Answers and Solutions

Work Done by Constant Force

1. (b) Work done by centripetal force is always zero, because force and instantaneous displacement are always perpendicular.
$W = \vec{F}.\vec{s} = Fs\cos\theta = Fs\cos(90°) = 0$

2. (a) Work = Force × Displacement (length)
If unit of force and length be increased by four times then the unit of energy will increase by 16 times.

3. (c) No displacement is there.

4. (d) Stopping distance $S \propto u^2$. If the speed is doubled then the stopping distance will be four times.

5. (c) $W = Fs\cos\theta \Rightarrow \cos\theta = \frac{W}{Fs} = \frac{25}{50} = \frac{1}{2} \Rightarrow \theta = 60°$

6. (b) Work done = Force × displacement
= Weight of the book × Height of the book shelf

7. (b) Work done does not depend on time.

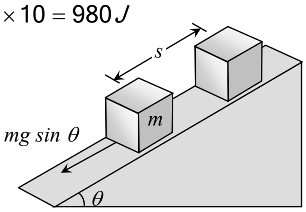
8. (c) $W = \vec{F}.\vec{s} = (5\hat{i} + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = 7\ J$

9. (a) $v = \frac{dx}{dt} = 3 - 8t + 3t^2$
$\therefore\ v_0 = 3\ m/s$ and $v_4 = 19\ m/s$
$W = \frac{1}{2}m(v_4^2 - v_0^2)$ (According to work energy theorem)
$= \frac{1}{2} \times 0.03 \times (19^2 - 3^2) = 5.28\ J$

10. (d) As the body moves in the direction of force therefore work done by gravitational force will be positive.
$W = Fs = mgh = 10 \times 9.8 \times 10 = 980\ J$

11. (d)

12. (b) $W = mg\sin\theta \times s$
$= 2 \times 10^3 \times \sin 15° \times 10$
$= 5.17\ kJ$



13. (d) $W = \vec{F}.\vec{s} = (5\hat{i} + 6\hat{j} - 4\hat{k}).(6\hat{i} + 5\hat{k}) = 30 - 20 = 10$ *units*

14. (b) $W = Fs = F \times \frac{1}{2}at^2$ $\left[\text{from } s = ut + \frac{1}{2}at^2\right]$
$\Rightarrow W = F\left[\frac{1}{2}\left(\frac{F}{m}\right)t^2\right] = \frac{F^2t^2}{2m} = \frac{25 \times (1)^2}{2 \times 15} = \frac{25}{30} = \frac{5}{6}\ J$

15. (b) Work done on the body = K.E. gained by the body
$Fs\cos\theta = 1 \Rightarrow F\cos\theta = \frac{1}{s} = \frac{1}{0.4} = 2.5\ N$

16. (b) Work done $= mgh = 10 \times 9.8 \times 1 = 98\ J$

17. (b)

18. (d) $s = \frac{t^2}{4}\ \therefore\ ds = \frac{t}{2}dt$
$F = ma = \frac{md^2s}{dt^2} = \frac{6d^2}{dt^2}\left[\frac{t^2}{4}\right] = 3\ N$
Now
$W = \int_0^2 F\,ds = \int_0^2 3\frac{t}{2}dt = \frac{3}{2}\left[\frac{t^2}{2}\right]_0^2 = \frac{3}{4}\left[(2)^2 - (0)^2\right] = 3\ J$

19. (d) Net force on body $= \sqrt{4^2 + 3^2} = 5\ N$
$\therefore\ a = F/m = 5/10 = 1/2\ m/s^2$
Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = 125\ J$

20. (d) $s = \frac{u^2}{2\mu g} = \frac{10 \times 10}{2 \times 0.5 \times 10} = 10\ m$

21. (d) $W = \vec{F}.\vec{s} = (3\hat{i} + 4\hat{j}).(3\hat{i} + 4\hat{j}) = 9 + 16 = 25\ J$

22. (d) Total mass = (50 + 20) = 70 *kg*
Total height = 20 × 0.25 = 5*m*
$\therefore$ Work done = *mgh* = 70 × 9.8 × 5 = 3430 *J*

23. (d) $W = \vec{F}.\vec{s} = (6\hat{i} + 2\hat{j} - 3\hat{k}).(2\hat{i} - 3\hat{j} + x\hat{k}) = 0$
$12 - 6 - 3x = 0 \Rightarrow x = 2$

24. (a) $W = \vec{F}.(\vec{r_2} - \vec{r_1}) = (4\hat{i} + \hat{j} + 3\hat{k})(11\hat{i} + 11\hat{j} + 15\hat{k})$
$W = 44 + 11 + 45 = 100$ *Joule*

25. (c) $W = (3\hat{i} + c\hat{j} + 2\hat{k}).(-4\hat{i} + 2\hat{j} + 3\hat{k}) = 6\ J$
$W = -12 + 2c + 6 = 6 \Rightarrow c = 6$

26. (a) Both part will have numerically equal momentum and lighter part will have more velocity.

27. (d) *Watt* and *Horsepower* are the unit of power

28. (b) Work = Force × Displacement
If force and displacement both are doubled then work would be four times.

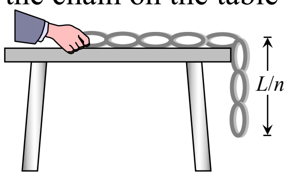
29. (d) $W = FS\cos\theta = 10 \times 4 \times \cos 60^\circ = 20\ Joule$

30. (a) $W = \vec{F}.\vec{s} = (5\hat{i} + 4\hat{j}).(6\hat{i} - 5\hat{j} + 3\hat{k}) = 30 - 20 = 10\ J$

31. (b) Fraction of length of the chain hanging from the table

$= \frac{1}{n} = \frac{60 cm}{200 cm} = \frac{3}{10} \Rightarrow n = \frac{10}{3}$

Work done in pulling the chain on the table



$W = \frac{mgL}{2n^2}$

$= \frac{4 \times 10 \times 2}{2 \times (10/3)^2} = 3.6\ J$

32. (c) When a force of constant magnitude which is perpendicular to the velocity of particle acts on a particle, work done is zero and hence change in kinetic energy is zero.

33. (a) The ball rebounds with the same speed. So change in it's Kinetic energy will be zero *i.e.* work done by the ball on the wall is zero.

34. (b) $W = \vec{F}.\vec{r} = (5\hat{i} + 3\hat{j} + 2\hat{k}).(2\hat{i} - \hat{j}) = 10 - 3 = 7\ J$

35. (a) K.E. acquired by the body = work done on the body

$K.E. = \frac{1}{2}mv^2 = Fs$ *i.e.* it does not depend upon the mass of the body although velocity depends upon the mass

$v^2 \propto \frac{1}{m}$ [If F and s are constant]

36. (d) $W = \vec{F}.\vec{s} = (4\hat{i} + 5\hat{j} + 0\hat{k}).(3\hat{i} + 0\hat{j} + 6\hat{k}) = 4 \times 3$ *units*

37. (a) As surface is smooth so work done against friction is zero. Also the displacement and force of gravity are perpendicular so work done against gravity is zero.

38. (c) Opposing force in vertical pulling = *mg*

But opposing force on an inclined plane is $mg \sin\theta$, which is less than *mg*.

39. (c) Velocity of fall is independent of the mass of the falling body.

40. (a) Work done $= \vec{F}.\vec{s}$

$= (6\hat{i} + 2\hat{j}) \cdot (3\hat{i} - \hat{j}) = 6 \times 3 - 2 \times 1 = 18 - 2 = 16\ J$

41. (c) When the ball is released from the top of tower then ratio of distances covered by the ball in first, second and third second

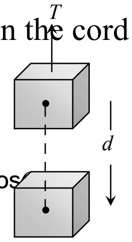
$h_I : h_{II} : h_{III} = 1 : 3 : 5 :$ [because $h_n \propto (2n - 1)$]

$\therefore$ Ratio of work done $mgh_I : mgh_{II} : mgh_{III} =$ 1:3:5

Work Done by Variable Force

1. (b) $W = \int_0^{x_1} F.dx = \int_0^{x_1} Cx\, dx = C\left[\frac{x^2}{2}\right]_0^{x_1} = \frac{1}{2}Cx_1^2$

2. (c) When the block moves vertically downward with acceleration $\frac{g}{4}$ then tension in the cord



$T = M\left(g - \frac{g}{4}\right) = \frac{3}{4}Mg$

Work done by the cord $= \vec{F}.\vec{s} = Fs\cos\theta$

$= Td\cos(180^\circ) = -\left(\frac{3Mg}{4}\right) \times d = -3Mg\frac{d}{4}$

3. (c) $W = \frac{F^2}{2k}$

If both springs are stretched by same force then $W \propto \frac{1}{k}$

As $k_1 > k_2$ therefore $W_1 < W_2$

i.e. more work is done in case of second spring.

4. (a) ΔP.E. $= \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 10[(0.25)^2 - (0.20)^2]$

$= 5 \times 0.45 \times 0.05 = 0.1\ J$

5. (a) $\frac{1}{2}kS^2 = 10\ J$ (given in the problem)

$\frac{1}{2}k[(2S)^2 - (S)^2] = 3 \times \frac{1}{2}kS^2 = 3 \times 10 = 30\ J$

6. (c) $U = \frac{F^2}{2k} \Rightarrow \frac{U_1}{U_2} = \frac{k_2}{k_1}$ (if force are same)

$\therefore \frac{U_1}{U_2} = \frac{3000}{1500} = \frac{2}{1}$

7. (d) Here $k = \frac{F}{x} = \frac{10}{1 \times 10^{-3}} = 10^4\ N/m$

$W = \frac{1}{2}kx^2 = \frac{1}{2} \times 10^4 \times (40 \times 10^{-3})^2 = 8\ J$

8. (d) $W = \int_0^5 F dx = \int_0^5 (7 - 2x + 3x^2)\, dx = [7x - x^2 + x^3]_0^5$

$= 35 - 25 + 125 = 135\ J$

9. (d) $S = \frac{t^3}{3}\ \therefore\ dS = t^2 dt$

$a=\frac{d^2S}{dt^2}=\frac{d^2}{dt^2}\left[\frac{t^3}{3}\right]=2t\ m/s^2$

Now work done by the force $W=\int_0^2 F.dS=\int_0^2 ma.dS$

$\int_0^2 3\times 2t\times t^2 dt=\int_0^2 6t^3 dt=\frac{3}{2}\left[t^4\right]_0^2=24\ J$

10. (b) $W=\frac{1}{2}kx^2$

If both wires are stretched through same distance then $W\propto k$. As $k_2=2k_1$ so $W_2=2W_1$

11. (b) $\frac{1}{2}mv^2=\frac{1}{2}kx^2\Rightarrow x=v\sqrt{\frac{m}{k}}=10\sqrt{\frac{0.1}{1000}}=0.1\ m$

12. (c) Force constant of a spring

$k=\frac{F}{x}=\frac{mg}{x}=\frac{1\times 10}{2\times 10^{-2}}\Rightarrow k=500\ N/m$

Increment in the length = 60 – 50 = 10 *cm*

$U=\frac{1}{2}kx^2=\frac{1}{2}500(10\times 10^{-2})^2=2.5\ J$

13. (b) $W=\frac{1}{2}k(x_2^2-x_1^2)=\frac{1}{2}\times 800\times(15^2-5^2)\times 10^{-4}=8\ J$

14. (c) $100=\frac{1}{2}kx^2$ (given)

$W=\frac{1}{2}k(x_2^2-x_1^2)=\frac{1}{2}k[(2x)^2-x^2]$

$=3\times\left(\frac{1}{2}kx^2\right)=3\times 100=300\ J$

15. (d) $U=\frac{1}{2}kx^2$ if x becomes 5 times then energy will become 25 times *i.e.* $4\times 25=100\ J$

16. (c) $W=\frac{1}{2}k(x_2^2-x_1^2)=\frac{1}{2}\times 5\times 10^3(10^2-5^2)\times 10^{-4}$

$=18.75\ J$

17. (a) The kinetic energy of mass is converted into potential energy of a spring

$\frac{1}{2}mv^2=\frac{1}{2}kx^2\Rightarrow x=\sqrt{\frac{mv^2}{k}}=\sqrt{\frac{0.5\times(1.5)^2}{50}}=0.15\ m$

18. (a) This condition is applicable for simple harmonic motion. As particle moves from mean position to extreme position its potential energy increases according to expression $U=\frac{1}{2}kx^2$ and accordingly kinetic energy decreases.

