Answers

Fundamentals of Vectors

1	d	2	b	3	c	4	d	5	d
6	a	7	a	8	b	9	b	10	d
11	d	12	d	13	a	14	b	15	c
16	c	17	a	18	b	19	c	20	c
21	d	22	d	23	b	24	d	25	b
26	b	27	a	28	a	29	a	30	d
31	a	32	b	33	a	34	a		

Addition and Subtraction of Vectors

1	a	2	b	3	d	4	b	5	b
6	a	7	b	8	a	9	d	10	b
11	d	12	c	13	a	14	c	15	c
16	c	17	c	18	c	19	c	20	b
21	a	22	d	23	d	24	a	25	c
26	b	27	b	28	a	29	b	30	a
31	c	32	c	33	c	34	d	35	a
36	c	37	d	38	a	39	c	40	d
41	a	42	b	43	d	44	d	45	a
46	c	47	d	48	a	49	a	50	c
51	c	52	a	53	d				

Multiplication of Vectors

1	c	2	b	3	d	4	a	5	a
6	b	7	c	8	b	9	b	10	d
11	b	12	d	13	c	14	d	15	c
16	c	17	b	18	c	19	b	20	a
21	a	22	c	23	a	24	b	25	c
26	d	27	d	28	b	29	b	30	b
31	d	32	c	33	d	34	b	35	d
36	b	37	a	38	b	39	a	40	a
41	d	42	d	43	c	44	b	45	a
46	a	47	a	48	d	49	d	50	a
51	b	52	b	53	d	54	a	55	c
56	d	57	a	58	b	59	c		

Lami's Theorem

1	c	2	a	3	b	4	c	5	b

Relative Velocity

1	b	2	b	3	c	4	c	5	d
6	a	7	c	8	c	9	d	10	ac
11	b	12	b	13	d	14	b		

Critical Thinking Questions

1	c	2	c	3	c	4	c	5	b
6	b	7	d	8	d	9	b	10	c
11	a	12	d	13	b	14	d		

Assertion and Reason

1	a	2	a	3	d	4	b	5	c
6	b	7	a	8	b	9	a	10	c
11	b	12	e	13	a	14	c	15	a
16	b	17	c	18	b	19	b	20	c
21	a	22	c						

Answers and Solutions

Fundamentals of Vectors

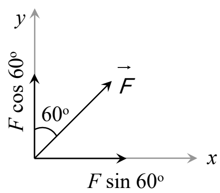
1. (d) As the multiple of $\hat{j}$ in the given vector is zero therefore this vector lies in *XZ* plane and projection of this vector on *y*-axis is zero.

2. (b) If a point have coordinate (*x, y, z*) then its position vector $= x\hat{i} + y\hat{j} + z\hat{k}$.

3. (c) Displacement vector $\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

$= (3-2)\hat{i} + (4-3)\hat{j} + (5-5)\hat{k} = \hat{i} + \hat{j}$

4. (d)



The component of force in vertical direction

$= F\cos\theta = F\cos 60^\circ = 5 \times \frac{1}{2} = 2.5\ N$

5. (d) $|B| = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25$

Unit vector in the direction of A will be $\hat{A} = \frac{3\hat{i}+4\hat{j}}{5}$

So required vector $= 25\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = 15\hat{i}+20\hat{j}$

6. (a) Let the components of $\vec{A}$ makes angles α, β and γ with *x, y* and *z* axis respectively then $\alpha = \beta = \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 3\cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

$\therefore A_x = A_y = A_z = A\cos\alpha = \frac{A}{\sqrt{3}}$

7. (a) $\vec{A} = 2\hat{i}+4\hat{j}-5\hat{k} \therefore |\vec{A}| = \sqrt{(2)^2+(4)^2+(-5)^2} = \sqrt{45}$

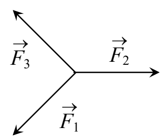
$\therefore \cos\alpha = \frac{2}{\sqrt{45}}, \quad \cos\beta = \frac{4}{\sqrt{45}}, \quad \cos\gamma = \frac{-5}{\sqrt{45}}$

8. (b) Unit vector along *y* axis $= \hat{j}$ so the required vector $= \hat{j} - [(\hat{i}-3\hat{j}+2\hat{k}) + (3\hat{i}+6\hat{j}-7\hat{k})]$

$= -4\hat{i} - 2\hat{j} + 5\hat{k}$

9. (b) $\vec{F}_3 = \vec{F}_1 + \vec{F}_2$

There should be minimum three coplaner vectors having different magnitude which should be added to give zero resultant

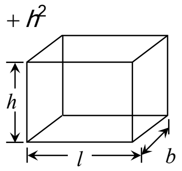


10. (d) Diagonal of the hall $= \sqrt{l^2+b^2+h^2}$

$= \sqrt{10^2+12^2+14^2}$

$= \sqrt{100+144+196}$

$= \sqrt{400} = 20m$



11. (d) Total angle $= 100 \times \frac{\pi}{50} = 2\pi$

So all the force will pass through one point and all forces will be balanced. *i.e.* their resultant will be zero.

12. (d) $\vec{r} = \vec{r}_2 - \vec{r}_1 = (-2\hat{i}-2\hat{j}+0\hat{k}) - (4\hat{i}-4\hat{j}+0\hat{k})$

$\Rightarrow \vec{r} = -6\hat{i}+2\hat{j}+0\hat{k}$

$\therefore |\vec{r}| = \sqrt{(-6)^2+(2)^2+0^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

13. (a) $\vec{P} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} \therefore |\vec{P}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

$\therefore$ It is a unit vector.

14. (b)

15. (c) $\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{\hat{i}+\hat{j}}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

16. (c) $\vec{R} = 3\hat{i}+\hat{j}+2\hat{k}$

$\therefore$ Length in *XY* plane $= \sqrt{R_x^2+R_y^2} = \sqrt{3^2+1^2}$ $= \sqrt{10}$

17. (a) If the angle between all forces which are equal and lying in one plane are equal then resultant force will be zero.

18. (b) $\vec{A} = \hat{i}+\hat{j} \Rightarrow |\vec{A}| = \sqrt{1^2+1^2} = \sqrt{2}$

$\cos\alpha = \frac{A_x}{|\vec{A}|} = \frac{1}{\sqrt{2}} = \cos 45° \therefore \alpha = 45°$

19. (c)

20. (c)

21. (d) All quantities are tensors.

22. (d) $\vec{P}+\vec{Q} = P\hat{P} + Q\hat{Q}$

23. (b) $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = -a\omega\sin\omega t\,\hat{i} + a\omega\cos\omega t\,\hat{j}$

As $\vec{r}.\vec{v} = 0$ therefore velocity of the particle is perpendicular to the position vector.

