{1}{\sqrt{k/2}} : \frac{1}{\sqrt{2k}} = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}$

25. (d) $T \propto \frac{1}{\sqrt{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{k_1}{k_2}} = \sqrt{\frac{k}{4k}} = \frac{1}{2} \Rightarrow T_2 = \frac{T_1}{2}$

26. (d) The time period of oscillation of a spring does not depend on gravity.

27. (b) In series combination



$\frac{1}{k_S} = \frac{1}{2k_1} + \frac{1}{k_2}$

$\Rightarrow$

$k_S = \left[\frac{1}{2k_1} + \frac{1}{k_2}\right]^{-1}$

28. (a) Work done in stretching (W) $\propto$ Stiffness of spring (*i.e.* k)

$\because\ k_A > k_B \Rightarrow W_A > W_B$

29. (a) When external force is applied, one spring gets extended and another one gets contracted by the same distance hence force due to two springs act in same direction.

i.e. $F = F_1 + F_2 \Rightarrow -kx = -k_1x - k_2x \Rightarrow k = k_1 + k_2$

30. (a) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} \Rightarrow$ $\frac{3}{2} = \sqrt{\frac{m+2}{m}} \Rightarrow$

$\frac{9}{4} = \frac{m+2}{m}$

$\Rightarrow m = \frac{8}{5}kg = 1.6\ kg$

31. (b) For series combination $k_{eq} = \frac{k_1k_2}{k_1+k_2}$

$F = k_{eq}x \Rightarrow mg = \left(\frac{k_1k_2}{k_1+k_2}\right)x \Rightarrow x = \frac{mg(k_1+k_2)}{k_1k_2}$

32. (d) $n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)m}}$

33. (b) Using $F = kx \Rightarrow 10g = k \times 0.25 \Rightarrow k = \frac{10g}{0.25} = 98 \times 4$

Now $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2}{4\pi^2}k$

$\Rightarrow m = \frac{\pi^2}{100} \times \frac{1}{4\pi^2} \times 98 \times 4 = 0.98\ kg$

34. (b) When spring is cut into *n* equal parts then spring constant of each part will be *nk* and so using $T \propto \frac{1}{\sqrt{k}}$, time period will be $T/\sqrt{n}$.

35. (b) By using $K \propto \frac{1}{l}$

Since one fourth length is cut away so remaining length is $\frac{3}{4}th$, hence *k* becomes $\frac{4}{3}$ times *i.e.*, $k' = \frac{4}{3}x$.

36. (d) $t_1 = 2\pi\sqrt{\frac{m}{K_1}}$ and $t_2 = 2\pi\sqrt{\frac{m}{K_2}}$

Equivalent spring constant for shown combination is

$K_1 + K_2$. So time period *t* is given by

$t = 2\pi\sqrt{\frac{m}{K_1+K_2}}$

By solving these equations we get $t^{-2} = t_1^{-2} + t_2^{-2}$

37. (c) $n = \frac{1}{2\pi}\sqrt{\frac{K_{effective}}{m}} = \frac{1}{2\pi}\sqrt{\frac{(K+2K)}{m}} = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$

38. (d) In series combination

$\frac{1}{k_S} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_2+k_1}{k_1k_2} \Rightarrow k_S = \frac{k_1k_2}{k_1+k_2}$.

39. (b) $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

In series, effective spring constant, $k = \frac{k_1k_2}{k_1+k_2}$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$ .....(ii)

Now, $t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{4\pi^2 m(k_1+k_2)}{k_1k_2}$

$t_1^2 + t_2^2 = T^2$. [Using equation (ii)]

40. (c) $\frac{1}{k_{eff}} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + ...$

$= \frac{1}{k}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right] = \frac{1}{k}\left(\frac{1}{1-1/2}\right) = \frac{2}{k}$

(By using sum of infinite geometrical progression $a + \frac{a}{r} + \frac{a}{r^2} + ...\infty$ sum (*S*) $= \frac{a}{1-r}$)

$\therefore k_{eff} = \frac{k}{2}$.

41. (a) $n \propto \sqrt{\frac{k}{m}}$

42. (b) $F = kx \Rightarrow mg = kx \Rightarrow m \propto kx$

Hence $\frac{m_1}{m_2} = \frac{k_1}{k_2} \times \frac{x_1}{x_2} \Rightarrow \frac{4}{6} = \frac{k}{k/2} \times \frac{1}{x_2}$

$\Rightarrow x_2 = 3\,cm$.