19. (c) Potential energy $U=\frac{1}{2}kx^2$

$\therefore U\propto x^2$ [if k = constant]

If elongation made 4 times then potential energy will become 16 times.

20. (b)

21. (d) $U\propto x^2 \Rightarrow \frac{U_2}{U_1}=\left(\frac{x_2}{x_1}\right)^2=\left(\frac{0.1}{0.02}\right)^2=25\ \therefore$

$U_2=25U$

22. (a) If x is the extension produced in spring.

$F=kx\Rightarrow x=\frac{F}{k}=\frac{mg}{k}=\frac{20\times 9.8}{4000}=4.9\ cm$

23. (a) $U=\frac{F^2}{2k}=\frac{T^2}{2k}$

24. (b) $U=A-Bx^2\Rightarrow F=-\frac{dU}{dx}=2Bx\Rightarrow F\propto x$

25. (d) Condition for stable equilibrium $F=-\frac{dU}{dx}=0$

$\Rightarrow -\frac{d}{dx}\left[\frac{a}{x^{12}}-\frac{b}{x^6}\right]=0\ \Rightarrow$

$-12ax^{-13}+6bx^{-7}=0$

$\Rightarrow \frac{12a}{x^{13}}=\frac{6b}{x^7}\Rightarrow\frac{2a}{b}=x^6\Rightarrow x=\sqrt[6]{\frac{2a}{b}}$

26. (d)Friction is a non-conservative force.

Conservation of Energy and Momentum

1. (c) $P=\sqrt{2mE}\ \therefore P\propto\sqrt{m}$ (if E = const.) $\therefore \frac{P_1}{P_2}=\sqrt{\frac{m_1}{m_2}}$

2. (c) Work in raising a box

= (weight of the box) × (height by which it is raised)

3. (a) $E=\frac{P^2}{2m}$ if P = constant then $E\propto\frac{1}{m}$

4. (a) Body at rest may possess potential energy.

5. (b) Due to theory of relativity.

6. (d) $E=\frac{P^2}{2m}\ \therefore\ E\propto P^2$

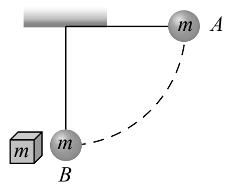
i.e. if P is increased n times then E will increase n^2 times.

7. (c)

8. (c) P.E. of bob at point A = mgl

This amount of energy will be converted into kinetic energy

$\therefore$ K.E. of bob at point B = mgl



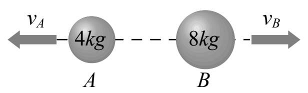
and as the collision between bob and block (of same mass) is elastic so after collision bob will come to rest and total Kinetic energy will be transferred to block. So kinetic energy of block = mgl

9. (b) According to conservation of momentum

Momentum of tank = Momentum of shell

$125000 \times v_{\text{tank}} = 25 \times 1000 \Rightarrow v_{\text{tank}} = 0.2$ *ft/sec.*

10. (d) As the initial momentum of bomb was zero, therefore after explosion two parts should possess numerically equal momentum



i.e. $m_A v_A = m_B v_B \Rightarrow 4 \times v_A = 8 \times 6 \Rightarrow v_A = 12\ m/s$

$\therefore$ Kinetic energy of other mass A, $= \frac{1}{2} m_A v_A^2$

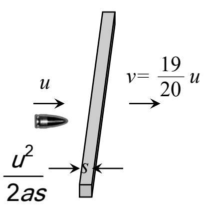
$= \frac{1}{2} \times 4 \times (12)^2 = 288\ J.$

11. (c) Let the thickness of one plank is s

if bullet enters with velocity u then it leaves with velocity

$v = \left(u - \frac{u}{20}\right) = \frac{19}{20} u$

from $v^2 = u^2 - 2as$



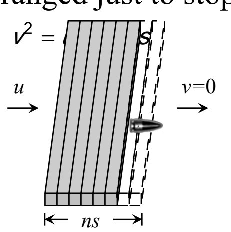
$\Rightarrow \left(\frac{19}{20} u\right)^2 = u^2 - 2as \Rightarrow \frac{400}{39} = \frac{u^2}{2as}$

Now if the n planks are arranged just to stop the bullet then again from $v^2 =$

$0 = u^2 - 2ans$

$\Rightarrow n = \frac{u^2}{2as} = \frac{400}{39}$

$\Rightarrow n = 10.25$



As the planks are more than 10 so we can consider $n = 11$

12. (b) Let h is that height at which the kinetic energy of the body becomes half its original value *i.e.* half of its kinetic energy will convert into potential energy

$\therefore mgh = \frac{490}{2} \Rightarrow 2 \times 9.8 \times h = \frac{490}{2} \Rightarrow h = 12.5 m.$

13. (c) $P = \sqrt{2mE}$. If E are same then $P \propto \sqrt{m}$

$\Rightarrow \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

14. (a) Let initial kinetic energy, $E_1 = E$

Final kinetic energy, $E_2 = E + 300\%$ of E = $4E$

As $P \propto \sqrt{E} \Rightarrow \frac{P_2}{P_1} = \sqrt{\frac{E_2}{E_1}} = \sqrt{\frac{4E}{E}} = 2 \Rightarrow P_2 = 2P_1$

$\Rightarrow P_2 = P_1 + 100\%$ of P_1

i.e. Momentum will increase by 100%.

15. (b) $P = \sqrt{2mE}$ if E are equal then $P \propto \sqrt{m}$

i.e. heavier body will possess greater momentum.

16. (c) Let $P_1 = P$, $P_2 = P_1 + 50\%$ of $P_1 = P_1 + \frac{P_1}{2} = \frac{3P_1}{2}$

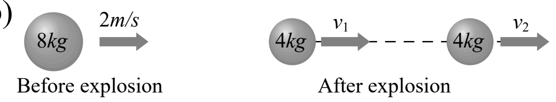
$E \propto P^2 \Rightarrow \frac{E_2}{E_1} = \left(\frac{P_2}{P_1}\right)^2 = \left(\frac{3P_1/2}{P_1}\right)^2 = \frac{9}{4}$

$\Rightarrow E_2 = 2.25E = E_1 + 1.25E_1$

$\therefore E_2 = E_1 + 125\%$ of E_1

i.e. kinetic energy will increase by 125%.

17. (b)



As the body splits into two equal parts due to internal explosion therefore momentum of system remains conserved *i.e.*

$8 \times 2 = 4v_1 + 4v_2 \Rightarrow v_1 + v_2 = 4$ …(i)

By the law of conservation of energy

Initial kinetic energy + Energy released due to explosion

= Final kinetic energy of the system

$\Rightarrow \frac{1}{2} \times 8 \times (2)^2 + 16 = \frac{1}{2} 4 v_1^2 + \frac{1}{2} 4 v_2^2$

$\Rightarrow v_1^2 + v_2^2 = 16$ …(ii)

By solving eq. (i) and (ii) we get $v_1 = 4$ and $v_2 = 0$

i.e. one part comes to rest and other moves in the same direction as that of original body.

18. (d) $P=\sqrt{2mE} \therefore P \propto \sqrt{E}$

i.e. if kinetic energy of a particle is doubled the its momentum will becomes $\sqrt{2}$ times.

19. (b) Potential energy = mgh

Potential energy is maximum when h is maximum

20. (c) If particle is projected vertically upward with velocity of $2m/s$ then it returns with the same velocity.

So its kinetic energy $=\frac{1}{2}mv^2=\frac{1}{2}\times 2\times (2)^2=4\ J$

21. (b)

22. (c) $E=\frac{P^2}{2m}$ if bodies possess equal linear momenta then

$E\propto \frac{1}{m}$ *i.e.* $\frac{E_1}{E_2}=\frac{m_2}{m_1}$

23. (d) $s\propto u^2$ *i.e.* if speed becomes double then stopping distance will become four times *i.e.* $8\times 4=32m$

24. (c) $s\propto u^2$ *i.e.* if speed becomes three times then distance needed for stopping will be nine times.

25. (a) $P=\sqrt{2mE} \therefore P\propto \sqrt{E}$

Percentage increase in $P=\frac{1}{2}$(percentage increase in E)

$=\frac{1}{2}(0.1\%)=0.05\%$

26. (c) Kinetic energy = $\frac{1}{2}mv^2$ $\therefore$ K.E. $\propto v^2$

If velocity is doubled then kinetic energy will become four times.

27. (d) $P=\sqrt{2mE} \therefore \frac{P_1}{P_2}=\sqrt{\frac{m_1}{m_2}}$ (if E = constant)

$\therefore \frac{P_1}{P_2}=\sqrt{\frac{3}{1}}$

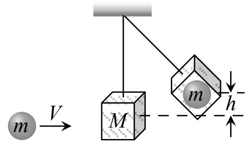
28. (d) In compression or extension of a spring work is done against restoring force.

In moving a body against gravity work is done against gravitational force of attraction.

It means in all three cases potential energy of the system increases.

But when the bubble rises in the direction of upthrust force then system works so the potential energy of the system decreases.

29. (a)



By the conservation of linear momentum

Initial momentum of sphere = Final momentum of system

$mV=(m+M)v_{sys.}$ ...(i)

If the system rises up to height h then by the conservation of energy

$\frac{1}{2}(m+M)v_{sys.}^2=(m+M)gh$ ...(ii)

$\Rightarrow v_{sys.}=\sqrt{2gh}$

Substituting this value in equation (i)

$V=\left(\frac{m+M}{m}\right)\sqrt{2gh}$

30. (b) $E=\frac{P^2}{2m}$. If momentum are same then $E\propto\frac{1}{m}$

$\therefore \frac{E_1}{E_2}=\frac{m_2}{m_1}=\frac{2m}{m}=\frac{2}{1}$

31. (d) $P=\sqrt{2mE}$. If kinetic energy are equal then $P\propto\sqrt{m}$

i.e., heavier body posses large momentum

As $M_1<M_2$ therefore $M_1V_1<M_2V_2$

32. (d) Condition for vertical looping $h=\frac{5}{2}r=5cm \therefore r=2$ c

33. (a) Max. K.E. of the system = Max. P.E. of the system

$\frac{1}{2}kx^2==\frac{1}{2}\times(16)\times(5\times 10^{-2})^2=2\times 10^{-2}\ J$

34. (d) $E=\frac{p^2}{2m}$ $\therefore$ $m\propto\frac{1}{E}$ (If momentum are constant)

$\frac{m_1}{m_2}=\frac{E_2}{E_1}=\frac{1}{4}$

35. (a) $P=\sqrt{2mE}$ $\therefore P\propto\sqrt{E}$ *i.e.* if kinetic energy becomes four time then new momentum will become twice.

36. (a) $E=\frac{P^2}{2m}$. If P = constant then $E\propto\frac{1}{m}$

i.e. kinetic energy of heavier body will be less. As the mass of gun is more than bullet therefore it possess less kinetic energy.