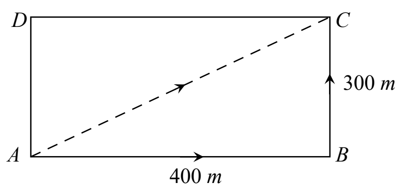
24. (d) Displacement, electrical and acceleration are vector quantities.

25. (b) Magnitude of unit vector = 1

$\Rightarrow \sqrt{(0.5)^2+(0.8)^2+c^2} = 1$

By solving we get $c = \sqrt{0.11}$

26. (b)



Displacement $\vec{AC} = \vec{AB} + \vec{BC}$

$AC = \sqrt{(AB)^2+(BC)^2} = \sqrt{(400)^2+(300)^2} = 500m$

Distance $= AB + BC = 400 + 300 = 700m$

27. (a) Resultant of vectors $\vec{A}$ and $\vec{B}$

$\vec{R} = \vec{A}+\vec{B} = 4\hat{i}+3\hat{j}+6\hat{k}-\hat{i}+3\hat{j}-8\hat{k}$

$\vec{R} = 3\hat{i}+6\hat{j}-2\hat{k}$

$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{3^2+6^2+(-2)^2}} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$

28. (a) $\phi = \vec{B}.\vec{A}$. In this formula $\vec{A}$ is a area vector.

29. (a) $\vec{r} = \vec{a} + \vec{b} + \vec{c} = 4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - \hat{k} = \hat{i} + \hat{j} - \hat{k}$

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$$

30. (d) $\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{9+16+25}{\sqrt{9+16+25}\sqrt{9+16+25}} = \frac{50}{50} = 1$

$\Rightarrow \cos\theta = 1 \ \therefore \ \theta = \cos^{-1}(1)$

31. (a) $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$

at $t = 0$, $\vec{r}_1 = 7\hat{k}$

at $t = 10\,\text{sec}$, $\vec{r}_2 = 300\hat{i} + 400\hat{j} + 7\hat{k}$,

$\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1 = 300\hat{i} + 400\hat{j}$

$|\overrightarrow{\Delta r}| = |\vec{r}_2 - \vec{r}_1| = \sqrt{(300)^2 + (400)^2} = 500m$

32. (b) Resultant of vectors $\vec{A}$ and $\vec{B}$

$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 8\hat{i} + 8\hat{j} = 12\hat{i} + 5\hat{j}$

$$\hat{R} = \frac{\vec{R}}{|R|} = \frac{12\hat{i} + 5\hat{j}}{\sqrt{(12)^2 + (5)^2}} = \frac{12\hat{i} + 5\hat{j}}{13}$$

33. (a) $\frac{\vec{A}.\vec{B}}{|\hat{i} + \hat{j}|} = \frac{(2\hat{i} + 3\hat{j})(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

34. (a) $\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{(3\hat{i} + 4\hat{j} + 5\hat{k})(3\hat{i} + 4\hat{j} - 5\hat{k})}{\sqrt{9+16+25}\sqrt{9+16+25}}$

$= \frac{9+16-25}{50} = 0$

$\Rightarrow \cos\theta = 0$, $\therefore \ \theta = 90°$

Addition and Subtraction of Vectors

1. (a) For 17 N both the vector should be parallel *i.e.* angle between them should be zero.

For 7 N both the vectors should be antiparallel *i.e.* angle between them should be 180°

For 13 N both the vectors should be perpendicular to each other *i.e.* angle between them should be 90°

2. (b) $\vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 6\hat{i} + 8\hat{j} = 10\hat{i} + 5\hat{j}$

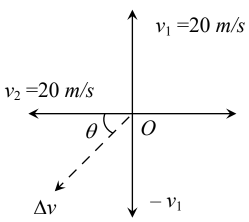
$|\vec{A} + \vec{B}| = \sqrt{(10)^2 + (5)^2} = 5\sqrt{5}$

$\tan\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$

3. (d) From figure

$\vec{v}_1 = 20\hat{j}$ and $\vec{v}_2 = -20\hat{i}$

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = -20(\hat{i} + \hat{j})$



$|\Delta\vec{v}| = 20\sqrt{2}$ and direction

$\theta = \tan^{-1}(1) = 45°$ *i.e.* *S–W*

4. (b) Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is

$\vec{n}_s = \hat{n}_1 + \hat{n}_2$ or $n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta$

$= 1 + 1 + 2\cos\theta$

Since it is given that n_s is also a unit vector, therefore $1 = 1 + 1 + 2\cos\theta \Rightarrow \cos\theta = -\frac{1}{2}$ $\therefore$

$\theta = 120°$

Now the difference vector is $\hat{n}_d = \hat{n}_1 - \hat{n}_2$ or

$n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1 + 1 - 2\cos(120°)$

$\therefore \ n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$

5. (b) $\vec{A} - 2\vec{B} + 3\vec{C} = (2\hat{i} + \hat{j}) - 2(3\hat{j} - \hat{k}) + 3(6\hat{i} - 2\hat{k})$

$= 2\hat{i} + \hat{j} - 6\hat{j} + 2\hat{k} + 18\hat{i} - 6\hat{k} = 20\hat{i} - 5\hat{j} - 4\hat{k}$

6. (a) $\vec{P}_1 = mv\sin\theta\,\hat{i} - mv\cos\theta\,\hat{j}$

and $\vec{P}_2 = mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}$

So change in momentum

$\overrightarrow{\Delta P} = \vec{P}_2 - \vec{P}_1 = 2mv\cos\theta\,\hat{j}$, $|\Delta\vec{P}| = 2mv\cos\theta$

7. (b) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

By substituting, $A = F, B = F$ and $R = F$ we get

$\cos\theta = \frac{1}{2} \therefore \theta = 120°$

8. (a)

9. (d) If two vectors $\vec{A}$ and $\vec{B}$ are given then the resultant $R_{max} = A + B = 7N$ and $R_{min} = 4 - 3 = 1N$

i.e. net force on the particle is between 1 N and 7 N.

10. (b) If $\vec{C}$ lies outside the plane then resultant force can not be zero.

