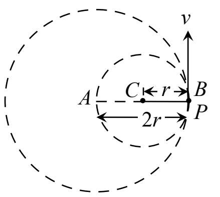
Answers and Solutions

Uniform Circular Motion

1. (c) $v = r\omega \Rightarrow \omega = \frac{v}{r}$ = constant [As v and r are constant]
2. (c) As time periods are equal therefore ratio of angular speeds will be same. $\omega = \frac{2\pi}{T}$
3. (b) $F = \frac{mv^2}{r} \Rightarrow F \propto v^2$. If v becomes double then F (tendency to overturn) will become four times.
4. (b) Work done by centripetal force is always zero.
5. (c) It is always directed in a direction of tangent to circle.
6. (c) Stone flies in the direction of instantaneous velocity due to inertia
7. (c) Centripetal acceleration $= \frac{v^2}{r} =$ constant. Direction keeps changing.
8. (c) Linear velocity, acceleration and force varies in direction.
9. (b) Angular velocity of particle P about point A,

$\omega_A = \frac{v}{r_{AB}} = \frac{v}{2r}$



Angular velocity of particle P about point C,

$\omega_C = \frac{v}{r_{BC}} = \frac{v}{r}$

Ratio $\frac{\omega_A}{\omega_C} = \frac{v/2r}{v/r} = \frac{1}{2}$.

10. (b)
11. (a) $F = \frac{mv^2}{r}$. If m and v are constants then $F \propto \frac{1}{r}$

$\therefore \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)$

12. (a) In uniform circular motion (constant angular velocity) kinetic energy remains constant but due to change in velocity of particle its momentum varies.
13. (c)
14. (a,c) Centripetal force $= \frac{mv^2}{r}$ and is directed always towards the centre of circle. Sense of rotation does not affect magnitude and direction of this centripetal force.
15. (a) When speed is constant in circular motion, it means work done by centripetal force is zero.
16. (d)
17. (a) This horizontal inward component provides required centripetal force.
18. (a) Thrust at the lowest point of concave bridge

$= mg + \frac{mv^2}{r}$

19. (d)
20. (a) Because the reaction on inner wheel decreases and becomes zero. So it leaves the ground first.
21. (b)
22. (a) $\frac{a_R}{a_r} = \frac{\omega_R^2 \times R}{\omega_r^2 \times r} = \frac{T_r^2}{T_R^2} \times \frac{R}{r} = \frac{R}{r}$ [As $T_r = T_R$]
23. (c) $\omega_{min} = \frac{2\pi}{60} \frac{Rad}{min}$ and $\omega_{hr} = \frac{2\pi}{12 \times 60} \frac{Rad}{min}$

$\therefore \frac{\omega_{min}}{\omega_{hr}} = \frac{2\pi/60}{2\pi/12 \times 60}$

24. (d) The particle performing circular motion flies off tangentially.
25. (a) The angle of banking, $\tan\theta = \frac{v^2}{rg}$

$\Rightarrow \tan 12° = \frac{(150)^2}{r \times 10} \Rightarrow r = 10.6 \times 10^3\ m = 10.6\ km$

26. (c) K.E. $= \frac{1}{2}mv^2$. Which is scalar, so it remains constant.
27. (b) $v = 72\ km/hour = 20\ m/\sec$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20 \times 20}{20 \times 10}\right) = \tan^{-1}(2)$

28. (a)
29. (d) $120\ rev/\min = 120 \times \frac{2\pi}{60}\ rad/\sec = 4\pi\ rad/\sec$

30. (c) In uniform circular motion, acceleration causes due to change in direction and is directed radially towards centre.

31. (b) Reaction on inner wheel $R_1 = \frac{1}{2}M\left[g - \frac{v^2 h}{ra}\right]$

Reaction on outer wheel $R_2 = \frac{1}{2}M\left[g + \frac{v^2 h}{ra}\right]$

where, r = radius of circular path, $2a$ = distance between two wheels and h = height of centre of gravity of car.

32. (d) Maximum tension $= m\omega^2 r = m \times 4\pi^2 \times n^2 \times r$

By substituting the values we get $T_{max} = 87.64\ N$

33. (d) $\frac{v^2}{rg} = \frac{h}{l} \Rightarrow v = \sqrt{\frac{rgh}{l}} = \sqrt{\frac{50 \times 1.5 \times 9.8}{10}} = 8.57\ m/s$

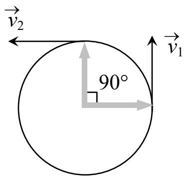
34. (b) $a = \omega^2 r = 4\pi^2 n^2 r = 4\pi^2 \times 1^2 \times 20 \times 10^3$

$\therefore$ a $= 8 \times 10^5\ m/sec^2$

35. (c)

36. (d) In 15 second's hand rotate through 90°.

Change in velocity $|\overrightarrow{\Delta v}| = 2v\sin(\theta/2)$



$= 2(r\omega)\sin(90°/2) = 2 \times 1 \times \frac{2\pi}{T} \times \frac{1}{\sqrt{2}}$

$= \frac{4\pi}{60\sqrt{2}} = \frac{\pi\sqrt{2}}{30}\frac{cm}{sec}$ [As T = 60 sec]

37. (c) Since $n = 2$, $\omega = 2\pi \times 2 = 4\pi\ rad/s^2$

So acceleration $= \omega^2 r = (4\pi)^2 \times \frac{25}{100} m/s^2 = 4\pi^2$

38. (b) $\omega^2 r = 4\pi^2 n^2 r = 4\pi^2\left(\frac{1200}{60}\right)^3 \times 30 = 4740\ m/s^2$

39. (a)

40. (c) Particles of cream are lighter so they get deposited near the centre of circular path.

41. (d) Radial force $= \frac{mv^2}{r} = \frac{m}{r}\left(\frac{p}{m}\right)^2 = \frac{p^2}{mr}$ [As $p = mv$]

42. (b) $\frac{mv^2}{r} \propto \frac{K}{r} \Rightarrow v \propto r^{\circ}$

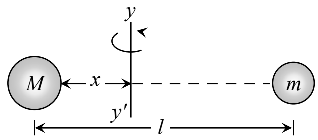
i.e. speed of the particle is independent of r.

43. (b) If the both mass are revolving about the axis yy' and tension in both the threads are equal then

$M\omega^2 x = m\omega^2(l - x)$

$\Rightarrow Mx = m(l - x)$

$\Rightarrow x = \frac{ml}{M + m}$



44. (b) $\tan\theta = \frac{v^2}{rg} = \frac{400}{20 \times 9.8} \Rightarrow \theta = 63.9°$

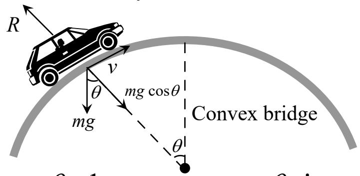
45. (d) In complete revolution change in velocity becomes zero so average acceleration will be zero.