37. (b) Potential energy of water = kinetic energy at turbine

$mgh=\frac{1}{2}mv^2\Rightarrow$

$v=\sqrt{2gh}=\sqrt{2\times 9.8\times 19.6}=19.6\,m/s$

38. (c) $p=\sqrt{2mE}$ $\therefore$ $\frac{p_1}{p_2}=\sqrt{\frac{m_1}{m_2}\frac{E_1}{E_2}}=\sqrt{\frac{2}{1}\times\frac{8}{1}}=\frac{4}{1}$

39. (a) The bomb of mass $12kg$ divides into two masses

m_1 and m_2 then $m_1+m_2=12$ ...(i)

and $\frac{m_1}{m_2}=\frac{1}{3}$ ...(ii)

by solving we get $m_1=3kg$ and $m_2=9kg$

Kinetic energy of smaller part = $\frac{1}{2}m_1v_1^2=216J$

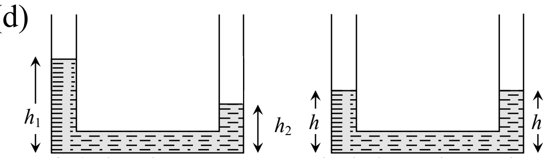
$\therefore$ $v_1^2=\frac{216\times 2}{3}\Rightarrow v_1=12m/s$

So its momentum = $m_1v_1=3\times 12=36\ kg\text{-}m/s$

As both parts possess same momentum therefore momentum of each part is $36\ kg\text{-}m/s$

40. (c) $P=\sqrt{2mE}$. If E are const. then $\frac{P_1}{P_2}=\sqrt{\frac{m_1}{m_2}}=\sqrt{\frac{4}{1}}=2$

41. (d)



If h is the common height when they are connected, by conservation of mass

$\rho A_1h_1+\rho A_2h_2=\rho h(A_1+A_2)$

$h=(h_1+h_2)/2$ [as $A_1=A_2=A$ given]

As $(h_1/2)$ and $(h_2/2)$ are heights of initial centre of gravity of liquid in two vessels., the initial potential energy of the system

$$U_i=(h_1A\rho)g\frac{h_1}{2}+(h_2A\rho)\frac{h_2}{2}=\rho gA\frac{(h_1^2+h_2^2)}{2} \quad \text{...(i)}$$

When vessels are connected the height of centre of gravity of liquid in each vessel will be $h/2$,

i.e. $(\frac{(h_1+h_2)}{4}$ [as $h=(h_1+h_2)/2$]

Final potential energy of the system

$$U_F=\left[\frac{(h_1+h_2)}{2}A\rho\right]g\left(\frac{h_1+h_2}{4}\right)$$

$$=A\rho g\left[\frac{(h_1+h_2)^2}{4}\right] \quad \text{...(ii)}$$

Work done by gravity

$$W=U_i-U_f=\frac{1}{4}\rho gA[2(h_1^2+h_2^2)-(h_1+h_2)^2]$$

$$=\frac{1}{4}\rho gA(h_1\sim h_2)^2$$

42. (c) $P=\sqrt{2mE}$. If m is constant then

$\frac{P_2}{P_1}=\sqrt{\frac{E_2}{E_1}}=\sqrt{\frac{1.22E}{E}}\Rightarrow\frac{P_2}{P_1}=\sqrt{1.22}=1.1$

$\Rightarrow P_2=1.1P_1\Rightarrow P_2=P_1+0.1P_1=P_1+10\%$ of P_1

So the momentum will increase by 10%

43. (b) $\Delta U=mgh=0.2\times 10\times 200=400\ J$

$\therefore$ Gain in K.E. = decrease in P.E. = 400 J.

44. (a) $E=\frac{P^2}{2m}$. If m is constant then $E\propto P^2$

$\Rightarrow\frac{E_2}{E_1}=\left(\frac{P_2}{P_1}\right)^2=\left(\frac{1.2P}{P}\right)^2=1.44$

$\Rightarrow E_2=1.44E_1=E_1+0.44E_1$

$E_2=E_1+44\%$ of E_1

i.e. the kinetic energy will increase by 44%

45. (a) $E=\frac{P^2}{2m}=\frac{(2)^2}{2\times 2}=1J$

46. (b) $\Delta U=mgh=20\times 9.8\times 0.5=98\ J$

47. (b) $E=\frac{P^2}{2m}=\frac{(10)^2}{2\times 1}=50\ J$

48. (b) Because 50% loss in kinetic energy will affect its potential energy and due to this ball will attain only half of the initial height.

49. (d) If there is no air drag then maximum height

$H = \frac{u^2}{2g} = \frac{14 \times 14}{2 \times 9.8} = 10\,m$

But due to air drag ball reaches up to height $8m$ only. So loss in energy

$= mg(10-8) = 0.5 \times 9.8 \times 2 = 9.8\,J$

50. (a) $1\,kcal = 10^3\,Calorie = 4200\,J = \frac{4200}{3.6 \times 10^6}\,kWh$

$\therefore\ 700\,kcal = \frac{700 \times 4200}{3.6 \times 10^6}\,kWh = 0.81\,kWh$

51. (b) $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1} = \sqrt{1.96} = 1.4\,m/s$

52. (a)

53. (c) Let m = mass of boy, M = mass of man

v = velocity of boy, V = velocity of man

$\frac{1}{2}MV^2 = \frac{1}{2}\left[\frac{1}{2}mv^2\right]$ .....(i)

$\frac{1}{2}M(V+1)^2 = 1\left[\frac{1}{2}mv^2\right]$ .....(ii)

Putting $m = \frac{M}{2}$ and solving $V = \frac{1}{\sqrt{2}-1}$

54. (d) $P = \sqrt{2mE} \Rightarrow \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

55. (d) $E = \frac{P^2}{2m} \Rightarrow E_2 = E_1\left(\frac{P_2}{P_1}\right)^2 = E_1\left(\frac{2P}{P}\right)^2$

$\Rightarrow E_2 = 4E = E + 3E = E + 300\%$ of E

56. (a) For first condition

Initial velocity = u, Final velocity = $u/2$, s = 3 cm

From $v^2 = u^2 - 2as \Rightarrow \left(\frac{u}{2}\right)^2 = u^2 - 2as \Rightarrow$

$a = \frac{3u^2}{8s}$

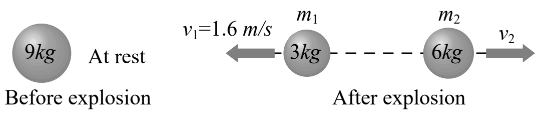
Second condition

Initial velocity = $u/2$, Final velocity = 0

From $v^2 = u^2 - 2ax \Rightarrow 0 = \frac{u^2}{4} - 2ax$

$\therefore\ x = \frac{u^2}{4 \times 2a} = \frac{u^2 \times 8s}{4 \times 2 \times 3u^2} = s/3 = 1\,cm$

57. (c)



As the bomb initially was at rest therefore

Initial momentum of bomb = 0

Final momentum of system = $m_1v_1 + m_2v_2$

As there is no external force

$\therefore\ m_1v_1 + m_2v_2 = 0 \Rightarrow 3 \times 1.6 + 6 \times v_2 = 0$

velocity of 6 kg mass $v_2 = 0.8\,m/s$ (numerically)

Its kinetic energy $= \frac{1}{2}m_2v_2^2$

$= \frac{1}{2} \times 6 \times (0.8)^2 = 1.92\,J$

58. (b) $P = \sqrt{2mE}$. $P \propto \sqrt{m}$ $\therefore\ \frac{P_1}{P_2} = \sqrt{\frac{1}{16}} = \frac{1}{4}$

59. (c) Potential energy of a body = 75% of 12 J

$mgh = 9\,J \Rightarrow h = \frac{9}{1 \times 10} = 0.9m$

Now when this mass allow to fall then it acquire velocity

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.9} = \sqrt{18}\,m/s$.

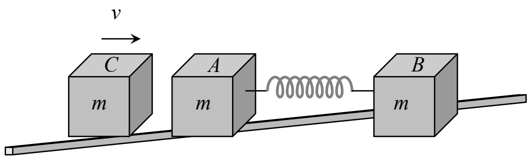
60. (a)

61. (b) Kinetic energy $E = \frac{P^2}{2m} = \frac{(Ft)^2}{2m} = \frac{F^2t^2}{2m}$ [As $P = F\,t$]

62. (b) Potential energy of spring = $\frac{1}{2}Kx^2$

$\therefore\ PE \propto x^2 \Rightarrow PE \propto a^2$

63. (a)



Initial momentum of the system (block C) = mv

After striking with A, the block C comes to rest and now both block A and B moves with velocity V, when compression in spring is maximum.

By the law of conservation of linear momentum

$mv = (m+m)V \Rightarrow V = \frac{v}{2}$

By the law of conservation of energy

K.E. of block C = K.E. of system + P.E. of system

$\frac{1}{2}mv^2 = \frac{1}{2}(2m)V^2 + \frac{1}{2}kx^2$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow x = v\sqrt{\frac{m}{2k}}$$

64. (c) $P = \sqrt{2mE}$ $\therefore$ $P \propto \sqrt{m} \Rightarrow \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m}{4m}} = \frac{1}{2}$

65. (d) $E = \frac{P^2}{2m} \Rightarrow E \propto \frac{1}{m} \Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1}$

66. (b) $E = \frac{P^2}{2m} = \frac{4}{2\times 3} = \frac{2}{3}J$

67. (d) Both fragment will possess the equal linear momentum

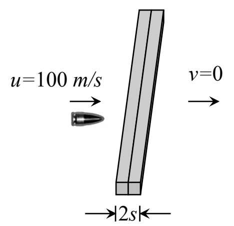
$m_1v_1 = m_2v_2 \Rightarrow 1\times 80 = 2\times v_2 \Rightarrow v_2 = 40\ m/s$

$\therefore$ Total energy of system $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$= \frac{1}{2}\times 1\times(80)^2 + \frac{1}{2}\times 2\times(40)^2$

$= 4800\ J = 4.8\ kJ$

68. (b)



Let the thickness of each plank is s. If the initial speed of bullet is 100 m/s then it stops by covering a distance $2s$

By applying $v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as$

$s = \frac{u^2}{2a}$ $s \propto u^2$ [If retardation is constant]

If the speed of the bullet is double then bullet will cover four times distance before coming to rest

i.e. $s_2 = 4(s_1) = 4(2s) \Rightarrow s_2 = 8s$

So number of planks required = 8

69. (a) $E = \frac{P^2}{2m}$ if P = constant then $E \propto \frac{1}{m}$

According to problem $m_1 > m_2$ $\therefore$ $E_1 < E_2$

70. (c) Kinetic energy $= \frac{1}{2}mv^2$

As both balls are falling through same height therefore they possess same velocity.

but $KE \propto m$ (If v = constant)

$\therefore \frac{(KE)_1}{(KE)_2} = \frac{m_1}{m_2} = \frac{2}{4} = \frac{1}{2}$

71. (b) $E = \frac{P^2}{2m}$ $\therefore$ $E \propto \frac{1}{m}$ (If P = constant)

i.e. the lightest particle will possess maximum kinetic energy and in the given option mass of electron is minimum.

72. (a) $P = E \Rightarrow mv = \frac{1}{2}mv^2 \Rightarrow v = 2\ m/s$

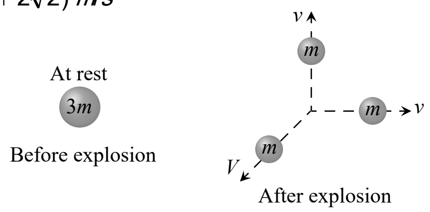
73. (c) Initial kinetic energy $E = \frac{1}{2}mv^2$ ...(i)

Final kinetic energy $2E = \frac{1}{2}m(v+2)^2$ ...(ii)

by solving equation (i) and (ii) we get

$v = (2 + 2\sqrt{2})\ m/s$

74. (c)



Initial momentum of $3m$ mass = 0 ...(i)

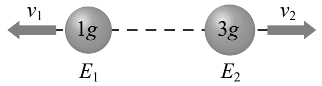
Due to explosion this mass splits into three fragments of equal masses.

Final momentum of system = $m\vec{V} + mv\hat{i} + mv\hat{j}$ ...(ii)

By the law of conservation of linear momentum

$m\vec{V} + mv\hat{i} + mv\hat{j} = 0 \Rightarrow \vec{V} = -v(\hat{i} + \hat{j})$

75. (c)



As the momentum of both fragments are equal therefore $\frac{E_1}{E_2} = \frac{m_2}{m_1} = \frac{3}{1}$ *i.e.* $E_1 = 3E_2$

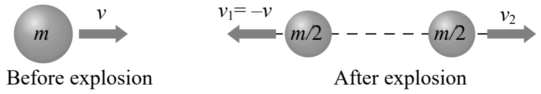
According to problem $E_1 + E_2 = 6.4\times 10^4\ J$ ...(i)

By solving equation (i) and (ii) we get

$E_1 = 4.8\times 10^4\ J$ and $E_2 = 1.6\times 10^4\ J$

76. (a)

77. (b)



Let the initial mass of body = m

Initial linear momentum = mv …(i)

When it breaks into equal masses then one of the fragment retrace back with same velocity

$\therefore$ Final linear momentum = $\frac{m}{2}(-v)+\frac{m}{2}(v_2)$ …(ii)

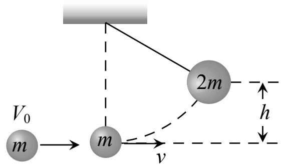
By the conservation of linear momentum

$\Rightarrow mv = \frac{-mv}{2}+\frac{mv_2}{2} \Rightarrow v_2 = 3v$

i.e. other fragment moves with velocity 3v in forward direction

78. (a)

79. (a)



Initial momentum of particle = mV_0

Final momentum of system (particle + pendulum) = 2mv

By the law of conservation of momentum

$\Rightarrow mV_0 = 2mv \Rightarrow$ Initial velocity of system v = $\frac{V_0}{2}$