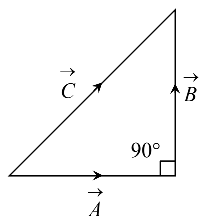
11. (d)

12. (c) $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90°} = \sqrt{F_1^2 + F_2^2}$

13. (a)

14. (c)

15. (c) $C = \sqrt{A^2 + B^2}$



The angle between A and B is $\frac{\pi}{2}$

16. (c) $\vec{R}=\vec{A}+\vec{B}=6\hat{i}+7\hat{j}+3\hat{i}+4\hat{j}=9\hat{i}+11\hat{j}$

$\therefore |\vec{R}|=\sqrt{9^2+11^2}=\sqrt{81+121}=\sqrt{202}$

17. (c) $R=\sqrt{12^2+5^2+6^2}=\sqrt{144+25+36}=\sqrt{205}=14.31\ m$

18. (c) $\vec{A}=3\hat{i}-2\hat{j}+\hat{k}$, $\vec{B}=\hat{i}-3\hat{j}+5\hat{k}$, $\vec{C}=2\hat{i}-\hat{j}+4\hat{k}$

$|\vec{A}|=\sqrt{3^2+(-2)^2+1^2}=\sqrt{9+4+1}=\sqrt{14}$

$|\vec{B}|=\sqrt{1^2+(-3)^2+5^2}=\sqrt{1+9+25}=\sqrt{35}$

$|\vec{A}|=\sqrt{2^2+1^2+(-4)^2}=\sqrt{4+1+16}=\sqrt{21}$

As $B=\sqrt{A^2+C^2}$ therefore ABC will be right angled triangle.

19. (c)

20. (b) $\vec{C}+\vec{A}=\vec{B}$.

The value of C lies between $A-B$ and $A+B$

$\therefore |\vec{C}|<|\vec{A}|$ or $|\vec{C}|<|\vec{B}|$

21. (a)

22. (d)

23. (d) Here all the three force will not keep the particle in equilibrium so the net force will not be zero and the particle will move with an acceleration.

24. (a) $A+B=16$ (given) ...(i)

$\tan\alpha=\frac{B\sin\theta}{A+B\cos\theta}=\tan 90°$

$\therefore\ A+B\cos\theta=0 \Rightarrow \cos\theta=\frac{-A}{B}$ ...(ii)

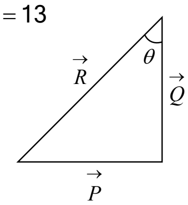
$8=\sqrt{A^2+B^2+2AB\cos\theta}$ ...(iii)

By solving eq. (i), (ii) and (iii) we get $A=6N$, $B=10N$

25. (c) $|\vec{P}|=5$, $|\vec{Q}|=12$ and $|\vec{R}|=13$

$\cos\theta=\frac{Q}{R}=\frac{12}{13}$

$\therefore\ \theta=\cos^{-1}\left(\frac{12}{13}\right)$



26. (b) $\frac{B}{2}=\sqrt{A^2+B^2+2AB\cos\theta}$ ...(i)

$\therefore\ \tan 90°=\frac{B\sin\theta}{A+B\cos\theta}\Rightarrow A+B\cos\theta=0$

$\therefore\ \cos\theta=-\frac{A}{B}$

Hence, from (i) $\frac{B^2}{4}=A^2+B^2-2A^2 \Rightarrow A=\sqrt{3}\frac{B}{2}$

$\Rightarrow \cos\theta=-\frac{A}{B}=-\frac{\sqrt{3}}{2}\ \therefore\ \theta=150°$

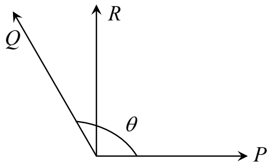
27. (b) $(\hat{i}-2\hat{j}+2\hat{k})+(2\hat{i}+\hat{j}-\hat{k})+\vec{R}=i$

$\therefore$ Required vector $\vec{R}=-2\hat{i}+\hat{j}-\hat{k}$

28. (a) Resultant $\vec{R}=\vec{P}+\vec{Q}+\vec{P}-\vec{Q}=2\vec{P}$

The angle between $\vec{P}$ and $2\vec{P}$ is zero.

29. (b)



$\Rightarrow \tan 90°=\frac{Q\sin\theta}{P+Q\cos\theta}\Rightarrow P+Q\cos\theta=0$

$\cos\theta=\frac{-P}{Q}\ \therefore\ \theta=\cos^{-1}\left(\frac{-P}{Q}\right)$

30. (a) According to problem $P+Q=3$ and $P-Q=1$

By solving we get $P=2$ and $Q=1\ \therefore\ \frac{P}{Q}=2\Rightarrow P=2Q$

31. (c)

32. (c)

33. (c)

34. (d) $F_1+F_2+F_3=0\Rightarrow 4\hat{i}+6\hat{j}+F_3=0$

$\therefore \vec{F_3}=-4\hat{i}-6\hat{j}$

35. (a) $\Delta v=2v\sin\left(\frac{\theta}{2}\right)=2\times v\times\sin 90°$

$=2\times 100=200\ km/hr$

36. (c)

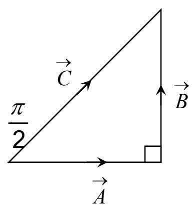
37. (d) Resultant velocity $=\sqrt{20^2+15^2}$

$=\sqrt{400+225}=\sqrt{625}=25\ km/hr$

38. (a) $C=\sqrt{A^2+B^2}$

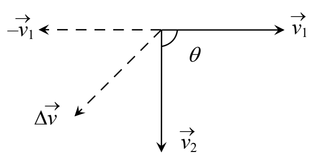
$=\sqrt{3^2+4^2}=5$

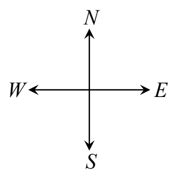
$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$



39. (c)

40. (d)





If the magnitude of vector remains same, only direction change by θ then

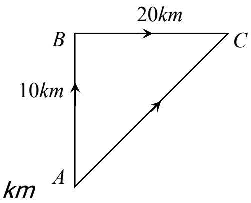
$\overrightarrow{\Delta v}=\overrightarrow{v_2}-\overrightarrow{v_1}$, $\overrightarrow{\Delta v}=\overrightarrow{v_2}+(-\overrightarrow{v_1})$

Magnitude of change in vector $|\overrightarrow{\Delta v}|=2v\sin\left(\frac{\theta}{2}\right)$

$|\overrightarrow{\Delta v}|=2\times 10\times\sin\left(\frac{90^\circ}{2}\right)=10\sqrt{2}=14.14\ m/s$

Direction is south-west as shown in figure.