46. (a) We know that $\tan\theta = \frac{v^2}{Rg}$ and $\tan\theta = \frac{h}{b}$

Hence $\frac{h}{b} = \frac{v^2}{Rg} \Rightarrow h = \frac{v^2 b}{Rg}$

47. (b)

48. (a) $R = mg\cos\theta - \frac{mv^2}{r}$



when θ decreases $\cos\theta$ increases *i.e.*, R increases.

49. (d) Tension in the string $T = m\omega^2 r = 4\pi^2 n^2 mr$

$\therefore\ T \propto n^2 \Rightarrow \frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow n_2 = 5\sqrt{\frac{2T}{T}} = 7\ rpm$

50. (b)

51. (a) $T = m\omega^2 r \Rightarrow 10 = 0.25 \times \omega^2 \times 0.1 \Rightarrow \omega = 20\ rad/s$

52. (c) $v = 36\frac{km}{h} = 10\frac{m}{s}\ \therefore\ F = \frac{mv^2}{r} = \frac{500 \times 100}{50} = 1000\ N.$

53. (a) $T = \frac{mv^2}{r} \Rightarrow 25 = \frac{0.25 \times v^2}{1.96} \Rightarrow v = 14\ m/s$

54. (b) Centripetal force $= mr\omega^2 = 5 \times 1 \times (2)^2 = 20\ N$

55. (a) $\frac{mv^2}{r} = \frac{k}{r^2} \Rightarrow mv^2 = \frac{k}{r}\ \therefore$ K.E.$= \frac{1}{2}mv^2 = \frac{k}{2r}$

P.E. $= \int F\,dr = \int \frac{k}{r^2}dr = -\frac{k}{r}$

$\therefore$ Total energy = K.E. + P.E. $= \frac{k}{2r} - \frac{k}{r} = -\frac{k}{2r}$

56. (d) Maximum tension $= \frac{mv^2}{r} = 16\ N$

$\Rightarrow \frac{16 \times v^2}{144} = 16 \Rightarrow v = 12\ m/s$

57. (a) The maximum velocity for a banked road with friction,

$v^2 = gr\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$

$\Rightarrow v^2 = 9.8 \times 1000 \times \left(\frac{0.5 + 1}{1 - 0.5 \times 1}\right) \Rightarrow v = 172\ m/s$

58. (d) $v = r\omega = \frac{r \times 2\pi}{T} = \frac{0.06 \times 2\pi}{60} = 6.28\ mm/s$

Magnitude of change in velocity $= |\vec{v}_2 - \vec{v}_1|$

$= \sqrt{v_1^2 + v_2^2} = 8.88\ mm/s$ (As $v_1 = v_2 = 6.28 mm/s$)

59. (a) Work done by centripetal force in uniform circular motion is always equal to zero.

60. (b) $v = r\omega = 20 \times 10\ cm/s = 2\ m/s$

61. (d) $v_{max} = \sqrt{\mu rg} = \sqrt{0.2 \times 100 \times 9.8} = 14 m/s$

62. (d) $F = mg - \frac{mv^2}{r}$

63. (a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} rad/s$

64. (b) $\omega = 2\pi n = \frac{2\pi \times 100}{60} = 10.47\ rad/s$

65. (d) Work done in circular motion is always zero.

66. (d) In complete revolution total displacement is zero so average velocity is zero

67. (c) $v_{max} = \sqrt{\mu rg} = \sqrt{0.75 \times 60 \times 9.8} = 21\ m/s$

68. (a) Distance covered in 'n' revolution = $n\ 2\pi r = n\pi D$

$\Rightarrow 2000\pi D = 9500$ [As n = 2000, distance = 9500 m]

$\Rightarrow D = \frac{9500}{2000 \times \pi} = 1.5\ m$

69. (c) Centripetal acceleration = $4\pi^2 n^2 r = 4\pi^2 \times (1) \times 0.4 = 1.6\pi^2$

70. (a)

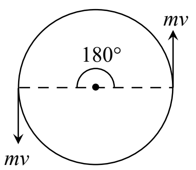
71. (b) Due to centrifugal force.

72. (d) As momentum is vector quantity

$\therefore$ change in momentum

$\Delta P = 2mv\sin(\theta/2)$

$= 2mv\sin(90) = 2mv$



But kinetic energy remains always constant so change in kinetic energy is zero.

73. (a) $\omega = \frac{v}{r} = \frac{100}{100} = 1\ rad/s$

74. (c) $\alpha = \frac{d\omega}{dt} = 0$ (As ω = constant)

75. (b) $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

76. (a) $a = 4\pi^2 n^2 r = 4\pi^2 \left(\frac{1}{2}\right)^2 \times 50 = 493\ cm/s^2$

77. (c) Maximum force of friction = centripetal force

$\frac{mv^2}{r} = \frac{100 \times (9)^2}{30} = 270\ N$

78. (a) $v = \sqrt{\mu rg} = \sqrt{0.4 \times 30 \times 9.8} = 10.84\ m/s$

79. (b) $v = r\omega = 0.5 \times 70 = 35\ m/s$

80. (a) $2\pi r = 34.3 \Rightarrow r = \frac{34.3}{2\pi}$ and $v = \frac{2\pi r}{T} = \frac{2\pi r}{\sqrt{22}}$

Angle of binding $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = 45°$

81. (c)

82. (a) $T = m\omega^2 r \Rightarrow$ $\omega \propto \sqrt{T}$ $\therefore$

$\frac{\omega_2}{\omega_1} = \sqrt{\frac{1}{4}} \Rightarrow \omega_2 = \frac{\omega_1}{2} = 5\ rpm$

83. (d) $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left[\frac{(14\sqrt{3})^2}{20\sqrt{3} \times 9.8}\right] = \tan^{-1}[\sqrt{3}] = 60°$

84. (c) Centripetal acceleration

$= 4\pi^2 n^2 r = 4\pi^2 \left(\frac{1}{2}\right)^2 \times 4 = 4\pi^2$

85. (b) Centripetal force = breaking force

$\Rightarrow m\omega^2 r =$ breaking stress × cross sectional area

$\Rightarrow m\omega^2 r = p \times A$ $\Rightarrow$

$\omega = \sqrt{\frac{p \times A}{mr}} = \sqrt{\frac{4.8 \times 10^7 \times 10^{-6}}{10 \times 0.3}}$

$\therefore\ \omega = 4\ rad/\sec$

86. (a) Because velocity is always tangential and centripetal acceleration is radial.

87. (c) T = tension, W = weight and F = centrifugal force.

88. (c) $\mu = \frac{v^2}{rg} = \frac{(4.9)^2}{4 \times 9.8} = 0.61$

89. (d) As body covers equal angle in equal time intervals. its angular velocity and hence magnitude of linear velocity is constant.

90. (b) $\omega = \frac{v}{r} = \frac{10}{100} = 0.1\, rad/s$

91. (a) $F = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{rF}{m}}$

92. (d) Electrostatic force provides necessary centripetal force for circular motion of electron.

93. (a) Acceleration $= \omega^2 r = \frac{v^2}{r} = \omega v = \frac{2\pi}{T} v$

94. (b) $v = \sqrt{\mu rg} = \sqrt{0.6 \times 150 \times 10} = 30\, m/s$

95. (b)

96. (c) $F = \frac{mv^2}{r} \Rightarrow F \propto v^2$ *i.e.* force will become 4 times.