$\therefore$ Initial K.E. of the system = $\frac{1}{2}(2m)v^2 = \frac{1}{2}(2m)\left(\frac{V_0}{2}\right)^2$

If the system rises up to height h then P.E. = $2mgh$

By the law of conservation of energy

$\frac{1}{2}(2m)\left(\frac{V_0}{2}\right)^2 = 2mgh \Rightarrow h = \frac{V_0^2}{8g}$

80. (d) $\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

81. (d) Change in momentum = Force × time

$P_2 - P_1 = F \times t = 0.2 \times 10 = 2$

$\Rightarrow P_2 = 2 + P_1 = 2 + 10 = 12 \, kg\text{-}m/s$

Increase in K.E. = $\frac{1}{2m}(P_2^2 - P_1^2) = \frac{1}{2\times 5}\left[(12)^2 - (10)^2\right]$

$= \frac{44}{10} = 4.4 \, J$

82. (b) $E \propto P^2$ (if m = constant)

Percentage increase in E = 2(Percentage increase in P)

= 2 × 0.01%=0.02%

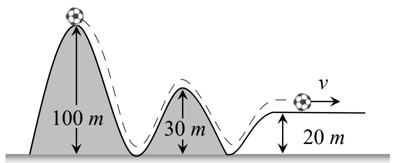
83. (c) 1 *amu* $= 1.66 \times 10^{-27} \, kg$

$E = mc^2 = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 1.5 \times 10^{-10} \, J$

84. (b) Change in gravitational potential energy = Elastic potential energy stored in compressed spring

$\Rightarrow mg(h + x) = \frac{1}{2}kx^2$

85. (c)



Ball starts from the top of a hill which is 100 m high and finally rolls down to a horizontal base which is 20 m above the ground so from the conservation of energy

$mg(h_1 - h_2) = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 10 \times (100 - 20)}$

$= \sqrt{1600} = 40 \, m/s.$

86. (c) When block of mass M collides with the spring its kinetic energy gets converted into elastic potential energy of the spring.

From the law of conservation of energy

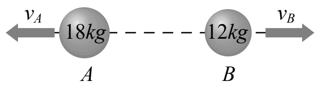
$\frac{1}{2}Mv^2 = \frac{1}{2}KL^2 \quad \therefore \quad v = \sqrt{\frac{K}{M}}L$

Where v is the velocity of block by which it collides with spring. So, its maximum momentum

$P = Mv = M\sqrt{\frac{K}{M}}\,L = \sqrt{MK}\,L$

After collision the block will rebound with same linear momentum.

87. (b)



According to law of conservation of linear momentum

$m_A v_A = m_B v_B = 18 \times 6 = 12 \times v_B \Rightarrow v_B = 9\ m/s$

K.E. of mass 12 kg, $E_B = \frac{1}{2} m_B v_B^2$

$= \frac{1}{2} \times 12 \times (9)^2 = 486\ J$

88. (c) Force = Rate of change of momentum

Initial momentum $\vec{P}_1 = mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}$

Final momentum $\vec{P}_2 = -mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}$

$\therefore\ \vec{F} = \frac{\Delta\vec{P}}{\Delta t} = \frac{-2mv\sin\theta}{2 \times 10^{-3}}$

Substituting $m = 0.1\ kg$, $v = 5\ m/s$, $\theta = 60°$

Force on the ball $\vec{F} = -250\sqrt{3}\ N$

Negative sign indicates direction of the force

Power

1. (a)

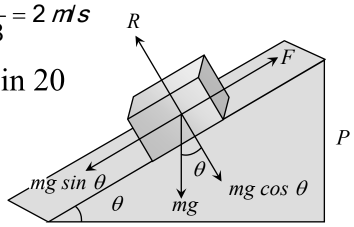
2. (d) $P = \vec{F}.\vec{v} = ma \times at = ma^2 t$ [as $u = 0$]

$= m\left(\frac{v_1}{t_1}\right)^2 t = \frac{mv_1^2 t}{t_1^2}$ $[\text{As } a = v_1 / t_1]$

3. (d) $v = 7.2\frac{km}{h} = 7.2 \times \frac{5}{18} = 2\ m/s$

Slope is given 1 in 20

$\therefore\ \sin\theta = \frac{1}{20}$



When man and cycle moves up then component of weight opposes it motion *i.e.* $F = mg\sin\theta$

So power of the man $P = F \times v = mg\sin\theta \times v$

$= 100 \times 9.8 \times \left(\frac{1}{20}\right) \times 2 = 98\ Watt$

4. (b) If a motor of 12 HP works for 10 days at the rate of 8 hr/day then energy consumption = power × time

$= 12 \times 746 \frac{J}{\sec} \times (80 \times 60 \times 60)\ \sec$

$= 12 \times 746 \times 80 \times 60 \times 60\ J = 2.5 \times 10^9\ J$

Rate of energy = $50 \frac{paisa}{kWh}$

i.e. $3.6 \times 10^6\ J$ energy cost 0.5 Rs

So 2.5 × 10^9 J energy cost = $\frac{2.5 \times 10^9}{2 \times 3.6 \times 10^6} = 358\ Rs$

5. (c) $P = Fv = 500 \times 3 = 1500\ W = 1.5\ kW$

6. (a) $P = Fv = F \times \frac{s}{t} = 40 \times \frac{30}{60} = 20\ W$

7. (b) $P = Fv = 4500 \times 2 = 9000\ W = 9\ kW$

8. (d) $P = \frac{\text{Workdone}}{\text{Time}} = \frac{mgh}{t} = \frac{300 \times 9.8 \times 2}{3} = 1960\ W$

9. (d) $P = \frac{mgh}{t} \Rightarrow m = \frac{p \times t}{gh} = \frac{2 \times 10^3 \times 60}{10 \times 10} = 1200\ kg$

As volume = $\frac{\text{mass}}{\text{density}} \Rightarrow v = \frac{1200 kg}{10^3\ kg/m^3} = 1.2 m^3$

Volume = $1.2 m^3 = 1.2 \times 10^3\ litre = 1200\ litre$

10. (c) $P = \frac{mgh}{t} = 10 \times 10^3 \Rightarrow t = \frac{200 \times 40 \times 10}{10 \times 10^3} = 8 \sec$

11. (c) Force required to move with constant velocity

$\therefore$ Power = FV

Force is required to oppose the resistive force R and also to accelerate the body of mass with acceleration a.

$\therefore$ Power = $(R + ma)V$

12. (d) $P = \frac{mgh}{t} = \frac{100 \times 9.8 \times 50}{50} = 980\ J/s$

13. (a) $P = \left(\frac{m}{t}\right) gh = 100 \times 10 \times 100 = 10^5\ W = 100\ kW$

14. (a) $p = \frac{mgh}{t} = \frac{200 \times 10 \times 200}{10} = 40\ kW$

15. (c) Volume of water to raise = 22380 l = $22380 \times 10^{-3} m^3$

$P = \frac{mgh}{t} = \frac{V\rho gh}{t} \Rightarrow t = \frac{V\rho gh}{P}$

$t = \frac{22380 \times 10^{-3} \times 10^3 \times 10 \times 10}{10 \times 746} = 15\ min$

16. (c) Force produced by the engine $F = \frac{P}{v} = \frac{30 \times 10^3}{30} = 10^3 N$

Acceleration=
$\frac{\text{Forward force by engine} - \text{resistive force}}{\text{mass of car}}$

$= \frac{1000-750}{1250} = \frac{250}{1250} = \frac{1}{5} m/s^2$

17. (b) $\text{Power} = \frac{\text{Work done}}{\text{time}} = \frac{\frac{1}{2}m(v^2-u^2)}{t}$

$P = \frac{1}{2} \times \frac{2.05\times10^6 \times [(25)^2-(5^2)]}{5\times60}$

$P = 2.05\times10^6\, W = 2.05\, MW$

18. (a) As truck is moving on an incline plane therefore only component of weight $(mg\sin\theta)$ will oppose the upward motion

Power = force × velocity = $mg\sin\theta \times v$

$= 30000\times10\times\left(\frac{1}{100}\right)\times\frac{30\times5}{18} = 25\,kW$

19. (c) $P = \frac{mgh}{t} \Rightarrow \frac{P_1}{P_2} = \frac{m_1}{m_2}\times\frac{t_2}{t_1}$

$\therefore \frac{P_1}{P_2} = \frac{60}{50}\times\frac{11}{12} = \frac{11}{10}$

20. (c) Power of a pump $= \frac{1}{2}\rho A v^3$

To get twice amount of water from same pipe v has to be made twice. So power is to be made 8 times.

21. (a) $p = \frac{mgh}{t} = \frac{80\times9.8\times6}{10} W = \frac{470}{746} HP = 0.63\, HP$

22. (b) $\text{Power} = \frac{\text{Work done}}{\text{time}} = \frac{\text{Increase in K.E.}}{\text{time}}$

$P = \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2}\times10^3\times(15)^2}{5} = 22500\, W$

23. (a) Motor makes 600 revolution per minute

$\therefore n = 600\frac{\text{revolution}}{\text{minute}} = 10\frac{rev}{\text{sec}}$

$\therefore$ Time required for one revolution $= \frac{1}{10} sec$

Energy required for one revolution = power × time

$= \frac{1}{4}\times746\times\frac{1}{10} = \frac{746}{40} J$

But work done = 40% of input

$= 40\% \times \frac{746}{40} = \frac{40}{100}\times\frac{746}{40} = 7.46\, J$

24. (a) Work output of engine = mgh = $100\times10\times10 = 10^4\, J$

Efficiency $(\eta) = \frac{\text{output}}{\text{input}}$ $\therefore$ Input energy = $\frac{\text{outupt}}{\eta}$

$= \frac{10^4}{60}\times100 = \frac{10^5}{6} J$

$\therefore$ Power = $\frac{\text{input energy}}{\text{time}} = \frac{10^5/6}{5} = \frac{10^5}{30} = 3.3\, kW$

25. (a) $P = \frac{\vec{F}.\vec{s}}{t} = \frac{(2\hat{i}+3\hat{j}+4\hat{k}).(3\hat{i}+4\hat{j}+5\hat{k})}{4} = \frac{38}{4} = 9.5\, W$

26. (a) $P = \frac{W}{t} = \frac{mgh}{t} = \frac{200\times10\times50}{10} = 10\times10^3\, W$

27. (a) Power of gun $= \frac{\text{Total K.E. of fired bullet}}{\text{time}}$

$= \frac{n\times\frac{1}{2}mv^2}{t} = \frac{360}{60}\times\frac{1}{2}\times2\times10^{-2}\times(100)^2 = 600\, W$

28. (a) Energy supplied to liquid per second by the pump

(As $h \equiv$ constant) $\frac{1}{2}\frac{mv^2}{t} = \frac{1}{2}\frac{V\rho v^2}{t} = \frac{1}{2}A\times\left(\frac{l}{t}\right)\times\rho\times v^2$ $\left[\frac{l}{t} = v\right]$

$= \frac{1}{2}A\times v\times\rho\times v^2 = \frac{1}{2}A\rho v^3$

29. (a) $\text{Power} = \frac{\text{workdone}}{\text{time}} = \frac{\text{pressure}\times\text{change in volume}}{\text{time}}$

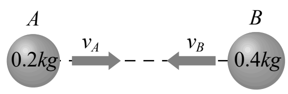
$= \frac{20000\times1\times10^{-6}}{1} = 2\times10^{-2} = 0.02\, W$

30. (c) $\text{Power} = \frac{W}{t}$. If W is constant then $P \propto \frac{1}{t}$

i.e. $\frac{P_1}{P_2} = \frac{t_2}{t_1} = \frac{20}{10} = \frac{2}{1}$

Elastic and Inelastic Collision

1. (a)
2. (a)
3. (c) According to law of conservation of linear momentum both pieces should possess equal momentum after explosion. As their masses are equal therefore they will possess equal speed in opposite direction.
4. (a)
5. (c)



Initial linear momentum of system = $m_A\vec{v}_A + m_B\vec{v}_B$

= 0.2 × 0.3 + 0.4 × v_B

Finally both balls come to rest

$\therefore$ final linear momentum = 0

By the law of conservation of linear momenum

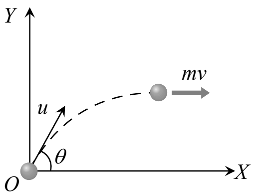
$$0.2 \times 0.3 + 0.4 \times v_B = 0$$

$\therefore$

$$v_B = -\frac{0.2 \times 0.3}{0.4} = -0.15\ m/s$$

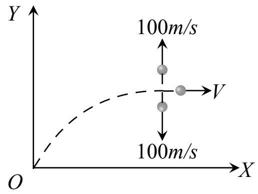
6. (c) For a collision between two identical perfectly elastic particles of equal mass, velocities after collision get interchanged.