41. (a) $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$



$AC=\sqrt{(AB)^2+(BC)^2}$

$=\sqrt{(10)^2+(20)^2}$

$=\sqrt{100+400}=\sqrt{500}=22.36\ km$

42. (b) $\cos\theta=\frac{\vec{F_1}.\vec{F_2}}{|F_1||F_2|}$

$=\frac{(5\hat{i}+10\hat{j}-20\hat{k}).(10\hat{i}-5\hat{j}-15\hat{k})}{\sqrt{25+100+400}\sqrt{100+25+225}}=\frac{50-50+300}{\sqrt{525}\sqrt{350}}$

$\Rightarrow\cos\theta=\frac{1}{\sqrt{2}}\ \therefore\ \theta=45^\circ$

43. (d) If two vectors A and B are given then Range of their resultant can be written as $(A-B)\le R\le(A+B)$.

i.e. $R_{max}=A+B$ and $R_{min}=A-B$

If B = 1 and A = 4 then their resultant will lies in between $3N$ and $5N$. It can never be $2N$.

44. (d) $A=3N$, $B=2N$ then $R=\sqrt{A^2+B^2+2AB\cos\theta}$

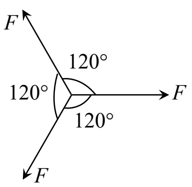
$R=\sqrt{9+4+12\cos\theta}$ ...(i)

Now $A=6N$, $B=2N$ then

$2R=\sqrt{36+4+24\cos\theta}$ ...(ii)

from (i) and (ii) we get $\cos\theta=-\frac{1}{2}\ \therefore\ \theta=120^\circ$

45. (a) In N forces of equal magnitude works on a single point and their resultant is zero then angle between any two forces is given $\theta=\frac{360}{N}=\frac{360}{3}=120^\circ$



If these three vectors are represented by three sides of triangle then they form equilateral triangle

46. (c) Resultant of two vectors $\vec{A}$ and $\vec{B}$ can be given by $\vec{R}=\vec{A}+\vec{B}$

$|\vec{R}|=|\vec{A}+\vec{B}|=\sqrt{A^2+B^2+2AB\cos\theta}$

If $\theta=0^\circ$ then $|\vec{R}|=A+B=|\vec{A}|+|\vec{B}|$

47. (d) $R_{max}=A+B=17$ when $\theta=0^\circ$

$R_{min}=A-B=7$ when $\theta=180^\circ$

by solving we get $A=12$ and $B=5$

Now when $\theta=90^\circ$ then $R=\sqrt{A^2+B^2}$

$\Rightarrow R=\sqrt{(12)^2+(5)^2}=\sqrt{169}=13$

48. (a) If two vectors are perpendicular then their dot product must be equal to zero. According to problem

$(\vec{A}+\vec{B}).(\vec{A}-\vec{B})=0\Rightarrow\vec{A}.\vec{A}-\vec{A}.\vec{B}+\vec{B}.\vec{A}-\vec{B}.\vec{B}=0$

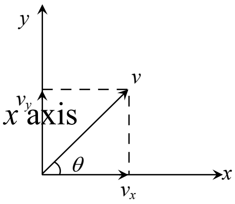
$\Rightarrow A^2-B^2=0\Rightarrow A^2=B^2$

$\therefore\ A=B$ *i.e.* two vectors are equal to each other in magnitude.

49. (a) $v_y=20$ and $v_x=10$

$\therefore$ velocity $\vec{v}=10\hat{i}+20\hat{j}$

direction of velocity with x axis



$\tan\theta=\frac{v_y}{v_x}=\frac{20}{10}=2$

$\therefore\ \theta=\tan^{-1}(2)$

50. (c) $R_{max}=A+B$ when $\theta=0^\circ\ \therefore\ R_{max}=12+8=20N$

51. (c) $R=\sqrt{A^2+B^2+2AB\cos\theta}$

If $A=B=P$ and $\theta=120^\circ$ then $R=P$

52. (a) Sum of the vectors $\vec{R}=5\hat{i}+8\hat{j}+2\hat{i}+7\hat{j}$ $=7\hat{i}+15\hat{j}$

magnitude of $\vec{R}=|\vec{R}|=\sqrt{49+225}=\sqrt{274}$

53. (d)

Multiplication of Vectors

1. (c) Given vectors can be rewritten as $\vec{A}=2\hat{i}+3\hat{j}+8\hat{k}$ and $\vec{B}=-4\hat{i}+4\hat{j}+\alpha\hat{k}$

Dot product of these vectors should be equal to zero because they are perpendicular.

$\therefore \quad \vec{A}.\vec{B} = -8 + 12 + 8\alpha = 0 \quad \Rightarrow \quad 8\alpha = -4 \quad \Rightarrow$

$\alpha = -1/2$

2. (b) Let $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} + \lambda\hat{k}$

$\vec{A}$ and $\vec{B}$ are parallel to each other

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ *i.e.* $\frac{2}{-4} = \frac{3}{-6} = \frac{-1}{\lambda} \Rightarrow \lambda = 2$.

3. (d) $W = \vec{F}.\vec{S} = FS\cos\theta$

$= 50 \times 10 \times \cos 60° = 50 \times 10 \times \frac{1}{2} = 250\ J$.

4. (a) $S = \vec{r_2} - \vec{r_1}$

$W = \vec{F}.\vec{S} = (4\hat{i} + \hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} + 15\hat{k})$

$= (4 \times 11 + 1 \times 11 + 3 \times 15) = 100\ J$.

5. (a) $(\vec{A} + \vec{B})$ is perpendicular to $(\vec{A} - \vec{B})$. Thus

$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$

or $A^2 + \vec{B}.\vec{A} - \vec{A}.\vec{B} - B^2 = 0$

Because of commutative property of dot product $\vec{A}\vec{B} = \vec{B}\vec{A}$

$\therefore A^2 - B^2 = 0$ or $A = B$

Thus the ratio of magnitudes *A*/*B* = 1

6. (b) Let $\vec{A}.(\vec{B} \times \vec{A}) = \vec{A}.\vec{C}$

Here $\vec{C} = \vec{B} \times \vec{A}$ Which is perpendicular to both vector

$\vec{A}$ and $\vec{B}$ $\quad \therefore \vec{A}.\vec{C} = 0$

7. (c) We know that $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$ because the angle between these two is always 90°.

But if the angle between $\vec{A}$ and $\vec{B}$ is 0 or π. Then $\vec{A} \times \vec{B} = \vec{B} \times \vec{A} = 0$.