97. (d) $v = \sqrt{\mu rg} = \sqrt{0.25 \times 40 \times 10} = 10\, m/s$

98. (d) Time period = 40 *sec*

No. of revolution $= \frac{\text{Total time}}{\text{Time period}}$

$= \frac{140\text{ sec}}{40\text{ sec}} = 3.5\, Rev.$

So, distance $= 3.5 \times 2\pi R = 3.5 \times 2\pi \times 10 = 220\, m$.

99. (a) $m 4\pi^2 n^2 r = 4 \times 10^{-13} \Rightarrow n = 0.08 \times 10^8\, cycles/\text{sec}$.

100. (b) Momentum changes by 2*mv* but kinetic energy remains same.

101. (c) $L = I\omega$. In U.C.M. ω=constant ∴ *L* = constant.

102. (c) $\because W = FS\cos\theta \therefore \theta = 90°$

103. (b)

104. (c) In uniform circular motion tangential acceleration remains zero but magnitude of radial acceleration remains constant.

105. (d) The inclination of person from vertical is given by,

$\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{50 \times 10} = \frac{1}{5} \therefore \theta = \tan^{-1}(1/5)$

106. (d) The centripetal force, $F = \frac{mv^2}{r} \Rightarrow r = \frac{mv^2}{F}$

$\therefore r \propto v^2$ or $v \propto \sqrt{r}$ (If *m* and *F* are constant),

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{1}{2}}$

107. (b) As the speed is constant throughout the circular motion therefore its average speed is equal to instantaneous speed.

108. (a) Linear velocity,

$v = \omega r = 2\pi n r = 2 \times 3.14 \times 3 \times 0.1 = 1.88\, m/s$

Acceleration, $a = \omega^2 r = (6\pi)^2 \times 0.1 = 35.5\, m/s^2$

Tension in string, $T = m\omega^2 r = 1 \times (6\pi)^2 \times 0.1 = 35.5\, N$

109. (a) $a = \frac{v^2}{r} = \frac{(400)^2}{160} = 10^3\, m/s^2 = 1\, km/s^2$

110. (b) $v_{max.} = \sqrt{\mu rg} = \sqrt{0.5 \times 40 \times 9.8} = 14\, m/s$

111. (b) $F = \frac{mv^2}{r} = \frac{500 \times 100}{50} = 10^3\, N$

112. (b) $F = m\left(\frac{4\pi^2}{T^2}\right)R$. If masses and time periods are same then $F \propto R \therefore F_1/F_2 = R_1/R_2$

113. (b) It is a vector quantity.

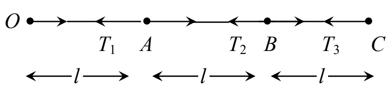
114. (a) $a = \frac{v^2}{r} = v\omega \Rightarrow a' = (2v)\left(\frac{\omega}{2}\right) = a$ *i.e.* remains constant.

115. (d) Tension in the string $T_0 = mR\omega_0^2$

In the second case $T = m(2R)(4\omega_0^2) = 8mR\omega_0^2 = 8T_0$

116. (b) Average velocity $= \frac{\text{Total displacement}}{\text{Total time}} = \frac{2\, m}{1\, s} = 2\, ms^{-1}$

117. (d) Let ω is the angular speed of revolution



$T_3 = m\omega^2 3l$

$T_2 - T_3 = m\omega^2 2l \Rightarrow T_2 = m\omega^2 5l$

$T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = m\omega^2 6l$

$T_3 : T_2 : T_1 = 3 : 5 : 6$

118. (b) $F = \frac{mv^2}{r}$. For same mass and same speed if radius is doubled then force should be halved.

119. (c) $a = \frac{v^2}{r} = \omega^2 r = 4\pi^2 n^2 r = 4\pi^2 \left(\frac{22}{44}\right)^2 \times 1$

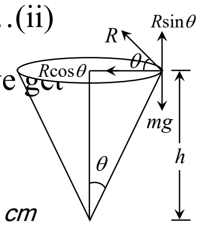
$= \pi^2 \, m/s^2$

and its direction is always along the radius and towards the centre.

120. (d) The particle is moving in circular path

From the figure, $mg = R\sin\theta$ ...(i)

$\frac{mv^2}{r} = R\cos\theta$ ...(ii)



From equation (i) and (ii) we get

$\tan\theta = \frac{rg}{v^2}$ but $\tan\theta = \frac{r}{h}$

$\therefore h = \frac{v^2}{g} = \frac{(0.5)^2}{10} = 0.025 m = 2.5 \, cm$

121. (a) Angular velocity $= \frac{2\pi}{T} = \frac{2\pi}{24} \, rad/hr$

$= \frac{2\pi}{86400} \, rad/s$

122. (d) $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.1047 \, rad/s$

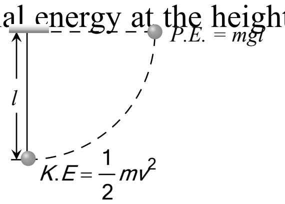
and $v = \omega r = 0.1047 \times 3 \times 10^{-2} = 0.00314 \, m/s$

Non-uniform Circular Motion

1. (d) Minimum speed at the highest point of vertical circular path $v = \sqrt{gR}$

2. (d) At highest point $\frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$

3. (d) Kinetic energy given to a sphere at lowest point = potential energy at the height of suspension



$\Rightarrow \frac{1}{2}mv^2 = mgl$

$\therefore v = \sqrt{2gl}$

4. (c) Due to less centrifugal force experienced by the bubbles.

5. (a) Critical velocity at highest point $= \sqrt{gR} = \sqrt{10 \times 1.6} = 4 m/s$

6. (c) Using relation $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta_1 = \frac{1}{2}(\alpha)(2)^2 = 2\alpha$ ...(i) (As $\omega_0 = 0, t = 2$ sec)

Now using same equation for $t = 4$ sec, $\omega_0 = 0$

$\theta_1 + \theta_2 = \frac{1}{2}\alpha(4)^2 = 8\alpha$ ...(ii)

From (i) and (ii), $\theta_1 = 2\alpha$ and $\theta_2 = 6\alpha$ $\therefore \frac{\theta_2}{\theta_1} = 3$

7. (a) $mg = 1 \times 10 = 10 N$, $\frac{mv^2}{r} = \frac{1 \times (4)^2}{1} = 16$

Tension at the top of circle $= \frac{mv^2}{r} - mg = 6 N$

Tension at the bottom of circle $= \frac{mv^2}{r} + mg = 26 N$

8. (d) For critical condition at the highest point $\omega = \sqrt{g/R}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{R/g} = 2 \times 3.14\sqrt{4/9.8} = 4$ sec.