7. (b)



Momentum of ball (mass m) before explosion at the highest point $= mv\hat{i} = mu\cos 60°\hat{i}$

$= m \times 200 \times \frac{1}{2}\hat{i} = 100\ m\hat{i}\ kgms^{-1}$



Let the velocity of third part after explosion is V

After explosion momentum of system $= \vec{P}_1 + \vec{P}_2 + \vec{P}_3$

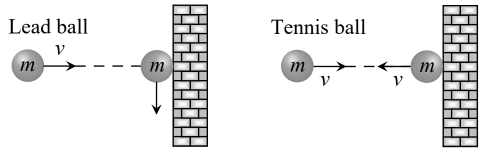
$= \frac{m}{3} \times 100\hat{j} - \frac{m}{3} \times 100\hat{j} + \frac{m}{3} \times V\hat{i}$

By comparing momentum of system before and after the explosion

$\frac{m}{3} \times 100\hat{j} - \frac{m}{3} \times 100\hat{j} + \frac{m}{3} V\hat{i} = 100m\hat{i} \Rightarrow V = 300\ m/s$

8. (c) Change in the momentum

= Final momentum – initial momentum



For lead ball $\Delta\vec{P}_{\text{lead}} = 0 - m\vec{v} = -m\vec{v}$

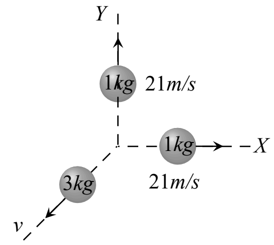
For tennis ball $\Delta\vec{P}_{\text{tennis}} = -m\vec{v} - m\vec{v} = -2m\vec{v}$

i.e. tennis ball suffers a greater change in momentum.

9. (c)

10. (d)

11. (d)



$P_x = m \times v_x = 1 \times 21 = 21\ kg\ m/s$

$P_y = m \times v_y = 1 \times 21 = 21\ kg\ m/s$

$\therefore$ Resultant $= \sqrt{P_x^2 + P_y^2} = 21\sqrt{2}\ kg\ m/s$

The momentum of heavier fragment should be numerically equal to resultant of $\vec{P}_x$ and $\vec{P}_y$.

$3 \times v = \sqrt{P_x^2 + P_y^2} = 21\sqrt{2}\ \therefore\ v = 7\sqrt{2} = 9.89\ m/s$

12. (b) We know that when heavier body strikes elastically with a lighter body then after collision lighter body will move with double velocity that of heavier body.

i.e. the ping pong ball move with speed of $2 \times 2 = 4\ m/s$

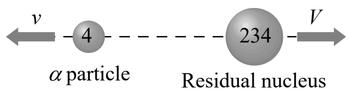
13. (d) Change in momentum $= m\vec{v}_2 - m\vec{v}_1$

$= -mv - mv = -2mv$

14. (c) $m_G = \frac{m_B v_B}{v_G}\ \frac{50 \times 10^{-3} \times 30}{1} = 1.5\ kg$

15. (d)

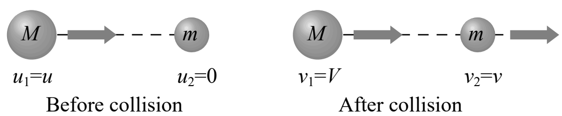
16. (a) Initially ^{238}U nucleus was at rest and after decay its part moves in opposite direction.



According to conservation of momentum

$4v + 234V = 238 \times 0 \Rightarrow V = -\frac{4v}{234}$

17. (c)

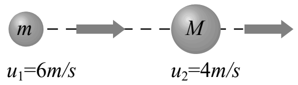


$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \frac{2m_1 u_1}{m_1 + m_2} = \frac{2Mu}{M+m} = \frac{2u}{1+\frac{m}{M}}$$

18. (c) Velocity exchange takes place when the masses of bodies are equal

19. (d) In perfectly elastic head on collision of equal masses velocities gets interchanged

20. (a)



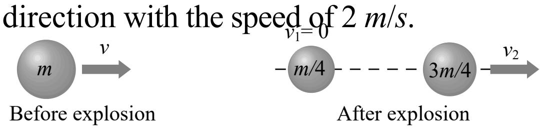
$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \frac{2m_2 u_2}{m_1 + m_2}$$

Substituting $m_1 = 0$, $v_1 = -u_1 + 2u_2$

$\Rightarrow v_1 = -6 + 2(4) = 2m/s$

i.e. the lighter particle will move in original direction with the speed of 2 *m/s*.

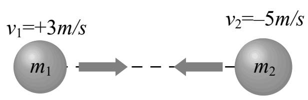
21. (d)



According to conservation of momentum

$$mv = \left(\frac{m}{4}\right) v_1 + \left(\frac{3m}{4}\right) v_2 \Rightarrow v_2 = \frac{4}{3} v$$

22. (d)

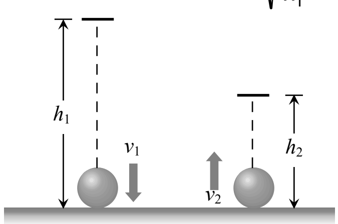


As $m_1 = m_2$ therefore after elastic collision velocities of masses get interchanged

i.e. velocity of mass $m_1 = -5\ m/s$

and velocity of mass $m_2 = +3\ m/s$

23. (b) If ball falls from height h_1 and bounces back up to height h_2 then $e = \sqrt{\frac{h_2}{h_1}}$



Similarly if the velocity of ball before and after collision are v_1 and v_2 respectively then $e = \frac{v_2}{v_1}$

So $\frac{v_2}{v_1} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{1.8}{5}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

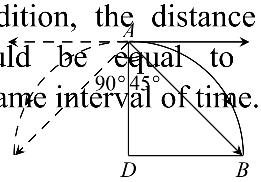
i.e. fractional loss in velocity $= 1 - \frac{v_2}{v_1} = 1 - \frac{3}{5} = \frac{2}{5}$

24. (a) $h_n = he^{2n} = 32\left(\frac{1}{2}\right)^4 = \frac{32}{16} = 2m$ (here $n = 2$, $e = 1/2$)

25. (c) As the body at rest explodes into two equal parts, they acquire equal velocities in opposite directions according to conservation of momentum.

When the angle between the radius vectors connecting the point of explosion to the fragments is 90°, each radius vector makes an angle 45° with the vertical.

To satisfy this condition, the distance of free fall *AD* should be equal to the horizontal range in same interval of time.



$AD = DB$

$AD = 0 + \frac{1}{2} \times 10t^2 = 5t^2$

$DB = ut = 10t$

$\therefore\ 5t^2 = 10t \Rightarrow t = 2\sec$

26. (a) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$ and

$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right) u_1 + \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_2$

on putting the values $v_1 = 6\ m/s$ and $v_2 = 12\ m/s$

27. (b) $F = \frac{dp}{dt} = m\frac{dv}{dt} = \frac{m \times 2v}{1/50} = \frac{2 \times 2 \times 100}{1/50} = 2 \times 10^4\ N$

28. (d) $h_n = he^{2n} = 1 \times e^{2\times 1} = 1 \times (0.6)^2 = 0.36m$

29. (d) $h_n = he^{2n}$, if $n = 2$ then $h_n = he^4$

30. (b) Impulse = change in momentum

$mv_2 - mv_1 = 0.1 \times 40 - 0.1 \times (-30)$

31. (b) In elastic head on collision velocities gets interchanged.

32. (a) Impulse = change in momentum = 2 *mv*

$= 2 \times 0.06 \times 4 = 0.48\ kg\ m/s$

33. (b) When ball falls vertically downward from height h_1 its velocity $\vec{v}_1 = \sqrt{2gh_1}$

and its velocity after collision $\vec{v}_2 = \sqrt{2gh_2}$

Change in momentum

$\Delta\vec{P} = m(\vec{v}_2 - \vec{v}_1) = m(\sqrt{2gh_1} + \sqrt{2gh_2})$

(because $\vec{v}_1$ and $\vec{v}_2$ are opposite in direction)

34. (a) Velocity of 50 *kg*. mass after 5 *sec* of projection $v = u - gt = 100 - 9.8 \times 5 = 51\ m/s$

At this instant momentum of body is in upward direction

$P_{\text{initial}} = 50 \times 51 = 2550\ kg-m/s$

After breaking 20 *kg* piece travels upwards with 150 *m/s* let the speed of 30 *kg* mass is *V*

$P_{\text{final}} = 20 \times 150 + 30 \times V$

By the law of conservation of momentum

$P_{\text{initial}} = P_{\text{final}}$

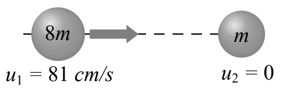
$\Rightarrow 2550 = 20 \times 150 + 30 \times V \Rightarrow V = -15\ m/s$

i.e. it moves in downward direction.

35. (c) Ratio in radius of steel balls = 1/2

So, ratio in their masses $= \frac{1}{8}$

Let $m_1 = 8m$ and $m_2 = m$



$v_2 = \frac{2m_1u_1}{m_1 + m_2} = \frac{2 \times 8m \times 81}{8m + m} = 144\ cm/s$

36. (a) After explosion *m* mass comes at rest and let Rest $(M - m)$ mass moves with velocity *v*.

By the law of conservation of momentum

$MV = (M - m)v \Rightarrow v = \frac{MV}{(M - m)}$

37. (c) As the ball bounces back with same speed so change in momentum = 2 *mv*

and we know that force = rate of change of momentum

i.e. force will act on the ball so there is an acceleration.

38. (d) According to conservation of momentum

$m_Bv_B + m_Gv_G = 0 \Rightarrow v_G = -\frac{m_Bv_B}{m_G}$

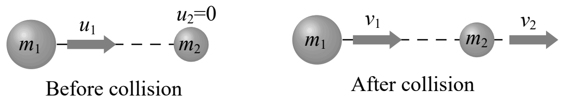
$v_G = \frac{-50 \times 10^{-3} \times 10^3}{5} = -10\ m/s$

39. (a) As 20% energy lost in collision therfore

$mgh_2 = 80\%$ of $mgh_1 \Rightarrow \frac{h_2}{h_1} = 0.8$

but $e = \sqrt{\frac{h_2}{h_1}} = \sqrt{0.8} = 0.89$

40. (b)



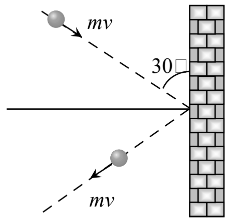
Before collision After collision

If target is at rest then final velocity of bodies are

$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$ …(i) and $v_2 = \frac{2m_1u_1}{m_1 + m_2}$ …(ii)

From (i) and (ii) $\frac{v_1}{v_2} = \frac{m_1 - m_2}{2m_1} = \frac{2}{5} \Rightarrow \frac{m_1}{m_2} = 5$

41. (b) F = Rate of change in momentum



$= \frac{2mv\sin\theta}{t}$

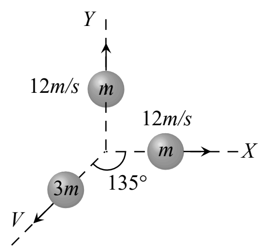
$= \frac{2 \times 10^{-1} \times 10\sin 30°}{0.1}$

[As $M \propto V \propto r$] $\therefore F = 10\,N$

42. (d) By the conservation of momentum

$40 \times 10 + (40) \times (-7) = 80 \times v \Rightarrow v = 1.5\ m/s$

43. (d)



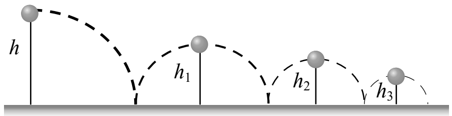
The momentum of third part will be equal and opposite to the resultant of momentum of rest two equal parts

let *V* is the velocity of third part.