8. (b) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= (1 \times 4 - 2 \times -2)\hat{i} + (2 \times 2 - 4 \times 3)\hat{j} + (3 \times -2 - 1 \times 2)\hat{k}$

$= 8\hat{i} - 8\hat{j} - 8\hat{k}$

$\therefore$ Magnitude of $\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| = \sqrt{(8)^2 + (-8)^2 + (-8)^2}$

$= 8\sqrt{3}$

9. (b) $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$= [(2 \times 4) - (3 \times -3)]\hat{i} + [(2 \times 3) - (3 \times 4)]\hat{j}$

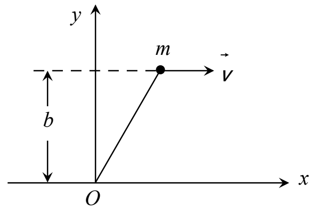
$+ [(3 \times -3) - (2 \times 2)]\hat{k} = 17\hat{i} - 6\hat{j} - 13\hat{k}$

10. (d) From the property of vector product, we notice that $\vec{C}$ must be perpendicular to the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ and $(\vec{A} + \vec{B})$ vector also, must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ must be perpendicular to $(\vec{A} + \vec{B})$ also but the cross product $(\vec{A} \times \vec{B})$ gives a vector $\vec{C}$ which can not be perpendicular to itself. Thus the last statement is wrong.

11. (b) We know that, Angular momentum

$\vec{L} = \vec{r} \times \vec{p}$ in terms of component becomes

$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$



As motion is in *x*-*y* plane (*z* = 0 and $P_z = 0$),

so $\vec{L} = \vec{k}(xp_y - yp_x)$

Here *x* = *vt*, *y* = *b*, $p_x = mv$ and $p_y = 0$

$\therefore \vec{L} = \vec{k}[vt \times 0 - bmv] = -mvb\hat{k}$

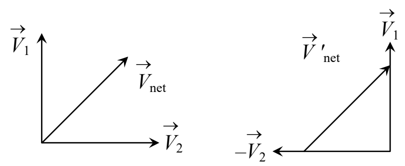
12. (d) $\vec{F_1}.\vec{F_2} = (2\hat{j} + 5\hat{k})(3\hat{j} + 4\hat{k})$

$= 6 + 20 = 20 + 6 = 26$

13. (c) Force *F* lie in the *x*-*y* plane so a vector along *z*-axis will be perpendicular to *F*.

14. (d) $\vec{A}\vec{B} = |\vec{A}|.|\vec{B}|.\cos\theta = \vec{A}.\vec{B}.\cos 90° = 0$

15. (c)



According to problem $|\vec{V_1} + \vec{V_2}| = |\vec{V_1} - \vec{V_2}|$

$\Rightarrow |\vec{V}_{net}| = |\vec{V}_{net}|$

So V_1 and V_2 will be mutually perpendicular.

16. (c) $W = \vec{F}.\vec{r} = (5\hat{i} + 3\hat{j})(2\hat{i} - \hat{j}) = 10 - 3 = 7\ J$.

17. (b) $\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{-2 + 6 - 4}{\sqrt{14}\sqrt{21}} = 0 \ \therefore \ \theta = 90°$

18. (c) $(\hat{i}+\hat{j}).(\hat{j}+\hat{k}) = 0+0+1+0 = 1$

$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{1}{\sqrt{2}\times\sqrt{2}} = \frac{1}{2} \therefore \theta = 60°$

19. (b) $P = \vec{F}.\vec{v} = 20\times 6 + 15\times(-4) + (-5)\times 3$

$= 120 - 60 - 15 = 120 - 75 = 45\, J/s$

20. (a) $\cos\theta = \frac{\vec{P}.\vec{Q}}{PQ} = 1 \therefore \theta = 0°$

21. (a) $W = \vec{F}.\vec{s} = (5\hat{i}+6\hat{j}+4\hat{k})(6\hat{i}-5\hat{k}) = 30 - 20 = 10\, J$

22. (c) $\vec{A}.\vec{B} = 0 \therefore \theta = 90°$

23. (a) $\vec{P}.\vec{Q} = 0 \therefore a^2 - 2a - 3 = 0 \Rightarrow a = 3$

24. (b) $W = \vec{F}.\vec{r} = (-2\hat{i}+15\hat{j}+6\hat{k})(10\hat{j}) = 150$

25. (c) $P_x = 2\cos t,\ P_y = 2\sin t \therefore \vec{P} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

$\vec{F} = \frac{d\vec{P}}{dt} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$

$\vec{F}.\vec{P} = 0 \therefore \theta = 90°$

26. (d) $|\vec{A}\times\vec{B}| = |(2\hat{i}+3\hat{j})\times(\hat{i}+4\hat{j})| = |5\hat{k}| = 5\ units$

27. (d)

28. (b) $\vec{A}\times\vec{B} = 0 \therefore \sin\theta = 0 \therefore \theta = 0°$

Two vectors will be parallel to each other.

29. (b) $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ are parallel and opposite to each other. So the angle will be π.

30. (b) Vector $(\vec{P}+\vec{Q})$ lies in a plane and vector $(\vec{P}\times\vec{Q})$ is perpendicular to this plane *i.e.* the angle between given vectors is $\frac{\pi}{2}$.

31. (d) $\sqrt{2^2+3^2+2\times 2\times 3\times\cos\theta} = 1$

By solving we get $\theta = 180° \therefore \vec{A}\times\vec{B} = 0$

32. (c) Dot product of two perpendicular vector will be zero.

33. (d) $\cos\theta = \frac{\vec{A}\vec{B}}{AB} = \frac{42+24-12}{\sqrt{36+36+9}\sqrt{49+16+16}} = \frac{56}{9\sqrt{71}}$

$\cos\theta = \frac{56}{9\sqrt{71}} \therefore \sin\theta = \frac{\sqrt{5}}{3}$ or $\theta = \sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$

34. (b) Direction of vector A is along z-axis $\therefore \vec{A} = a\hat{k}$

Direction of vector B is towards north $\therefore \vec{B} = b\hat{j}$

Now $\vec{A}\times\vec{B} = a\hat{k}\times b\hat{j} = ab(-\hat{j})$

$\therefore$ The direction is $\vec{A}\times\vec{B}$ is along west.

35. (d) $\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2} \therefore \theta = 60°$

36. (d) $\overrightarrow{AB} = (4\hat{i}+5\hat{j}+6\hat{k}) - (3\hat{i}+4\hat{j}+5\hat{k}) = \hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{CD} = (4\hat{i}+6\hat{j}) - (7\hat{i}+9\hat{j}+3\hat{k}) = -3\hat{i}-3\hat{j}-3\hat{k}$

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel, because its cross-products is 0.