9. (b) $mg = 20 N$ and $\frac{mv^2}{r} = \frac{2 \times (4)^2}{1} = 32 N$

It is clear that 52 N tension will be at the bottom of the circle. Because we know that $T_{Bottom} = mg + \frac{mv^2}{r}$

10. (b) $h = \frac{5}{2}R = \frac{5}{2}\left(\frac{D}{2}\right) = \frac{5D}{4}$

11. (b) Net acceleration in nonuniform circular motion,

$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left(\frac{900}{500}\right)^2} = 2.7 \, m/s^2$

a_t = tangential acceleration

a_c = centripetal acceleration $= \frac{v^2}{r}$

12. (b) $T = mg + \frac{mv^2}{l} = mg + 2mg = 3mg$

where $v = \sqrt{2gl}$ from $\frac{1}{2}mv^2 = mgl$

13. (a) $T_{max} = m\omega_{max}^2 r + mg \Rightarrow \frac{T_{max}}{m} = \omega^2 r + g$

$\Rightarrow \frac{30}{0.5} - 10 = \omega^2{}_{max} r \Rightarrow$

$\omega_{max} = \sqrt{\frac{50}{r}} = \sqrt{\frac{50}{2}} = 5 \, rad/s$

14. (b)

15. (b) Because here tension is maximum.

16. (a) Max. tension that string can bear = 3.7 *kgwt* = 37*N*

Tension at lowest point of vertical loop $= mg + m\omega^2 r$

$= 0.5 \times 10 + 0.5 \times \omega^2 \times 4 = 5 + 2\omega^2$

$\therefore 37 = 5 + 2\omega^2 \Rightarrow \omega = 4$ rad/s.

17. (c)

18. (c) $\omega = \frac{d\theta}{dt} = \frac{d}{dt}(2t^3 + 0.5) = 6t^2$

at t = 2 *s*, $\omega = 6 \times (2)^2 = 24\, rad/s$

19. (a) When body is released from the position *p* (inclined at angle θ from vertical) then velocity at mean position

$v = \sqrt{2gl(1-\cos\theta)}$

$\therefore$ Tension at the lowest point = $mg + \frac{mv^2}{l}$

$= mg + \frac{m}{l}[2gl(1-\cos 60)] = mg + mg = 2mg$

20. (a)

21. (c) Tension = Centrifugal force + weight $= \frac{mv^2}{r} + mg$

22. (a) $v_{min} = \sqrt{5\,gr} = 17.7\, m/\sec$

23. (d)

24. (c) $v = \sqrt{2gl(1-\cos\theta)} = \sqrt{2 \times 9.8 \times 2(1-\cos 60°)}$

$= 4.43\, m/s$

25. (b) Increment in angular velocity $\omega = 2\pi(n_2 - n_1)$

$\omega = 2\pi(1200 - 600)\frac{rad}{min} = \frac{2\pi \times 600}{60}\frac{rad}{s} = 20\pi\frac{rad}{s}$

26. (d) $\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.8}{0.2}} = 7\, rad/s$

27. (a)

28. (d) In non-uniform circular motion particle possess both centripetal as well as tangential acceleration.

29. (c)

30. (b) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2\, m/s$

31. (d) $T = mg + m\omega^2 r = m\{g + 4\pi^2 n^2 r\}$

$= m\left\{g + \left(4\pi^2\left(\frac{n}{60}\right)^2 r\right)\right\} = m\left\{g + \left(\frac{\pi^2 n^2 r}{900}\right)\right\}$

32. (d) $h = \frac{5}{2}r \Rightarrow r = \frac{2}{5} \times h = \frac{2}{5} \times 5 = 2\, metre$

33. (d) In the given condition friction provides the required centripetal force and that is constant. *i.e.* $m\omega^2 r$=constant

$\Rightarrow r \propto \frac{1}{\omega^2} \therefore r_2 = r_1\left(\frac{\omega_1}{\omega_2}\right)^2 = 9\left(\frac{1}{3}\right)^2 = 1\, cm$

34. (b) By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha(2\pi n) \Rightarrow \alpha = \frac{3}{4}\frac{\omega_0^2}{4\pi \times 36}$, ($n$ = 36) ..(i)

Now let fan completes total n' revolution from the starting to come to rest

$0 = \omega_0^2 - 2\alpha(2\pi n') \Rightarrow n' = \frac{\omega_0^2}{4\alpha\pi}$

substituting the value of α from equation (i)

$n' = \frac{\omega_0^2}{4\pi}\frac{4 \times 4\pi \times 36}{3\omega_0^2} = 48$ revolution

Number of rotation = 48 – 36 = 12

35. (b) $v = \sqrt{3gr}$ and $a = \frac{v^2}{r} = \frac{3gr}{r} = 3g$

36. (d) Tension at mean position, $mg + \frac{mv^2}{r} = 3mg$

$v = \sqrt{2gl}$ …(i)

and if the body displaces by angle θ with the vertical then $v = \sqrt{2gl(1-\cos\theta)}$ …(ii)

Comparing (i) and (ii), $\cos\theta = 0 \Rightarrow \theta = 90°$

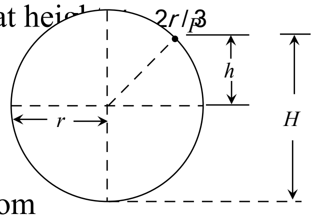
37. (c) Tension, $T = \frac{mv^2}{r} + mg\cos\theta$

For, $\theta = 30°, T_1 = \frac{mv^2}{r} + mg\cos 30°$

$\theta = 60°, T_2 = \frac{mv^2}{r} + mg\cos 60° \therefore T_1 > T_2$

38. (c) As we know for hemisphere the particle will leave the sphere at height $2r/3$

$h = \frac{2}{3} \times 21 = 14\, m$



but from the bottom

$H = h + r = 14 + 21 = 35\ metre$

39. (c) $x = \alpha t^3$ and $y = \beta t^3$ (given)

$v_x = \frac{dx}{dt} = 3\alpha t^2$ and $v_y = \frac{dy}{dt} = 3\beta t^2$

Resultant velocity $= v = \sqrt{v_x^2 + v_y^2} = 3t^2\sqrt{\alpha^2 + \beta^2}$

40. (b)

41. (d) Tension at the top of the circle, $T = m\omega^2 r - mg$

$T = 0.4 \times 4\pi^2 n^2 \times 2 - 0.4 \times 9.8 = 115.86\ N$

42. (c) Minimum angular velocity $\omega_{min} = \sqrt{g/R}$

$\therefore T_{max} = \frac{2\pi}{\omega_{min}} = 2\pi\sqrt{\frac{R}{g}} == 2\pi\sqrt{\frac{2}{10}} = 2\sqrt{2} \cong 3s$

43. (a) $|\overrightarrow{\Delta v}| = 2v\sin(\theta/2) = 2v\sin\left(\frac{90}{2}\right) = 2v\sin 45 = v\sqrt{2}$

44. (a) In this problem it is assumed that particle although moving in a vertical loop but its speed remain constant.