By the conservation of linear momentum

$3m \times V = m \times 12\sqrt{2} \Rightarrow V = 4\sqrt{2}\ m/s$

44. (a)



Particle falls from height h then formula for height covered by it in nth rebound is given by

$$h_n = he^{2n}$$

where e = coefficient of restitution, n = No. of rebound

Total distance travelled by particle before rebounding has stopped

$$H = h + 2h_1 + 2h_2 + 2h_3 + 2h_n + \dots\dots$$

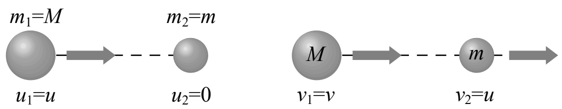
$$= h + 2he^2 + 2he^4 + 2he^6 + 2he^8 + \dots\dots$$

$$= h + 2h(e^2 + e^4 + e^6 + e^8 + \dots\dots)$$

$$= h + 2h\left[\frac{e^2}{1-e^2}\right] = h\left[1 + \frac{2e^2}{1-e^2}\right] = h\left(\frac{1+e^2}{1-e^2}\right)$$

45. (d) Due to the same mass of A and B as well as due to elastic collision velocities of spheres get interchanged after the collision.

46. (a)



Before collision After collision

From the formulae $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

We get $v = \left(\frac{M - m}{M + m}\right)u$

47. (a) Momentum conservation

$$5 \times 10 + 20 \times 0 = 5 \times 0 + 20 \times v \Rightarrow v = 2.5\, m/s$$

48. (d) Due to elastic collision of bodies having equal mass, their velocities get interchanged.

49. (c)

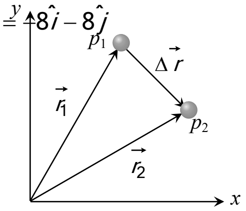
50. (b) $m_1 = 2\,kg$ and $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 = \frac{u_1}{4}$ (given)

By solving we get $m_2 = 1.2\,kg$

51. (c)

52. (d) It is clear from figure that the displacement vector $\Delta\vec{r}$ between particles p_1 and p_2 is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = 8\hat{i} - 8\hat{j}$$



$$|\Delta\vec{r}| = \sqrt{(-8)^2 + (-8)^2} = 8\sqrt{2} \qquad \dots\dots\text{(i)}$$

Now, as the particles are moving in same direction $(\because \vec{v}_1 \text{ and } \vec{v}_2 \text{ are} + ve)$, the relative velocity is given by

$$\vec{v}_{rel} = \vec{v}_2 - \vec{v}_1 = (\alpha - 4)\hat{i} + 4\hat{j}$$

$$\vec{v}_{rel} = \sqrt{(\alpha - 4)^2 + 16} \qquad \dots\dots\text{(ii)}$$

Now, we know $|\vec{v}_{rel}| = \frac{|\Delta\vec{r}|}{t}$

Substituting the values of $\vec{v}_{rel}$ and $|\Delta\vec{r}|$ from equation (i) and (ii) and $t = 2s$, then on solving we get $\alpha = 8$

53. (b) Fractional decrease in kinetic energy of neutron

$$= 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \qquad [\text{As } m_1 = 1 \text{ and } m_2 = 2]$$

$$= 1 - \left(\frac{1-2}{1+2}\right)^2 = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

54. (a)

55. (b) When target is very light and at rest then after head on elastic collision it moves with double speed of projectile *i.e.* the velocity of body of mass m will be $2v$.

56. (a) In head on elastic collision velocity get interchanged (if masses of particle are equal). *i.e.* the last ball will move with the velocity of first ball *i.e* 0.4 *m/s*

57. (a) By the principle of conservation of linear momentum,

$$Mv = mv_1 + mv_2 \Rightarrow Mv = 0 + (M - m)v_2 \Rightarrow$$

$$v_2 = \frac{Mv}{M - m}$$

58. (a) Since bodies exchange their velocities, hence their masses are equal so that $\frac{m_A}{m_B} = 1$

59. (d) $mgh =$ initial potential energy

$mgh' =$ final potential energy after rebound

As 40% energy lost during impact

$\therefore mgh' = 60\%$ of mgh

$$\Rightarrow h' = \frac{60}{100} \times h = \frac{60}{100} \times 10 = 6\, m$$

60. (c)

61. (a) Fractional loss $= \frac{\Delta U}{U} = \frac{mg(h - h')}{mgh} = \frac{2 - 1.5}{2} = \frac{1}{4}$

62. (c) $\frac{\Delta K}{K} = \left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\right] = \left[1 - \left(\frac{m - 2m}{m + 2m}\right)^2\right] = \frac{8}{9}$

$\Delta K = \frac{8}{9} K$ *i.e.* loss of kinetic energy of the colliding body is $\frac{8}{9}$ of its initial kinetic energy.

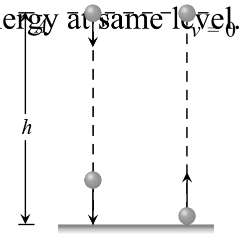
63. (d)

64. (a) $mgh = \frac{80}{100} \times mg \times 100 \Rightarrow h = 80\ m$

65. (a) Let ball is projected vertically downward with velocity v from height h

Total energy at point $A = \frac{1}{2}mv^2 + mgh$

During collision loss of energy is 50% and the ball rises up to same height. It means it possess only potential energy at same level.



$50\%\left(\frac{1}{2}mv^2 + mgh\right) = mgh$

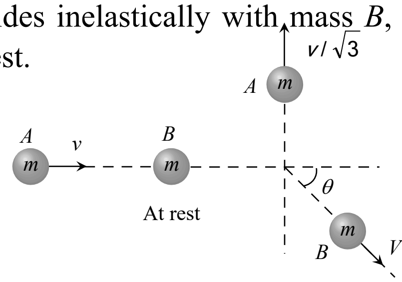
$\frac{1}{2}\left(\frac{1}{2}mv^2 + mgh\right) = mgh$

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20}$

$\therefore v = 20\ m/s$

66. (a) $h_n = he^{2n}$ after third collision $h_3 = he^6$ [as n = 3]

67. (a) Let mass A moves with velocity v and collides inelastically with mass B, which is at rest.



According to problem mass A moves in a perpendicular direction and let the mass B moves at angle θ with the horizontal with velocity v.

Initial horizontal momentum of system (before collision) = mv ....(i)

Final horizontal momentum of system (after collision) = $mV \cos\theta$ ....(ii)

From the conservation of horizontal linear momentum $mv = mV\cos\theta \Rightarrow v = V\cos\theta$ ...(iii)

Initial vertical momentum of system (before collision) is zero.

Final vertical momentum of system $\frac{mv}{\sqrt{3}} - mV\sin\theta$

From the conservation of vertical linear momentum $\frac{mv}{\sqrt{3}} - mV\sin\theta = 0 \Rightarrow \frac{v}{\sqrt{3}} = V\sin\theta$ ...(iv)

By solving (iii) and (iv)

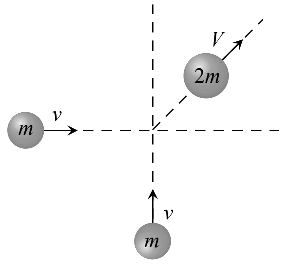
$v^2 + \frac{v^2}{3} = V^2(\sin^2\theta + \cos^2\theta)$

$\Rightarrow \frac{4v^2}{3} = V^2 \Rightarrow V = \frac{2}{\sqrt{3}}v.$

68. (d) Angle will be 90° if collision is perfectly elastic

Perfectly Inelastic Collision

1. (c)



Initial momentum of the system

$\vec{P}_i = mv\hat{i} + mv\hat{j}$

$|\vec{P}_i| = \sqrt{2}mv$

Final momentum of the system = $2mV$

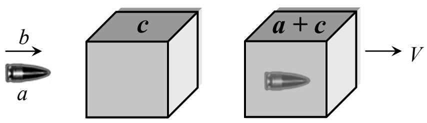
By the law of conservation of momentum

$\sqrt{2}mv = 2mV \Rightarrow V = \frac{v}{\sqrt{2}}$

2. (b)

3. (c)

4. (b)



Initially bullet moves with velocity b and after collision bullet get embedded in block and both move together with common velocity.

By the conservation of momentum

$\Rightarrow a \times b + 0 = (a + c)\, V \Rightarrow V = \frac{ab}{a+c}$

5. (d) Initially mass 10 *gm* moves with velocity 100 *cm/s*

$\therefore$ Initial momentum $= 10 \times 100 = 1000 \frac{gm \times m}{sec}$

After collision system moves with velocity $v_{sys.}$ then

Final momentum $= (10+10) \times v_{sys.}$

By applying the conservation of momentum

$10000 = 20 \times v_{sys.} \Rightarrow v_{sys.} == 50\ cm/s$

If system rises upto height *h* then

$h = \frac{v_{sys.}^2}{2g} = \frac{50 \times 50}{2 \times 1000} = \frac{2.5}{2} = 1.25\ cm$

6. (b)

7. (c)

8. (c) $m_1 v_1 - m_2 v_2 = (m_1 + m_2) v$

$\Rightarrow 2 \times 3 - 1 \times 4 = (2+1)\, v \Rightarrow v = \frac{2}{3}\ m/s$

9. (c) Initial momentum of the system $= mv - mv = 0$

As body sticks together $\therefore$ final momentum $= 2mV$

By conservation of momentum $2mV = 0$ $\therefore$ $V = 0$

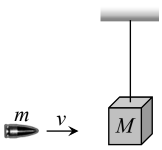
10. (a) If initially second body is at rest then

Initial momentum $= mv$

Final momentum $= 2mV$

By conservation of momentum $2mV = mv \Rightarrow V = \frac{v}{2}$

11. (d)



Initial momentum $= mv$

Final momentum $= (m+M)V$

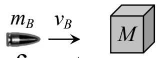
By conservation of momentum $mv = (m+M)V$

$\therefore$ Velocity of (bag + bullet) system $V = \frac{mv}{M+m}$

$\therefore$ Kinetic energy $= \frac{1}{2}(m+M)\, V^2$

$= \frac{1}{2}(m+M)\left(\frac{mv}{M+m}\right)^2 = \frac{1}{2}\frac{m^2 v^2}{M+m}$

12. (b)



Initial K.E. of system = K.E. of the bullet $= \frac{1}{2} m_B v_B^2$

By the law of conservation of linear momentum

$m_B v_B + 0 = m_{sys.} \times v_{sys.}$

$\Rightarrow v_{sys.} = \frac{m_B v_B}{m_{sys.}} = \frac{50 \times 10}{50 + 950} = 0.5\ m/s$

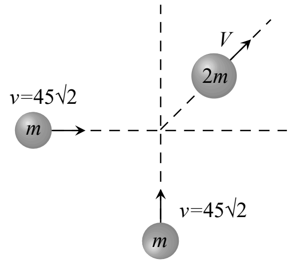
Fractional loss in K.E. $= \frac{\frac{1}{2} m_B v_B^2 - \frac{1}{2} m_{sys.} v_{sys.}^2}{\frac{1}{2} m_B v_B^2}$

By substituting $m_B = 50 \times 10^{-3}\ kg,\ v_B = 10\ m/s$

$m_{sys.} = 1 kg,\ v_s = 0.5\ m/s$ we get

Fractional loss $= \frac{95}{100}$ $\therefore$ Percentage loss = 95%

13. (b)



Initial momentum

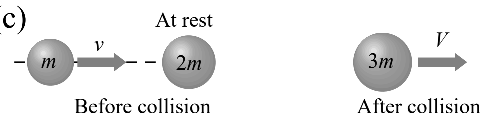
$\vec{P} = m45\sqrt{2}\,\hat{i} + m45\sqrt{2}\,\hat{j} \Rightarrow |\vec{P}| = m \times 90$

Final momentum $2m \times V$

By conservation of momentum $2m \times V = m \times 90$

$\therefore V = 45\ m/s$

14. (c)



Initial momentum $= mv$

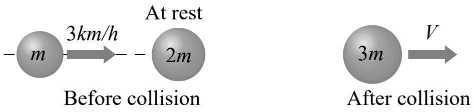
Final momentum $= 3mV$

By the law of conservation of momentum

$mv = 3mV$

$\therefore\ V = v/3$

15. (c)



Initial momentum = $m \times 3 + 2m \times 0 = 3m$

Final momentum = $3m \times V$

By the law of conservation of momentum

$3m = 3m \times V \Rightarrow V = 1\ km/h$

16. (d) Loss in K.E. = (initial K.E. – Final K.E.) of system

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)V^2$

$= \frac{1}{2}3 \times (32)^2 + \frac{1}{2} \times 4 \times (5)^2 - \frac{1}{2} \times (3+4) \times (5)^2$

= 986.5 *J*

17. (a) Momentum of earth-ball system remains conserved.