43. (b) Initially when 1 *kg* mass is suspended then by using $F = kx \Rightarrow mg = kx \Rightarrow$ $k = \frac{mg}{x} = \frac{1 \times 10}{5 \times 10^{-2}} = 200\frac{N}{m}$

Further, the angular frequency of oscillation of 2 *kg* mass is $\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{200}{2}} = 10\,rad/sec$

Hence, $v_{max} = a\omega = (10 \times 10^{-2}) \times 10 = 1\,m/s$

44. (a) $U = \frac{F^2}{2K} \Rightarrow U \propto \frac{1}{K} \Rightarrow \frac{U_1}{U_2} = \frac{K_2}{K_1} = 2$

45. (b) $U = \frac{1}{2}Kx^2$ but $T = Kx$

So energy stored $= \frac{1}{2}\frac{(Kx)^2}{K} = \frac{1}{2}\frac{T^2}{K}$

46. (a) System is equivalent to parallel combination of springs $\therefore K_{eq} = K_1 + K_2 = 400$ and

$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{0.25}{400}} = \frac{\pi}{20}$

47. (b) By cutting spring in four equal parts force constant (*K*) of each parts becomes four times $\left(\because k \propto \frac{1}{l}\right)$ so by using $T = 2\pi\sqrt{\frac{m}{K}}$; time period will be half *i.e.* $T' = T/2$

48. (d) $T \propto \sqrt{m} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} \Rightarrow \frac{5}{3} = \sqrt{\frac{M+m}{M}}$

$\Rightarrow \frac{25}{9} = \frac{M+m}{M} \Rightarrow \frac{m}{M} = \frac{16}{9}$

49. (c) $v_{max} = a\omega = a\frac{2\pi}{T}$

$\Rightarrow a = \frac{v_{max}T}{2\pi} = \frac{15\times 628\times 10^{-3}}{2\times 3.14} = 1.5\,cm$

50. (c) $Kx = mg \Rightarrow \frac{m}{K} = \frac{x}{g}$

So $T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{0.2}{9.8}} = \frac{2\pi}{7}$ sec

51. (b) $\omega = \sqrt{k/m} = \sqrt{\frac{4.84}{0.98}} = 2.22\ rad/\text{sec}$

52. (d) When spring is cut into two equal parts then spring constant of each part will be $2K$ and so using $n \propto \sqrt{K}$, new frequency will be $\sqrt{2}$ times *i.e.* $f_2 = \sqrt{2}\,f_1$.

53. (d)

54. (a) With mass m_2 alone, the extension of the spring l is given as $m_2 g = kl$ ...(i)

With mass $(m_1 + m_2)$, the extension l is given by

$(m_1 + m_2)g = k(l + \Delta l)$ ....(ii)

The increase in extension is Δl which is the amplitude of vibration. Subtracting (i) from (ii), we get

$m_1 g = k\Delta l$ or $\Delta l = \frac{m_1 g}{k}$

55. (b) Angular velocity $\omega = \sqrt{\left(\frac{k}{m}\right)} = \sqrt{\left(\frac{10}{10}\right)} = 1$

Now $v = \omega\sqrt{a^2 - y^2} \Rightarrow y^2 = a^2 - \frac{v^2}{\omega^2}$

$= (0.5)^2 - \frac{(0.4)^2}{1^2}$

$\Rightarrow y^2 = 0.9 = y = 0.3\,m$

Superposition of S.H.M.'s and Resonance

1. (c) Resultant amplitude $= \sqrt{3^2 + 4^2} = 5$

2. (c) $y = A\sin PT + B\cos PT$

Let $A = r\cos\theta,\ B = r\sin\theta$

$\Rightarrow y = r\sin(PT + \theta)$ which is the equation of SHM.