Tension at lowest point $T_{max} = \frac{mv^2}{r} + mg$

Tension at highest point $T_{min} = \frac{mv^2}{r} - mg$

$\frac{T_{max}}{T_{min}} = \frac{\frac{mv^2}{r} + mg}{\frac{mv^2}{r} - mg} = \frac{5}{3}$

by solving we get, $v = \sqrt{4gr} = \sqrt{4 \times 9.8 \times 2.5}$

$= \sqrt{98}\ m/s$

45. (d) There is no relation between centripetal and tangential acceleration. Centripetal acceleration is must for circular motion but tangential acceleration may be zero.

46. (d) Angular momentum is a axial vector. It is directed always in a fix direction (perpendicular to the plane of rotation either outward or inward), if the sense of rotation remain same.

47. (a) Difference in kinetic energy $= 2mgr = 2 \times 1 \times 10 \times 1 = 20\ J$

48. (d) Angular acceleration $= \frac{d^2\theta}{dt^2} = 2\theta_2$

Horizontal Projectile Motion

1. (b) $R_{max} = \frac{u^2}{g} = 16 \times 10^3 \Rightarrow u = 400\ m/s$

2. (c) Due to constant velocity along horizontal and vertical downward force of gravity stone will hit the ground following parabolic path.

3. (b) Because the vertical components of velocities of both the bullets are same and equal to zero and $t = \sqrt{\frac{2h}{g}}$.

4. (c) The pilot will see the ball falling in straight line because the reference frame is moving with the same horizontal velocity but the observer at rest will see the ball falling in parabolic path.

5. (b) Due to air resistance, it's horizontal velocity will decrease so it will fall behind the aeroplane.

6. (c) Because horizontal velocity is same for coin and the observer. So relative horizontal displacement will be zero.

7. (c) Horizontal displacement of the bomb

AB = Horizontal velocity × time available

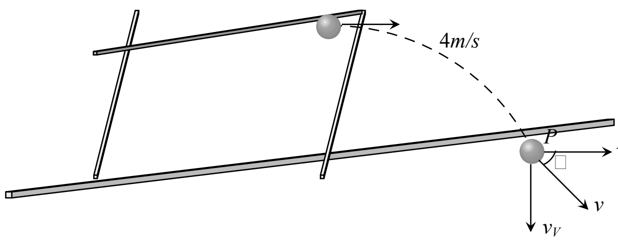
$AB = u \times \sqrt{\frac{2h}{g}} = 600 \times \frac{5}{18} \times \sqrt{\frac{2 \times 1960}{9.8}} = 3.33\ Km.$

8. (a,c) Vertical component of velocity of ball at point P

$v_V = 0 + gt = 10 \times 0.4 = 4\ m/s$

Horizontal component of velocity = initial velocity

$\Rightarrow v_H = 4\ m/s$



So the speed with which it hits the ground

$v = \sqrt{v_H^2 + v_V^2} = 4\sqrt{2}\ m/s$

and $\tan\theta = \frac{v_V}{v_H} = \frac{4}{4} = 1 \Rightarrow \theta = 45°$

It means the ball hits the ground at an angle of 45° to the horizontal.

Height of the table $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.4)^2 = 0.8\,m$

Horizontal distance travelled by the ball from the edge of table $h = ut = 4 \times 0.4 = 1.6\,m$

9. (b) $S = u \times \sqrt{\frac{2h}{g}} = 100 \times \sqrt{\frac{2 \times 490}{9.8}} = 1000\,m = 1\,km$

10. (c) $S = u \times \sqrt{\frac{2h}{g}} \Rightarrow 10 = u\sqrt{2 \times \frac{5}{10}} \Rightarrow u = 10\,m/s$

11. (d) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 396.9}{9.8}} \simeq 9$ secand

$u = 720\,km/hr = 200\,m/s$

$\therefore R = u \times t = 200 \times 9 = 1800\,m$

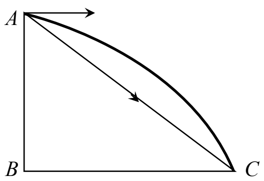
12. (a) For both cases $t = \sqrt{\frac{2h}{g}} =$ constant.

Because vertical downward component of velocity will be zero for both the particles.

13. (c)

14. (a) The horizontal distance covered by bomb,

$BC = v_H \times \sqrt{\frac{2h}{g}} = 150\sqrt{\frac{2 \times 80}{10}} = 660\,m$

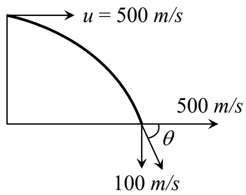


$\therefore$ The distance of target from dropping point of bomb,

$AC = \sqrt{AB^2 + BC^2} = \sqrt{(80)^2 + (600)^2} = 605.3\,m$

15. (a) Horizontal component of velocity $v_x = 500$ m/s

and vertical components of velocity while striking the ground.



$v_y = 0 + 10 \times 10 = 100\,m/s$

$\therefore$ Angle with which it strikes the ground.

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) = \tan^{-1}\left(\frac{1}{5}\right)$

16. (b) Area in which bullet will spread $= \pi r^2$

For maximum area, $r = R_{max} = \frac{v^2}{g}$ [when $\theta = 45^\circ$]

Maximum area $\pi R_{max}^2 = \pi\left(\frac{v^2}{g}\right)^2 = \frac{\pi v^4}{g^2}$

Oblique Projectile Motion

1. (d) $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore R \propto u^2$. If initial velocity be doubled then range will become four times.

2. (c) $H = \frac{u^2 \sin^2\theta}{2g}$ $\therefore H \propto u^2$. If initial velocity be doubled then maximum height reached by the projectile will quadrupled.

3. (a) An external force by gravity is present throughout the motion so momentum will not be conserved.

4. (a) Range $= \frac{u^2 \sin 2\theta}{g}$; when $\theta = 90^\circ$, $R = 0$ *i.e.* the body will fall at the point of projection after completing one dimensional motion under gravity.

5. (c) $R = 4H\cot\theta$.

When $R = H$ then $\cot\theta = 1/4 \Rightarrow \theta = \tan^{-1}(4)$

6. (c) Because there is no accelerating or retarding force available in horizontal motion.

7. (a) Direction of velocity is always tangent to the path so at the top of trajectory, it is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means angle between $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

8. (d) $R = 4H\cot\theta$ if $\theta = 45^\circ$ then $R = 4H\cot(45^\circ) = 4H$

9. (c) $v_y = \frac{dy}{dt} = 8 - 10t$, $v_x = \frac{dx}{dt} = 6$

at the time of projection *i.e.* $v_y = \frac{dy}{dt} = 8$ and $v_x = 6$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10\,m/s$

10. (b) The angle of projection is given by $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

11. (a) $a_x = \frac{d}{dt}(v_x) = 0$, $a_y = \frac{d}{dt}(v_y) = -10\,m/s^2$

$\therefore$ Net acceleration $a = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + 10^2} = 10 m/s^2$

12. (b) $R_{15^\circ} = \frac{u^2 \sin(2\times 15^\circ)}{g} = \frac{u^2}{2g} = 1.5\, km$