37. (a) $W = \vec{F}\,\vec{S} = (4\hat{i}+5\hat{j})(3\hat{i}+6\hat{j}) = 12$

38. (b) $|\vec{A}\times\vec{B}| = \vec{A}.\vec{B} \Rightarrow AB\sin\theta = AB\cos\theta \Rightarrow \tan\theta = 1$

$\therefore \theta = 45°$

39. (a)

40. (a) $\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 0 & 4 & -3\end{vmatrix} = \hat{i}(6-8) - \hat{j}(-3) + 4\hat{k}$

$-2\vec{i} + 3\vec{j} + 4\vec{k}$

$|\vec{v}| = \sqrt{(-2)^2+(3)^2+4^2} = \sqrt{29}\ unit$

41. (d) $\vec{a}.\vec{b} = 0$ *i.e.* $\vec{a}$ and $\vec{b}$ will be perpendicular to each other

$\vec{a}.\vec{c} = 0$ *i.e.* $\vec{a}$ and $\vec{c}$ will be perpendicular to each other

$\vec{b}\times\vec{c}$ will be a vector perpendicular to both $\vec{b}$ and $\vec{c}$

So $\vec{a}$ is parallel to $\vec{b}\times\vec{c}$

42. (d) Area $= \left|2\hat{i}\times 2\hat{j}\right| = \left|4\hat{k}\right| = 4\ unit.$

43. (c) $\vec{A} = 2\hat{i}+2\hat{j}-\hat{k}$ and $\vec{B} = 6\hat{i}-3\hat{j}+2\hat{k}$

$\vec{C} = \vec{A}\times\vec{B} = (2\hat{i}+2\hat{j}-\hat{k})\times(6\hat{i}-3\hat{j}+2\hat{k})$

$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & -1\\ 6 & -3 & 2\end{vmatrix} = \hat{i} - 10\hat{j} - 18\hat{k}$

Unit vector perpendicular to both $\vec{A}$ and $\vec{B}$

$= \frac{\hat{i}-10\hat{j}-18\hat{k}}{\sqrt{1^2+10^2+18^2}} = \frac{\hat{i}-10\hat{j}-18\hat{k}}{5\sqrt{17}}$

44. (b) $\vec{A} = \hat{j}+3\hat{k},\ \vec{B} = \hat{i}+2\hat{j}-\hat{k}$

$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 3\\ 1 & 2 & -1\end{vmatrix} = -7\hat{i}+3\hat{j}-\hat{k}$

Hence area $= |\vec{C}| = \sqrt{49+9+1} = \sqrt{59}\ sq\,unit$

45. (a) $\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -\hat{j} - 2\hat{k}$

i.e. the angular momentum is perpendicular to *x*-axis.

46. (a) $\vec{A} \times \vec{B}$ is a vector perpendicular to plane $\vec{A} + \vec{B}$ and hence perpendicular to $\vec{A} + \vec{B}$.

47. (a) $\vec{\tau} = \vec{r} \times \vec{F} = (7\hat{i} + 3\hat{j} + \hat{k})(-3\hat{i} + \hat{j} + 5\hat{k})$

$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = 14\hat{i} - 38\hat{j} + 16\hat{k}$

48. (d) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B}) = \vec{A} \times \vec{A} - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} - \vec{B} \times \vec{B}$

$= 0 - \vec{A} \times \vec{B} + \vec{B} \times \vec{A} - 0 = \vec{B} \times \vec{A} + \vec{B} \times \vec{A} = 2(\vec{B} \times \vec{A})$

49. (d) For perpendicular vector $\vec{A}.\vec{B} = 0$

$\Rightarrow (5\hat{i} + 7\hat{j} - 3\hat{k}).(2\hat{i} + 2\hat{j} - a\hat{k}) = 0$

$\Rightarrow 10 + 14 + 3a = 0 \Rightarrow a = -8$

50. (a) $\text{Mass} = \dfrac{\text{Force}}{\text{Acceleration}} = \dfrac{|\vec{F}|}{a}$

$= \dfrac{\sqrt{36 + 64 + 100}}{1} = 10\sqrt{2}\ kg$

51. (a) Area of parallelogram $= \vec{A} \times \vec{B}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = (8)\hat{i} + (8)\hat{j} - (8)\hat{k}$

Magnitude $= \sqrt{64 + 64 + 64} = 8\sqrt{3}$

52. (b) Radius vector $\vec{r} = \vec{r_2} - \vec{r_1} = (2\hat{i} - 3\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} + \hat{k})$

$\therefore \vec{r} = -4\hat{j}$

Linear momentum $\vec{p} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{L} = \vec{r} \times \vec{p} = (-4\hat{j}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & 0 \\ 2 & 3 & -1 \end{vmatrix} = 4\hat{i} - 8\hat{k}$

53. (d) $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

54. (a)

55. (c) $\vec{A}.\vec{B} = AB\cos\theta$

In the problem $\vec{A}.\vec{B} = -AB$ i.e. $\cos\theta = -1 \therefore \theta = 180°$

i.e. $\vec{A}$ and $\vec{B}$ acts in the opposite direction.

56. (d) $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A}.\vec{B})$

$AB\sin\theta = \sqrt{3}AB\cos\theta \Rightarrow \tan\theta = \sqrt{3} \therefore \theta = 60°$

Now $|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{A^2 + B^2 + 2AB\left(\frac{1}{2}\right)} = (A^2 + B^2 + AB)^{1/2}$

57. (a) $W = \vec{F}.\vec{s} = (3\hat{i} + c\hat{j} + 2\hat{k}).(-4\hat{i} + 2\hat{j} - 3\hat{k}) = -12 + 2c - 6$

Work done $= 6J$ (given)

$\therefore -12 + 2c - 6 = 6 \Rightarrow c = 12$

58. (b) $W = \vec{F}.\vec{s} = (5\hat{i} + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = 7\ J$

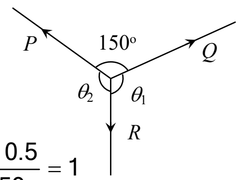
59. (c) $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

for parallel vectors $\theta = 0°$ or $180°$, $\sin\theta = 0$

$\therefore \vec{A} \times \vec{B} = \hat{0}$

Lami's Theorem

1. (c) $\dfrac{P}{\sin\theta_1} = \dfrac{Q}{\sin\theta_2} = \dfrac{R}{\sin 150°}$



$\Rightarrow \dfrac{1.93}{\sin\theta_1} = \dfrac{R}{\sin 150°}$

$\Rightarrow R = \dfrac{1.93 \times \sin 150°}{\sin\theta_1} = \dfrac{1.93 \times 0.5}{0.9659} = 1$

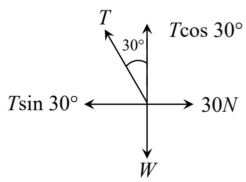
2. (a) According to Lami's theorem

$\dfrac{P}{\sin\alpha} = \dfrac{Q}{\sin\beta} = \dfrac{R}{\sin\gamma}$

3. (b)

4. (c)

5. (b)



From the figure $T\sin 30^o = 30$ …(i)

$T\cos 30^o = W$ …(ii)

By solving equation (i) and (ii) we get

$W = 30\sqrt{3}N$ and $T = 60N$

Relative Velocity

1. (b) The two car (say *A* and *B*) are moving with same velocity, the relative velocity of one (say B) with respect to the other $A, \vec{v}_{BA} = \vec{v}_B - \vec{v}_A = v - v = 0$

So the relative separation between them (= 5 *km*) always remains the same.