3. (c) $y = a(\cos\omega t + \sin\omega t) = a\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\omega t + \frac{1}{\sqrt{2}}\sin\omega t\right]$

$= a\sqrt{2}[\sin 45^\circ \cos\omega t + \cos 45^\circ \sin\omega t]$

$= a\sqrt{2}\sin(\omega t + 45^\circ) \Rightarrow$ Amplitude $= a\sqrt{2}$

4. (c) If first equation is $y_1 = a_1\sin\omega t \Rightarrow \sin\omega t = \frac{y_1}{a_1}$ ... (i)

then second equation will be $y_2 = a_2\sin\left(\omega t + \frac{\pi}{2}\right)$

$= a_2\left[\sin\omega t\cos\frac{\pi}{2} + \cos\omega t\sin\frac{\pi}{2}\right] = a_2\cos\omega t$

$\Rightarrow \cos\omega t = \frac{y_2}{a_2}$ ... (ii)

By squaring and adding equation (i) and (ii)

$\sin^2\omega t + \cos^2\omega t = \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2}$

$\Rightarrow \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2} = 1$; This is the equation of ellipse.

5. (a) If $y_1 = a_1\sin\omega t$ and $y_2 = a_2\sin(\omega t + \pi)$

$\Rightarrow \frac{y_1}{a_1} + \frac{y_2}{a_2} = 0 \Rightarrow y_2 = -\frac{a_2}{a_1}y_1$

This is the equation of straight line.

6. (c) If $y_1 = a_1\sin\omega t$ and $y_2 = a_2\sin(\omega t + 0) = a_2\sin\omega t$

$\Rightarrow \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2} - \frac{2y_1y_2}{a_1a_2} = 0 \Rightarrow y_2 = \frac{a_2}{a_1}y_1$

This is the equation of straight line.

7. (d) For given relation

Resultant amplitude $= \sqrt{4^2 + 4^2} = 4\sqrt{2}$

8. (a) $x = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$

$= 5\sqrt{2}\sin 2\pi t + 5\sqrt{2}\cos 2\pi t$

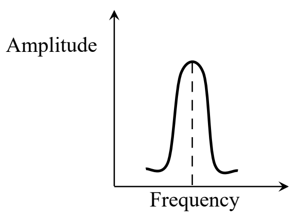
$x = 5\sqrt{2}\sin 2\pi t + 5\sqrt{2}\sin\left(2\pi t + \frac{\pi}{2}\right)$

Phase difference between constituent waves $\phi = \frac{\pi}{2}$

$\therefore$ Resultant amplitude $A = \sqrt{(5\sqrt{2})^2 + (5\sqrt{2})^2} = 10\ cm$.

9. (b)

10. (d) Less damping force gives a taller and narrower resonance peak



11. (b) $A = \frac{c}{a + b - c}$; when $b = 0$, $a = c$ amplitude

$A \to \infty$. This corresponds to resonance.

12. (c) Energy of particle is maximum at resonant frequency *i.e.,* $\omega_2 = \omega_o$. For amplitude resonance (amplitude maximum) frequency of driver force $\omega = \sqrt{\omega_o^2 - b^2 2m^2} \Rightarrow \omega_1 \neq \omega_o$

13. (a)

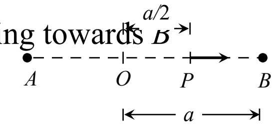
14. (c)

Critical Thinking Questions

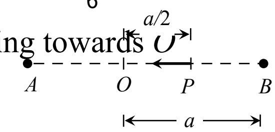
1. (d) $y = a\sin(\omega t + \phi_0)$. According to the question

$y = \frac{a}{2} \Rightarrow \frac{a}{2} = a\sin(\omega t + \phi_0) \Rightarrow (\omega t + \phi_0) = \phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

Physical meaning of $\phi = \frac{\pi}{6}$: Particle is at point *P* and it is going towards *B*



Physical meaning of $\phi = \frac{5\pi}{6}$: Particle is at point *P* and it is going towards *O*



So phase difference $\Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3} = 120°$

2. (b) $x = 12\sin\omega t - 16\sin^3\omega t = 4[3\sin\omega t - 4\sin^3\omega t]$

$= 4[\sin 3\omega t]$ (By using $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$)

$\therefore$ maximum acceleration $A_{max} = (3\omega)^2 \times 4 = 36\omega^2$

3. (b,c) Harmonic oscillator has some initial elastic potential energy and amplitude of harmonic variation of energy is $\frac{1}{2}Ka^2 = \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = 100\,J$

This is the maximum kinetic energy of the oscillator. Thus $K_{max} = 100\,J$

This energy is added to initial elastic potential energy may give maximum mechanical energy to have value $160\,J$.