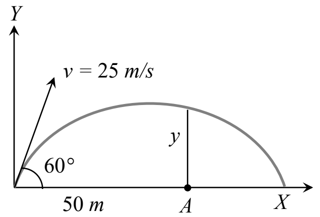
$R_{45^\circ} = \frac{u^2 \sin(2\times 45^\circ)}{g} = \frac{u^2}{g} = 1.5 \times 2 = 3\, km$

13. (a) Horizontal component of velocity

$v_x = 25\cos 60^\circ = 12.5\, m/s$

Vertical component of velocity

$v_y = 25 \sin 60^\circ = 12.5\sqrt{3}\, m/s$



Time to cover 50 m distance $t = \frac{50}{12.5} = 4 \sec$

The vertical height y is given by

$y = v_y t - \frac{1}{2} g t^2 = 12.5\sqrt{3} \times 4 - \frac{1}{2} \times 9.8 \times 16 = 8.2\, m$

14. (a) For vertical upward motion $h = ut - \frac{1}{2} g t^2$

$5 = (25 \sin\theta) \times 2 - \frac{1}{2} \times 10 \times (2)^2$

$\Rightarrow 25 = 50 \sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

15. (c) For angle $(45^\circ - \theta)$,

$R = \frac{u^2 \sin(90^\circ - 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

For angle $(45^\circ + \theta)$,

$R = \frac{u^2 \sin(90^\circ + 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

16. (b) Range is given by $R = \frac{u^2 \sin 2\theta}{g}$

On moon $g_m = \frac{g}{6}$. Hence $R_m = 6R$

17. (c) For greatest height $\theta = 90^\circ$

$H_{\max} = \frac{u^2 \sin^2(90^\circ)}{2g} = \frac{u^2}{2g} = h$ (given)

$R_{\max} = \frac{u^2 \sin^2 2(45^\circ)}{g} = \frac{u^2}{g} = 2h$

18. (c) $R = 4H\cot\theta$, if $R = 4H$ then $\cot\theta = 1 \Rightarrow \theta = 45^\circ$

19. (b) $E = E\cos^2\theta = E\cos^2(45^\circ) = \frac{E}{2}$

20. (b)

21. (b)

22. (d) Acceleration through out the projectile motion remains constant and equal to g.

23. (c)

24. (c) Time of flight $= \frac{2u\sin\theta}{g} = \frac{2\times 50 \times \sin 30}{10} = 5s$

25. (b) Change in momentum $= 2mu\sin\theta$

$= 2 \times 0.5 \times 98 \times \sin 30 = 45\, N\text{-}s$

26. (d) $R = 4H\cot\theta$, if $R = 3H$ then $\cot\theta = \frac{3}{4} \Rightarrow$

$\theta = 53^\circ 8'$

27. (c) Became vertical downward displacement of both (barrel and bullet) will be equal.

28. (b) As $H = \frac{u^2 \sin^2\theta}{2g}$ $\therefore$ $\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin\theta_2} = \frac{\sin^2 30^\circ}{\sin^2 60} =$

$\frac{1/4}{3/4} = \frac{1}{3}$

29. (d) $R = \frac{v^2 \sin 2\theta}{g} \Rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{v^2}\right)$

30. (a) $T = \frac{2u\sin\theta}{g} = 10 \sec \Rightarrow u\sin\theta = 50\, m/s$

$\therefore H = \frac{u^2 \sin^2\theta}{2g} = \frac{(u\sin\theta)^2}{2g} = \frac{50\times 50}{2\times 10} = 125m$

31. (b) For complementary angles range will be equal.

32. (b) $R = \frac{u^2 \sin 2\theta}{g} = \frac{(500)^2 \times \sin 30^\circ}{10} = 12.5 \times 10^3\, m$

33. (a) $T = \frac{2u\sin\theta}{g} \Rightarrow u = \frac{T\times g}{2\sin\theta} = \frac{2\times 9.8}{2\times \sin 30} = 19.6\, m/s$

34. (c) $R = \frac{u^2 \sin 2\theta}{g} = R \propto u^2$. So if the speed of projection doubled, the range will becomes four times,

i.e., $4 \times 50 = 200\, m$

35. (c) Range will be equal for complementary angles.

36. (a) When the angle of projection is very far from 45° then range will be minimum.

37. (a) $H = \frac{u^2 \sin^2\theta}{2g}$ and $T = \frac{2u\sin\theta}{g}$

So $\frac{H}{T^2} = \frac{u^2\sin^2\theta / 2g}{4u^2\sin^2\theta / g^2} = \frac{g}{8} = \frac{5}{4}$

38. (a) $H_1 = \frac{u^2 \sin^2\theta}{2g}$ and $H_2 = \frac{u^2 \sin^2(90-\theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$

$H_1 H_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g} = \frac{(u^2\sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$

$\therefore R = 4\sqrt{H_1 H_2}$

39. (d) Standard equation of projectile motion

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Comparing with given equation

$A = \tan\theta$ and $B = \frac{g}{2u^2\cos^2\theta}$

So $\frac{A}{B} = \frac{\tan\theta \times 2u^2\cos^2\theta}{g} = 40$

(As $\theta = 45°, u = 20\, m/s, g = 10\, m/s^2$)

40. (b) Range $= \frac{u^2 \sin 2\theta}{g}$. It is clear that range is proportional to the direction (angle) and the initial speed.

41. (c) $\frac{2u\sin\theta}{g} = 2\sec \Rightarrow u\sin\theta = 10$

$\therefore H = \frac{u^2\sin^2\theta}{2g} = \frac{100}{2g} = 5m$

42. (b) Only horizontal component of velocity $(u\cos\theta)$.

43. (a) For complementary angles range is same.

44. (b) $T = \frac{2u\sin\theta}{g} = \frac{2\times 9.8\times \sin 30}{9.8} = 1\,s$

45. (a) $x = 36t \therefore v_x = \frac{dx}{dt} = 36\, m/s$

$y = 48t - 4.9t^2 \therefore v_y = 48 - 9.8t$

at $t = 0$ $v_x = 36$ and $v_y = 48\, m/s$

So, angle of projection $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

Or $\theta = \sin^{-1}(4/5)$

46. (b) For same range angle of projection should be θ and $90-\theta$

So, time of flights $t_1 = \frac{2u\sin\theta}{g}$ and

$t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$

By multiplying $= t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$

$t_1 t_2 = \frac{2}{g}\frac{(u^2\sin 2\theta)}{g} = \frac{2R}{g} \Rightarrow t_1 t_2 \propto R$

47. (c) Instantaneous velocity of rising mass after t sec will be $v_t = \sqrt{v_x^2 + v_y^2}$

where $v_x = v\cos\theta =$ Horizontal component of velocity

$v_y = v\sin\theta - gt =$ Vertical component of velocity

$v_t = \sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$

$v_t = \sqrt{v^2 + g^2t^2 - 2v\sin\theta\, gt}$

48. (d) Maximum range $= \frac{u^2}{g} = 100\, m$

Maximum height $= \frac{u^2}{2g} = \frac{100}{2} = 50\, m$

49. (c) $R_{max} = \frac{u^2}{g} = 100 \Rightarrow u = 10\sqrt{10} = 32\, m/s$

50. (c) Since horizontal component of velocity is constant, hence momentum is constant.