Now if the velocity of car (say *C*) moving in opposite direction to *A* and *B*, is $\vec{v}_C$ relative to ground then the velocity of car *C* relative to *A* and B will be $\vec{v}_{rel.} = \vec{v}_C - \vec{v}$

But as $\vec{v}$ is opposite to v_C

So $v_{rel} = v_c - (-30) = (v_C + 30)\, km/hr.$

So, the time taken by it to cross the cars *A* and *B* $t = \frac{d}{v_{rel}} \Rightarrow \frac{4}{60} = \frac{5}{v_C + 30}$

$\Rightarrow v_C = 45\, km/hr.$

2. (b) When the man is at rest *w.r.t.* the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground.

Here $\vec{v}_{rg}$ = velocity of rain with respect to the ground

$\vec{v}_{mg}$ = velocity of the man with respect to the ground.

and $\vec{v}_{rm}$ = velocity of the rain with respect to the man,

We have $\vec{v}_{rg} = \vec{v}_{rm} + \vec{v}_{mg}$ ......(i)

Taking horizontal components equation (i) gives

$v_{rg} \sin 30° = v_{mg} = 10\, km/hr$

or $v_{rg} = \frac{10}{\sin 30°} = 20\, km/hr$

3. (c) Taking vertical components equation (i) gives $v_{rg} \cos 30° = v_{rm} = 20\frac{\sqrt{3}}{2} = 10\sqrt{3}\, km/hr$

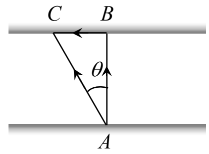
4. (c) Relative velocity = (3*i* + 4*j*) – (– 3*i* – 4*j*) = 6*i* + 8*j*

5. (d) Relative velocity of parrot with respect to train

$= 5 - (-10) = 5 + 10 = 15\, m/sec$

Time taken by the parrot $= \frac{d}{v_{rel.}} = \frac{150}{15} = 10\, sec.$

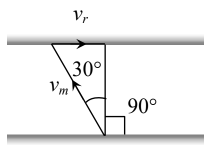
6. (a)



For shortest time, swimmer should swim along *AB*, so he will reach at point *C* due to the velocity of river.

i.e. he should swim due north.

7. (c)



$\sin 30^o = \frac{v_r}{v_m} = \frac{1}{2} \Rightarrow v_r = \frac{v_m}{2} = \frac{0.5}{2} = 0.25\, m/s$

8. (c) $\vec{v}_B + \vec{v}_A = \vec{v}_B + \vec{v}_A = 80 + 65 = 145\, km/hr$

9. (d) Relative speed of police with respect to thief

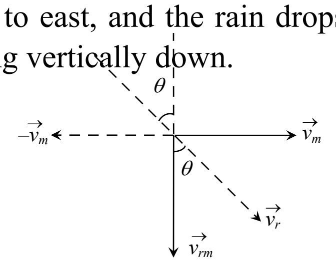
$= 10 - 9 = 1\, m/s$

Instantaneous separation = 100 *m*

Time = $\frac{\text{distance}}{\text{veclotiy}} = \frac{100}{1} = 100\, sec.$

10. (a,c)

11. (b) A man is sitting in a bus and travelling from west to east, and the rain drops are appears falling vertically down.



v_m = velocity of man

v_r = Actual velocity of rain which is falling at an angle θ with vertical

v_{rm} = velocity of rain *w.r.t.* to moving man

If the another man observe the rain then he will find that actually rain falling with velocity v_r at an angle going from west to east.

12. (b) Boat covers distance of 16*km* in a still water in 2 hours.

i.e. $v_B = \frac{16}{2} = 8\,km/hr$

Now velocity of water $\Rightarrow v_w = 4\,km/hr$.

Time taken for going upstream

$t_1 = \frac{8}{v_B - v_w} = \frac{8}{8-4} = 2hr$

(As water current oppose the motion of boat)

Time taken for going down stream

$t_2 = \frac{8}{v_B + v_w} = \frac{8}{8+4} = \frac{8}{12}hr$

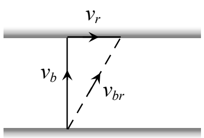
(As water current helps the motion of boat)

$\therefore$ Total time $= t_1 + t_2 = \left(2 + \frac{8}{12}\right)hr$ or 2*hr* 40*min*

13. (d) Relative velocity $= 10 + 5 = 15\,m/s$.

Time taken by the bird to cross the train $= \frac{120}{15} = 8\sec$

14. (b) $\vec{v}_{br} = \vec{v}_b + \vec{v}_r$



$\Rightarrow v_{br} = \sqrt{v_b^2 + v_r^2}$

$\Rightarrow 10 = \sqrt{8^2 + v_r^2}$

$\Rightarrow v_r = 6\,km/hr$.

Critical Thinking Questions

1. (c) $\sin^2\alpha + \sin^2\beta + \sin\gamma$

$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 3 - 1 = 2$

2. (c) If vectors are of equal magnitude then two vectors can give zero resultant, if they works in opposite direction. But if the vectors are of different magnitudes then minimum three vectors are required to give zero resultant.

3. (c)

4. (c) Let *P* be the smaller force and *Q* be the greater force then according to problem –

$P + Q = 18$ ......(i)

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12$ ....(ii)

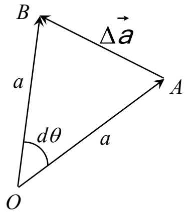
$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90 = \infty$

$\therefore P + Q\cos\theta = 0$ ....(iii)

By solving (i), (ii) and (iii) we will get $P = 5$, and $Q = 13$

5. (b) From the figure $|\overrightarrow{OA}| = a$ and $|\overrightarrow{OB}| = a$

Also from triangle rule $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \Delta\vec{a}$



$\Rightarrow |\Delta\vec{a}| = AB$

Using angle $= \frac{\text{arc}}{\text{radius}}$

$\Rightarrow AB = a\,.\,d\theta$

So $|\Delta\vec{a}| = a d\theta$

Δa means change in magnitude of vector *i.e.* $|\overrightarrow{OB}| - |\overrightarrow{OA}| \Rightarrow a - a = 0$