18. (b) $v = 36\ km/h = 10\ m/s$

By law of conservation of momentum

$2 \times 10 = (2+3)\ V \Rightarrow V = 4\ m/s$

Loss in K.E. $= \frac{1}{2} \times 2 \times (10)^2 - \frac{1}{2} \times 5 \times (4)^2 = 60\ J$

19. (d) Initial momentum = $\vec{P} = mv\hat{i} + mv\hat{j}$

$|\vec{P}| = \sqrt{2}mv$

Final momentum = $2m \times V$

By the law of conservation of momentum

$2m \times V = \sqrt{2}\ mv \Rightarrow V = \frac{v}{\sqrt{2}}$

In the problem $v = 10 m/s$ (given) $\therefore\ V = \frac{10}{\sqrt{2}} = 5\sqrt{2}\ m/s$

20. (a) Because in perfectly inelastic collision the colliding bodies stick together and move with common velocity

21. (b) $m_1v_1 + m_2v_2 = (m_1 + m_2)v_{sys.}$

$20 \times 10 + 5 \times 0 = (20+5)\ v_{sys.} \Rightarrow v_{sys.} = 8\ m/s$

K.E. of composite mass $= \frac{1}{2}(20+5) \times (8)^2 = 800\ J$

22. (c) According to law of conservation of momentum.

Momentum of neutron = Momentum of combination

$\Rightarrow 1.67 \times 10^{-27} \times 10^8 = (1.67 \times 10^{-27} + 3.34 \times 10^{-27})\ v$

$\therefore\ v = 3.33 \times 10^7\ m/s$

23. (b)

24. (c) Loss in kinetic energy

$= \frac{1}{2}\frac{m_1m_2(u_1 - u_2)^2}{m_1 + m_2} = \frac{1}{2}\left(\frac{40 \times 60}{40 + 60}\right)(4-2)^2 = 48\ J$

25. (b) By momentum conservation before and after collision.

$m_1V + m_2 \times 0 = (m_1 + m_2)v \Rightarrow v = \frac{m_1}{m_1 + m_2}V$

i.e. Velocity of system is less than *V*.

26. (a) By conservation of momentum, $mv + M \times 0 = (m + M)$

Velocity of composite block $V = \left(\frac{m}{m+M}\right)v$

K.E. of composite block $= \frac{1}{2}(M+m)V^2$

$= \frac{1}{2}(M+m)\left(\frac{m}{M+m}\right)^2 v^2 = \frac{1}{2}mv^2\left(\frac{m}{m+M}\right)$

27. (b)

28. (d) Velocity of combined mass, $v = \frac{m_1v_1 - m_2v_2}{m_1 + m_2}$

$= \frac{0.1 \times 1 - 0.4 \times 0.1}{0.5} = 0.12\ m/s$

$\therefore$ Distance travelled by combined mass

$= v \times t =$ 0.12 × 10 = 1.2 *m*.

29. (c) Loss in K.E. = $\frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$

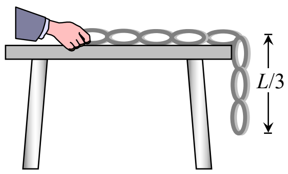
$= \frac{4 \times 6}{2 \times 10} \times (12 - 0)^2 = 172.8\ J$

30. (d) In case of perfectly inelastic collision, the bodies stick together after impact.

Critical Thinking Questions

1. (c) By the conservation of momentum in the absence of external force total momentum of the system (ball + earth) remains constant.

2. (d)



$W = \frac{MgL}{2n^2} = \frac{MgL}{2(3)^2} = \frac{MgL}{18}$ (n = 3 given)

3. (b) Gravitational force is a conservative force and work done against it is a point function *i.e.* does not depend on the path.

4. (b) Here $\frac{mv^2}{r} = \frac{K}{r^2}$ $\therefore$ K.E. $= \frac{1}{2}mv^2 = \frac{K}{2r}$

$U = -\int_{\infty}^{r} F.dr = -\int_{\infty}^{r}\left(-\frac{K}{r^2}\right)dr = -\frac{K}{r}$

Total energy $E = \text{K.E.} + \text{P.E.} = \frac{K}{2r} - \frac{K}{r} = -\frac{K}{2r}$

5. (c) $x = (t-3)^2 \Rightarrow v = \frac{dx}{dt} = 2(t-3)$

at $t = 0$; $v_1 = -6\,m/s$ and at $t = 6$ sec, $v_2 = 6\,m/s$

so, change in kinetic energy $= W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = 0$

6. (c) While moving from (0,0) to (a,0)

Along positive x-axis, y = 0 $\therefore \vec{F} = -kx\hat{j}$

i.e. force is in negative y-direction while displacement is in positive x-direction.

$\therefore W_1 = 0$

Because force is perpendicular to displacement

Then particle moves from $(a,0)$ to (a,a) along a line parallel to y-axis $(x = +a)$ during this $\vec{F} = -k(y\hat{i} + a\hat{j})$

The first component of force, $-ky\hat{i}$ will not contribute any work because this component is along negative x-direction $(-\hat{i})$ while displacement is in positive y-direction (a,0) to (a,a). The second component of force *i.e.* $-ka\hat{j}$ will perform negative work

$\therefore W_2 = (-ka\hat{j})(a\hat{j}) = (-ka)(a) = -ka^2$

So net work done on the particle $W = W_1 + W_2$

$= 0 + (-ka^2) = -ka^2$

7. (a) Gain in potential energy $\Delta U = \frac{mgh}{1 + \frac{h}{R}}$

If $h = R$ then $\Delta U = \frac{mgR}{1 + \frac{R}{R}} = \frac{1}{2}mgR$

8. (c) Stopping distance $= \frac{\text{kinetic energy}}{\text{retarding force}} \Rightarrow s = \frac{1}{2}\frac{mu^2}{F}$

If lorry and car both possess same kinetic energy and retarding force is also equal then both come to rest in the same distance.

9. (d) Potential energy of the particle $U = k(1 - e^{-x^2})$

Force on particle $F = \frac{-dU}{dx} = -k[-e^{-x^2} \times (-2x)]$

$F = -2kxe^{-x^2} = -2kx\left[1 - x^2 + \frac{x^4}{2!} - ......\right]$

For small displacement $F = -2kx$

$\Rightarrow F \propto -x$ *i.e.* motion is simple harmonic motion.

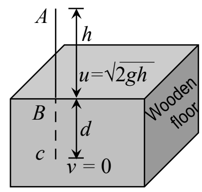
10. (b) Kinetic energy acquired by the body

= Force applied on it × Distance covered by the body

K.E. = $F \times d$

If F and d both are same then K.E. acquired by the body will be same

11. (c) Let the blade stops at depth d into the wood.



$v^2 = u^2 + 2aS$

$\Rightarrow 0 = (\sqrt{2gh})^2 + 2(g-a)d$

by solving $a = \left(1 + \frac{h}{d}\right)g$

So the resistance offered by the wood $= mg\left(1 + \frac{h}{d}\right)$

12. (d) Because linear momentum is vector quantity where as kinetic energy is a scalar quantity.

13. (c) $P = Fv = mav = m\left(\frac{dv}{dt}\right)v \Rightarrow \frac{P}{m}dt = v\,dv$

$\Rightarrow \frac{P}{m} \times t = \frac{v^2}{2} \Rightarrow v = \left(\frac{2P}{m}\right)^{1/2}(t)^{1/2}$

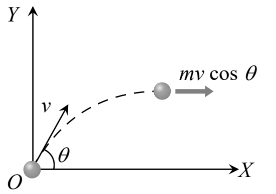
Now $s = \int v\,dt = \int \left(\frac{2P}{m}\right)^{1/2} t^{1/2}dt$

$\therefore s = \left(\frac{2P}{m}\right)^{1/2}\left[\frac{2t^{3/2}}{3}\right] \Rightarrow s \propto t^{3/2}$

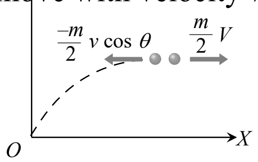
14. (a) Shell is fired with velocity v at an angle θ with the horizontal.

So its velocity at the highest point
= horizontal component of velocity = $v\cos\theta$

So momentum of shell before explosion = $mv\cos\theta$



When it breaks into two equal pieces and one piece retrace its path to the canon, then other part move with velocity V.



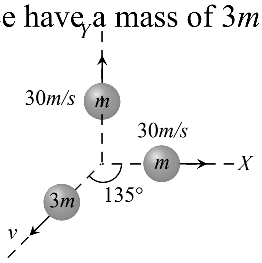
So momentum of two pieces after explosion

$$=\frac{m}{2}(-v\cos\theta)+\frac{m}{2}V$$

By the law of conservation of momentum

$$mv\cos\theta=\frac{-m}{2}v\cos\theta+\frac{m}{2}V \Rightarrow V=3v\cos\theta$$

15. (a) Let two pieces are having equal mass m and third piece have a mass of $3m$.



According to law of conservation of linear momentum. Since the initial momentum of the system was zero, therefore final momentum of the system must be zero *i.e.* the resultant of momentum of two pieces must be equal to the momentum of third piece. We know that if two particle possesses same momentum and angle in between them is 90° then resultant will be given by $P\sqrt{2}=mv\sqrt{2}=m30\sqrt{2}$

Let the velocity of mass $3m$ is V. So $3mV=30m\sqrt{2}$

$\therefore\ V=10\sqrt{2}$ and angle 135° from either.

(as it is clear from the figure)

16. (c) The momentum of the two-particle system, at $t = 0$ is

$$\vec{P}_i=m_1\vec{v}_1+m_2\vec{v}_2$$

Collision between the two does not affect the total momentum of the system.

A constant external force $(m_1+m_2)g$ acts on the system.

The impulse given by this force, in time t = 0 to $t=2t_0$ is $(m_1+m_2)g\times 2t_0$

$\therefore$ |Change in momentum in this interval

$$=|m_1\vec{v}'_1+m_2\vec{v}'_2-(m_1\vec{v}_1+m_2\vec{v}_2)|=2(m_1+m_2)gt_0$$

17. (b) If the masses are equal and target is at rest and after collision both masses moves in different direction. Then angle between direction of velocity will be 90°, if collision is elastic.

18. (d) K.E. of colliding body before collision $=\frac{1}{2}mv^2$

After collision its velocity becomes

$$v'=\frac{(m_1-m_2)}{(m_1+m_2)}v=\frac{m}{3m}v=\frac{v}{3}$$

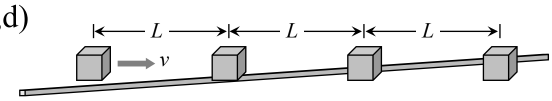
$\therefore$ K.E. after collision $\frac{1}{2}mv'^2=\frac{1}{2}\frac{mv^2}{9}$

Ratio of kinetic energy =

$$\frac{\text{K.E.}_{\text{before}}}{\text{K.E.}_{\text{after}}}=\frac{\frac{1}{2}mv^2}{\frac{1}{2}\frac{mv^2}{9}}=9:1$$

19. (c)

20. (b,d)



Since collision is perfectly inelastic so all the blocks will stick together one by one and move in a form of combined mass.

Time required to cover a distance 'L' by first block $=\frac{L}{v}$

Now first and second block will stick together and move with $v/2$ velocity (by applying conservation of momentum) and combined system will take time $\frac{L}{v/2} = \frac{2L}{v}$ to reach up to block third.

Now these three blocks will move with velocity $v/3$ and combined system will take time $\frac{L}{v/3} = \frac{3L}{v}$ to reach upto the block fourth.

So, total time $= \frac{L}{v} + \frac{2L}{v} + \frac{3L}{v} + ... \frac{(n-1)L}{v}$

$= \frac{n(n-1)L}{2v}$

and velocity of combined system having n blocks as $\frac{v}{n}$.

Graphical questions

1. (c) At time t_1 the velocity of ball will be maximum and it goes on decreasing with respect to time.

At the highest point of path its velocity becomes zero, then it increases but direction is reversed

This explanation match with graph (c).

2. (a) Work done = area between the graph and position axis

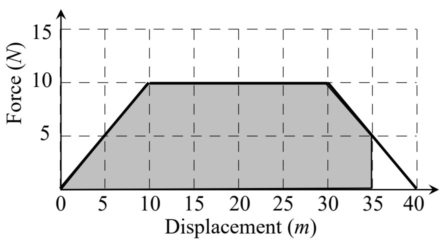
$W = 10 \times 1 + 20 \times 1 - 20 \times 1 + 10 \times 1 = 20\, erg$

3. (a) Spring constant $k = \frac{F}{x} =$ Slope of curve

$\therefore\ k = \frac{4-1}{30} = \frac{3}{30} = 0.1\, kg/cm$

4. (b) As the area above the time axis is numerically equal to area below the time axis therefore net momentum gained by body will be zero because momentum is a vector quantity.