51. (a) Time of flight = $\frac{2u\sin\theta}{g} = \frac{2u_y}{g} = \frac{2\times u_{vertical}}{g}$

52. (a) Person will catch the ball if its velocity will be equal to horizontal component of velocity of the ball.

$\frac{v_0}{2} = v_0\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

53. (b) $H = \frac{u^2\sin^2\theta}{2g}$ and $T = \frac{2u\sin\theta}{g} \Rightarrow T^2 = \frac{4u^2\sin^2\theta}{g^2}$

$\therefore \frac{T^2}{H} = \frac{8}{g} \Rightarrow T = \sqrt{\frac{8H}{g}} = 2\sqrt{\frac{2H}{g}}$

54. (d) $R = 4H\cot\theta$, if $R = 4\sqrt{3}H$ then $\cot\theta = \sqrt{3} \Rightarrow \theta = 30°$

55. (c) The vertical component of velocity of projection $= -50\sin 30° = -25\, m/s$

If t be the time taken to reach the ground,

$h = ut + \frac{1}{2}gt^2 \Rightarrow 70 = -25t + \frac{1}{2}\times 10t^2$

$\Rightarrow 70 = -25t + 5t^2 \Rightarrow t^2 - 5t - 14 = 0 \Rightarrow t = -2s$ and $7s$

Since, $t = -2\,s$ is not valid $\therefore t = 7\,s$

56. (c) $H_{max} = \frac{u^2\sin^2\theta}{2g}$

According to problem $\frac{u_1^2 \sin^2 45°}{2g} = \frac{u_2^2 \sin^2 60°}{2g}$

$\Rightarrow \frac{u_1^2}{u_2^2} = \frac{\sin^2 60°}{\sin^2 45°} \Rightarrow \frac{u_1}{u_2} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \sqrt{\frac{3}{2}}$.

57. (c)

58. (d) $R = 4H\cot\theta$, if $\theta = 45°$ then $R = 4H \Rightarrow \frac{R}{H} = \frac{4}{1}$

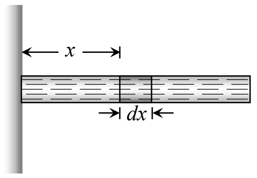
59. (b) $R_{\max} = \frac{u^2}{g} = 400m$ (For $\theta = 45°$)

$H_{\max} = \frac{u^2}{2g} = \frac{400}{2} = 200m$ (For $\theta = 90°$)

Critical Thinking Questions

1. (c,d) In the given condition, the particle undergoes uniform circular motion and for uniform circular motion the velocity and acceleration vector changes continuously but kinetic energy is constant for every point.

2. (a) $dM = \left(\frac{M}{L}\right)dx$



force on 'dM' mass is

$dF = (dM)\omega^2 x$

By integration we can get the force exerted by whole liquid

$\Rightarrow F = \int_0^L \frac{M}{L}\omega^2 x\,dx = \frac{1}{2}M\omega^2 L$

3. (b) According to given problem $\frac{1}{2}mv^2 = as^2$

$\Rightarrow v = s\sqrt{\frac{2a}{m}}$

So $a_R = \frac{v^2}{R} = \frac{2as^2}{mR}$ ...(i)

Further more as $a_t = \frac{dv}{dt} = \frac{dv}{ds}\cdot\frac{ds}{dt} = v\frac{dv}{ds}$ ...(ii)

(By chain rule)

Which in light of equation (i) *i.e.* $v = s\sqrt{\frac{2a}{m}}$ yields

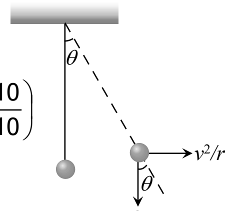
$a_t = \left[s\sqrt{\frac{2a}{m}}\right]\left[\sqrt{\frac{2a}{m}}\right] = \frac{2as}{m}$ ...(iii)

So that $a = \sqrt{a_R^2 + a_t^2} = \sqrt{\left[\frac{2as^2}{mR}\right]^2 + \left[\frac{2as}{m}\right]^2}$

Hence $a = \frac{2as}{m}\sqrt{1 + [s/R]^2}$

$\therefore F = ma = 2as\sqrt{1 + [s/R]^2}$

4. (c) $\tan\theta = \frac{v^2/r}{g} = \frac{v^2}{rg}$



$\therefore \theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{10\times 10}{10\times 10}\right)$

$\therefore \theta = \tan^{-1}(1) = 45°$

5. (b) Here the tangential acceleration also exits which requires power.

Given that $a_C = k^2rt^2$ and $a_C = \frac{v^2}{r}$ $\therefore$

$\frac{v^2}{r} = k^2rt^2$

or $v^2 = k^2r^2t^2$ or $v = krt$

Tangential acceleration $a = \frac{dv}{dt} = kr$

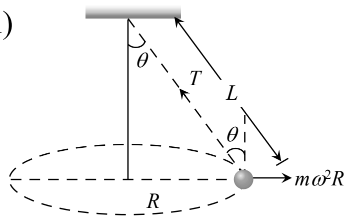
Now force $F = m\times a = mkr$

So power $P = F\times v = mkr \times krt = mk^2r^2t$

6. (d) $T\sin\theta = M\omega^2 R$ ...(i)

$T\sin\theta = M\omega^2 L\sin\theta$ ...(ii)

From (i) and (ii)



$T = M\omega^2 L$

$= M4\pi^2n^2L$

$= M4\pi^2\left(\frac{2}{\pi}\right)^2 L$

$= 16\,ML$

7. (d) Since the maximum tension T_B in the string moving in the vertical circle is at the bottom and minimum tension T_T is at the top.

$\therefore T_B = \frac{mv_B^2}{L} + mg$ and $T_T = \frac{mv_T^2}{L} - mg$

$\therefore \frac{T_B}{T_T} = \frac{\frac{mv_B^2}{L} + mg}{\frac{mv_T^2}{L} - mg} = \frac{4}{1}$ or $\frac{v_B^2 + gL}{v_T^2 - gL} = \frac{4}{1}$

or $v_B^2 + gL = 4v_T^2 - 4gL$ but $v_B^2 = v_T^2 + 4gL$

$\therefore v_T^2 + 4gL + gL = 4v_T^2 - 4gL \Rightarrow 3v_T^2 = 9gL$

$\therefore v_T^2 = 3\times g\times L = 3\times 10\times\frac{10}{3}$ or $v_T = 10\,m/\sec$

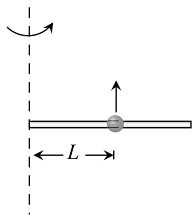
8. (a) For particle *P*, motion between *A* and *C* will be an accelerated one while between *C* and

B a retarded one. But in any case horizontal component of it's velocity will be greater than or equal to *v* on the other hand in case of particle *Q*, it is always equal to *v*. horizontal displacement of both the particles are equal, so $t_P < t_Q$.