So $\Delta a = 0$

6. (b) $R_{net} = R + \sqrt{R^2 + R^2} = R + \sqrt{2}R = R(\sqrt{2} + 1)$

7. (d)

8. (d) $\Delta v = 2v\sin\left(\frac{90°}{2}\right) = 2v\sin 45° = 2v \times \frac{1}{\sqrt{2}} = \sqrt{2}v$

$= \sqrt{2} \times r\omega = \sqrt{2} \times 1 \times \frac{2\pi}{60} = \frac{\sqrt{2}\pi}{`30}\,cm/s$

9. (b) $\Delta v = 2v\sin\left(\frac{\theta}{2}\right) = 2 \times 5 \times \sin 45° = \frac{10}{\sqrt{2}}$

$\therefore a = \frac{\Delta v}{\Delta t} = \frac{10/\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\,m/s^2$

10. (c) For motion of the particle from (0, 0) to (*a*, 0)

$\vec{F} = -K(0\hat{i} + a\hat{j}) \Rightarrow \vec{F} = -Ka\hat{j}$

Displacement $\vec{r} = (a\hat{i} + 0\hat{j}) - (0\hat{i} + 0\hat{j}) = a\hat{i}$

So work done from (0, 0) to (*a*, 0) is given by

$W = \vec{F}.\vec{r} = -Ka\hat{j}.a\hat{i} = 0$

For motion (*a*, 0) to (*a*, *a*)

$\vec{F} = -K(a\hat{i} + a\hat{j})$ and displacement

$\vec{r} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j}) = a\hat{j}$

So work done from (*a*, 0) to (*a*, *a*) $W = \vec{F}.\vec{r}$

$= -K(a\hat{i} + a\hat{j}).a\hat{j} = -Ka^2$

So total work done $= -Ka^2$

11. (a) Given $\overrightarrow{OA} = \vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\overrightarrow{OB} = \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$$\therefore (\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

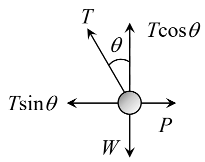
$= (12-2)\hat{i} + (4+6)\hat{j} + (3+12)\hat{k}$

$= 10\hat{i} + 10\hat{j} + 15\hat{k} \Rightarrow |\vec{a} \times \vec{b}| = \sqrt{10^2 + 10^2 + 15^2}$

$= \sqrt{425} \quad = 5\sqrt{17}$

Area of $\Delta OAB = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{5\sqrt{17}}{2}$ *sq.unit.*

12. (d)



As the metal sphere is in equilibrium under the effect of three forces therefore $\vec{T} + \vec{P} + \vec{W} = 0$

From the figure $T\cos\theta = W$ …(i)

$T\sin\theta = P$ …(ii)

From equation (i) and (ii) we get $P = W\tan\theta$

and $T^2 = P^2 + W^2$

13. (b)

14. (d)

Assertion and Reason

1 (a) Cross product of two vectors is perpendicular to the plane containing both the vectors.

2 (a) $\cos\theta = \frac{(\hat{i}+\hat{j}).(\hat{i})}{|\hat{i}+\hat{j}||\hat{i}|} = \frac{1}{\sqrt{2}}$. Hence $\theta = 45°$.

3 (d) $\frac{\vec{A} \times \vec{B}}{\vec{A}.\vec{B}} = \frac{AB\sin\theta\,\hat{n}}{AB\cos\theta} = \tan\theta\,\hat{n}$

where $\hat{n}$ is unit vector perpendicular to both $\vec{A}$ and $\vec{B}$.

However $\frac{|\vec{A} \times \vec{B}|}{\vec{A}.\vec{B}} = \tan\theta$

4 (b) $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 + 2AB\cos\theta$

Hence $\cos\theta = 0$ which gives $\theta = 90°$

Also vector addition is commutative.

Hence $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

5 (c) $\vec{v} = \vec{\omega} \times \vec{r}$

The expression $\vec{\omega} = \vec{v} \times \vec{r}$ is wrong.

6 (b) For giving a zero resultant, it should be possible to represent the given vectors along the sides of a closed polygon and minimum number of sides of a polygon is three.

7 (a) Since velocities are in opposite direction, therefore $v_{AB} = |\vec{v}_A - \vec{v}_B| = v_A + v_B$.

Which is greater than v_A or v_B

8 (b) Vector addition of two vectors is commutative *i.e.* $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

9 (a)

10 (c) Cross-product of two vectors is anticommutative.

i.e. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

11 (b)

12 (e) If a vector quantity has zero magnitude then it is called a null vector. That quantity may have some direction even if its magnitude is zero.

13 (a) Let $\vec{P}$ and $\vec{Q}$ are two vectors in opposite direction, then their sum $\vec{P} + (-\vec{Q}) = \vec{P} - \vec{Q}$

If $\vec{P} = \vec{Q}$ then sum equal to zero.

14 (c) If two vectors are in opposite direction, then they cannot be like vectors.

15 (a) If θ be the angle between two vectors $\vec{A}$ and $\vec{B}$, then their scalar product, $\vec{A}.\vec{B} = AB\cos\theta$

If $\theta = 90°$ then $\vec{A}.\vec{B} = 0$

i.e. if $\vec{A}$ and $\vec{B}$ are perpendicular to each other then their scalar product will be zero.

16 (b) We can multiply any vector by any scalar.

For example, in equation $\vec{F} = m\vec{a}$ mass is a scalar quantity, but acceleration is a vector quantity.

17 (c) If two vectors equal in magnitude are in opposite direction, then their sum will be a null vector.

A null vector has direction which is intermediate (or depends on direction of initial vectors) even its magnitude is zero.

18 (b) $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta = 0$

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = 0$

If $\vec{A}$ and $\vec{B}$ are not null vectors then it follows that $\sin\theta$ and $\cos\theta$ both should be zero simultaneously. But it cannot be possible so it is essential that one of the vector must be null vector.

19 (b)

20 (c) The resultant of two vectors of unequal magnitude given by $R=\sqrt{A^2+B^2+2AB\cos\theta}$ cannot be zero for any value of θ.

21 (a) $\vec{A}.\vec{B}=\vec{B}.\vec{C} \Rightarrow AB\cos\theta_1 = BC\cos\theta_2$

$\therefore A = C$, only when $\theta_1 = \theta_2$

So when angle between $\vec{A}$ and $\vec{B}$ is equal to angle between $\vec{B}$ and $\vec{C}$ only then $\vec{A}$ equal to $\vec{C}$

22 (c) Since vector addition is commutative, therefore $\vec{A}+\vec{B}=\vec{B}+\vec{A}$.