5. (c)



Work done = (Shaded area under the graph between

$x = 0$ to $x = 35\ m) = 287.5\ J$

6. (a) Work done = Area covered in between force displacement curve and displacement axis

= Mass × Area covered in between acceleration-displacement curve and displacement axis.

$= 10 \times \frac{1}{2}(8 \times 10^{-2} \times 20 \times 10^{-2})$

$= 8 \times 10^{-2}\ J$

7. (c) Work done = Gain in potential energy

Area under curve = mgh

$\Rightarrow \frac{1}{2} \times 11 \times 100 = 5 \times 10 \times h$

$\Rightarrow h = 11m$

8. (d) Initial K.E. of the body = $\frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 4 = 50\ J$

Work done against resistive force

= Area between F-x graph

$= \frac{1}{2} \times 4 \times 20 = 40\, J$

Final K.E. = Initial K.E. – Work done against resistive force

$= 50 - 40 = 10\ J$

9. (d) Area between curve and displacement axis

$= \frac{1}{2} \times (12 + 4) \times 10 = 80\ J$

In this time body acquire kinetic energy = $\frac{1}{2}mv^2$

by the law of conservation of energy

$\frac{1}{2}mv^2 = 80\, J$

$\Rightarrow \frac{1}{2} \times 0.1 \times v^2 = 80$

$\Rightarrow v^2 = 1600$

$\Rightarrow v = 40\ m/s$

10. (a) Work done = Area under curve and displacement axis

= Area of trapezium

$= \frac{1}{2}\times$(sum of two parallel lines) × distance between them

$= \frac{1}{2}(10+4)\times(2.5-0.5)$

$= \frac{1}{2}14\times 2 = 14\,J$

As the area actually is not trapezium so work done will be more than 14 *J i.e.* approximately 16 *J*

11. (a) As particle is projected with some velocity therefore its initial kinetic energy will not be zero.

As it moves downward under gravity then its velocity increases with time K.E. $\propto v^2 \propto t^2$ (As $v \propto t$)

So the graph between kinetic energy and time will be parabolic in nature.

12. (a) From the graph it is clear that force is acting on the particle in the region *AB* and due to this force kinetic energy (velocity) of the particle increases. So the work done by the force is positive.

13. (d) $F = \frac{-dU}{dx} \Rightarrow dU = -F\,dx$

$\Rightarrow U = -\int_0^x (-Kx + ax^3)dx = \frac{kx^2}{2} - \frac{ax^4}{4}$

$\therefore$ We get $U = 0$ at $x = 0$ and $x = \sqrt{2k/a}$

and also U = negative for $x > \sqrt{2k/a}$.

So $F = 0$ at $x = 0$

i.e. slope of $U - x$ graph is zero at $x = 0$.

14. (b) Work done = Area enclosed by $F - x$ graph

$= \frac{1}{2}\times(3+6)\times 3 = 13.5\ J$

15. (c) As slope of problem graph is positive and constant upto certain distance and then it becomes zero.

So from $F = \frac{-dU}{dx}$, up to distance *a*, F = constant (negative) and becomes zero suddenly.

16. (d) Work done = change in kinetic energy

$W = \frac{1}{2}mv^2$ $\therefore$ $W \propto v^2$ graph will be parabolic in nature

17. (a) Potential energy increases and kinetic energy decreases when the height of the particle increases it is clear from the graph (*a*).

18. (c) $P = \sqrt{2mE}$ it is clear that $P \propto \sqrt{E}$

So the graph between P and $\sqrt{E}$ will be straight line.

but graph between $\frac{1}{P}$ and $\sqrt{E}$ will be hyperbola

19. (b) When particle moves away from the origin then at position $x = x_1$ force is zero and at $x > x_1$, force is positive (repulsive in nature) so particle moves further and does not return back to original position.

i.e. the equilibrium is not stable.

Similarly at position $x = x_2$ force is zero and at $x > x_2$, force is negative (attractive in nature)

So particle return back to original position *i.e.* the equilibrium is stable.

20. (c) $F = \frac{-dU}{dx}$ it is clear that slope of $U - x$ curve is zero at point *B* and *C*. $\therefore$ $F = 0$ for point *B* and *C*

21. (a) Work done = area under curve and displacement axis

$= 1\times 10 - 1\times 10 + 1\times 10 = 10\ J$

22. (a) When the length of spring is halved, its spring constant will becomes double. (because $k \propto \frac{1}{x} \propto \frac{1}{L} \therefore k \propto \frac{1}{L}$)

Slope of force displacement graph gives the spring constant (k) of spring.

If k becomes double then slope of the graph increases *i.e.* graph shifts towards force-axis.

23. (a) Kinetic energy $E = \frac{1}{2}mv^2 \Rightarrow E \propto v^2$

graph will be parabola symmetric to E-axis.

24. (c) Change in momentum = Impulse

= Area under force-time graph

$\therefore$ $mv =$ Area of trapezium

$\Rightarrow mv = \frac{1}{2}\left(T + \frac{T}{2}\right)F_0$

$\Rightarrow mv = \frac{3T}{4}F_0 \Rightarrow F_0 = \frac{4mu}{3T}$

25. (c) When body moves under action of constant force then kinetic energy acquired by the body K.E. = $F \times S$

$\therefore$ KE $\propto$ S (If F = constant)

So the graph will be straight line.

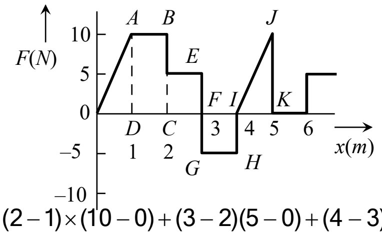
26. (a) When the distance between atoms is large then interatomic force is very weak. When they come closer, force of attraction increases and at a particular distance force becomes zero. When they are further brought closer force becomes repulsive in nature.

This can be explained by slope of $U - x$ curve shown in graph (a).

27. (b) Work done = area under F-x graph

= area of rectangle $ABCD$ + area of rectangle $LCEF$

+ area of rectangle $GFIH$ + area of triangle IJK



$= (2-1)\times(10-0) + (3-2)(5-0) + (4-3)(-5-0)$

$+ \frac{1}{2}(5-4)(10-0) = 15\ J$

28. (a) $U = -\int F dx = -\int kx\, dx = -k\frac{x^2}{2}$

This is the equation of parabola symmetric to U axis in negative direction

Assertion and Reason

1. (a) The work done, $W = \vec{F}.\vec{s} = Fs\cos\theta$, when a person walk on a horizontal road with load on his head then $\theta = 90°$.

Hence $W = Fs\cos 90° = 0$

Thus no work is done by the person.

2. (d) In a round trip work done is zero only when the force is conservative in nature.

Force is always required to move a body in a conservative or non-conservative field

3. (e) When a body slides down on inclined plane, work done by friction is negative because it opposes the motion (θ = 180° between force and displacement)

If $\theta < 90°$ then W=positive because $W = F.s\cos\theta$

4. (a) Since the gaseous pressure and the displacement (of piston) are in the same direction. Therefore $\theta = 0°$

$\therefore$ Work done = $Fs\cos\theta = Fs =$ Positive

Thus during expansion work done by gas is positive.

5. (d) When two bodies have same momentum then lighter body possess more kinetic energy because $E = \frac{P^2}{2m}$

$\therefore E \propto \frac{1}{m}$ when P = constant

6. (b) $P = \vec{F}.\vec{v}$ and unit of power is *Watt*.

7. (c) Change in kinetic energy = work done by net force.

This relationship is valid for particle as well as system of particles.

8. (a) The work done on the spring against the restoring force is stored as potential energy in both conditions when it is compressed or stretched.

9. (c) The gravitational force on the comet due to the sun is a conservative force. Since the work done by a conservative force over a closed path is always zero (irrespective of the nature of path), the work done by the gravitational forces over every complete orbit of the comet is zero.

10. (e) Rate of change of momentum is proportional to external forces acting on the system. The total momentum of whole system remain constant when no external force is acted upon it.

Internal forces can change the kinetic energy of the system.

11. (a) When the water is at the top of the fall it has potential energy *mgh* (where *m* is the mass of the water and *h* is the height of the fall). On falling, this potential energy is converted into kinetic energy, which further converted into heat energy and so temperature of water increases.

12. (b) The power of the pump is the work done by it per sec.

$\therefore$ Power $=\frac{\text{work}}{\text{time}}=\frac{mgh}{t}=\frac{100\times10\times100}{10}$

$=10^4\,W=10\,kW$

Also 1 *Horse power* (*hp*) =746 *W*.

13. (c) For conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. This is known as law of conservation of mechanical energy. According to this law,

Kinetic energy + Potential energy = constant

or, $\Delta K+\Delta U=0$ or, $\Delta K=-\Delta U$

14. (e) When the force retards the motion, the work done is negative.

Work done depends on the angle between force and displacement $W=Fs\cos\theta$

15. (d) In an elastic collision both the momentum and kinetic energy remains conserved. But this rule is not for individual bodies, but for the system of bodies before and after the collision. While collision in which there occurs some loss of kinetic energy is called inelastic collision. Collision in daily life are generally inelastic. The collision is said to be perfectly inelastic, if two bodies stick to each other.

16. (d) A body can have energy without having momentum if it possess potential energy but if body possess momentum then it must posses kinetic energy. Momentum and energy have different dimensions.

17. (e) Work done and power developed is zero in uniform circular motion only.

18. (a) $K=\frac{1}{2}mv^2$ $\therefore$ $K\propto v^2$

If velocity is doubled then K.E. will be quadrupled.

19. (a) In a quick collision, time *t* is small. As $F\times t=$ constant, therefore, force involved is large, *i.e.* collision is more violent in comparison to slow collision.

20. (a) From, definition, work done in moving a body against a conservative force is independent of the path followed.

21. (c) When we supply current through the cell, chemical reactions takes place, so chemical energy of cell is converted into electrical energy. If a large amount of current is drawn from wire for a long time only then wire get heated.

22. (e) Potential energy $U=\frac{1}{2}kx^2$ *i.e.* $U\propto x^2$

This is a equation of parabola, so graph between *U* and *x* is a parabola, not straight line.

23. (c) When two bodies of same mass undergo an elastic collision, their velocities get interchanged after collision. Water and heavy water are hydrogenic materials containing protons having approximately the same mass as that of a neutron. When fast moving neutrons collide with protons,

the neutrons come to rest and protons move with the velocity of that of neutrons.

24. (a) From Einstein equation $E = mc^2$

it can be observed that if mass is conserved then only energy is conserved and vice versa. Thus, both cannot be treated separately.

25. (b) If two protons are brought near one another, work has to be done against electrostatic force because same charge repel each other. This work done is stored as potential energy in the system.

26. (a) $E = \frac{P^2}{2m}$. In firing momentum is conserved

$\therefore E \propto \frac{1}{m}$

So $\frac{E_{\text{gun}}}{E_{\text{bullet}}} = \frac{m_{\text{bullet}}}{m_{\text{gun}}}$

27. (a) K.E. of one bullet = k $\therefore$ K.E. of n bullet = nk

According to law of conservation of energy, the kinetic energy of bullets be equal to the work done by machine gun per sec.

28. (d) Work done in the motion of a body over a closed loop is zero only when the body is moving under the action of conservative forces (like gravitational or electrostatic forces). *i.e.* work done depends upon the nature of force.

29. (a) If roads of the mountain were to go straight up, the slope θ would have been large, the frictional force $\mu mg\cos\theta$ would be small. Due to small friction, wheels of vehicle would slip. Also for going up a large slope, a greater power shall be required.

30. (a) The rise in temperature of the soft steel is an example of transferring energy into a system by work and having it appear as an increase in the internal energy of the system. This works well for the soft steel because it is soft. This softness results in a deformation of the steel under blow of the hammer. Thus the point of application of the force is displaced by the hammer and positive work is done on the steel. With the hard steel, less deformation occur, thus, there is less displacement of point of application of the force and less work done on the steel. The soft steel is therefore better in absorbing energy from the hammer by means of work and its temperature rises more rapidly.