9. (a) Let the bead starts slipping after time *t*

For critical condition

Frictional force provides the centripetal force



$m\omega^2 L = \mu R = \mu m \times a_t = \mu L m\alpha$

$\Rightarrow m(\alpha t)^2 L = \mu mL\alpha \Rightarrow t = \sqrt{\dfrac{\mu}{\alpha}}$ (As $\omega = \alpha t$)

10. (a) Normal reaction at the highest point

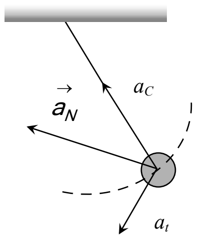
$R = \dfrac{mv^2}{r} - mg$

Reaction is inversely proportional to the radius of the curvature of path and radius is minimum for path depicted in (a).

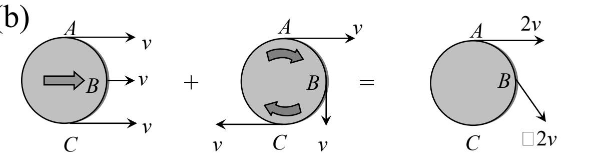
11. (c) $a_c =$ centripetal acceleration

$a_t =$ tangential acceleration

$a_N =$ net acceleration = Resultant of a_c and a_t



12. (b)



Pure translation + Pure Rotation = Rolling without Slipping

13. (d) $\dfrac{1}{2}mu^2 - \dfrac{1}{2}mv^2 = mgL$

$\Rightarrow v = \sqrt{u^2 - 2gL}$

$|\vec{v} - \vec{u}| = \sqrt{u^2 + v^2} = \sqrt{u^2 + u^2 - 2gL} = \sqrt{2(u^2 - gL)}$

14. (a) When driver applies brakes and the car covers distance *x* before coming to rest, under the effect of retarding force *F*

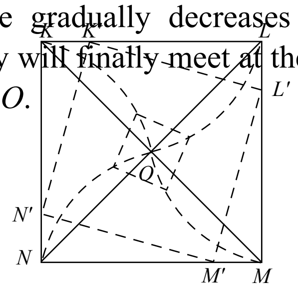
then $\dfrac{1}{2}mv^2 = Fx \Rightarrow x = \dfrac{mv^2}{2F}$

But when he takes turn then $\dfrac{mv^2}{r} = F \Rightarrow$ $r = \dfrac{mv^2}{F}$

It is clear that $x = r/2$

i.e. by the same retarding force the car can be stopped in a less distance if the driver apply breaks. This retarding force is actually a friction force.

15. (a) It is obvious from considerations of symmetry that at any moment of time all of the persons will be at the corners of square whose side gradually decreases (see fig.) and so they will finally meet at the centre of the square *O*.



The speed of each person along the line joining his initial position and *O* will be $v\cos 45 = v/\sqrt{2}$.

As each person has displacement $d\cos 45 = d/\sqrt{2}$ to reach the centre, the four persons will meet at the centre of the square *O* after time.

$\therefore\ t = \dfrac{d/\sqrt{2}}{v/\sqrt{2}} = \dfrac{d}{v}$

16. (a,b) $x = a\cos(pt)$ and $y = b\sin(pt)$ (given)

$\therefore\ \cos pt = \dfrac{x}{a}$ and $\sin pt = \dfrac{y}{b}$

By squaring and adding

$\cos^2(pt) + \sin^2(pt) = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Hence path of the particle is ellipse.

Now differentiating *x* and *y* *w.r.t.* time

$v_x = \dfrac{dx}{dt} = \dfrac{d}{dt}(a\cos(pt)) = -ap\sin(pt)$

$v_y = \frac{dy}{dt} = \frac{d}{dt}(b\sin(pt)) = bp\cos(pt)$

$\therefore\ \vec{v} = v_x\hat{i} + v_y\hat{j} = -ap\sin(pt)\hat{i} + bp\cos(pt)\hat{j}$

Acceleration

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[-ap\sin(pt)\hat{i} + bp\cos(pt)\hat{j}]$

$\vec{a} = -ap^2\cos(pt)\,\hat{i} - bp^2\sin(pt)\hat{j}$

Velocity at $t = \frac{\pi}{2p}$

$\vec{v} = -ap\sin p\left(\frac{\pi}{2p}\right)\hat{i} + bp\cos p\left(\frac{\pi}{2p}\right)\hat{j} = -ap\,\hat{i}$

Acceleration at $t = \frac{\pi}{2p}$

$\vec{a} = ap^2\cos p\left(\frac{\pi}{2p}\right)\hat{i} - bp^2\sin p\left(\frac{\pi}{2p}\right)\hat{j} = -bp^2\hat{j}$

As $\vec{v}\,.\,\vec{a} = 0$

Hence velocity and acceleration are perpendicular to each other at $t = \frac{\pi}{2p}$.

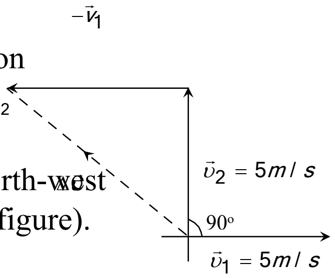
17. (b) $\Delta\vec{\upsilon} = \vec{\upsilon}_2 - \vec{\upsilon}_1 = \sqrt{\upsilon_1^2 + \upsilon_2^2 - 2\upsilon_1\upsilon_2\cos 90^o}$

$= \sqrt{5^2 + 5^2} = 5\sqrt{2}$

Average acceleration

$= \frac{\Delta\upsilon}{\Delta t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2

Directed toward north-west (As clear from the figure).



Graphical Questions

1. (d) $R = \frac{u^2\sin 2\theta}{g} = \frac{2u_x v_y}{g}$

$\therefore$ Range $\propto$ horizontal initial velocity (u_x)

In path 4 range is maximum so football possess maximum horizontal velocity in this path.

2. (a) If air resistance is taken into consideration then range and maximum height, both will decrease.

3. (b)

4. (c)

5. (d)

Assertion and Reason

1. (e) At the highest point, vertical component of velocity becomes zero so there will be only horizontal velocity and it is perpendicular to the acceleration due to gravity.

2. (a) $H = \frac{u^2\sin^2\theta}{2g}$ *i.e.* it is independent of mass of projectile.

3. (c) $R = \frac{u^2\sin 2\theta}{g}$ $\therefore R_{\max} = \frac{u^2}{g}$ when $\theta = 45°$ $\therefore$ $R_{\max} \propto u^2$

Height $H = \frac{u^2\sin^2\theta}{2g} \Rightarrow H_{\max} = \frac{u^2}{2g}$ when $\theta = 90°$

It is clear that $H_{\max} = \frac{R_{\max}}{2}$

4. (c) Horizontal range depends upon angle of projection and it is same for complementary angles *i.e.* θ and $(90 - \theta)$.

5. (b) We know $R = 4H\cot\theta$

if $R = H$ then $\cot\theta = \left(\frac{1}{4}\right)$ or tan $\theta = (4)$

and $R = \frac{u^2\sin 2\theta}{g}$ $\therefore$ $R \propto \frac{u^2}{g}$

6. (d) $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

7. (d) If a body is projected from a place above the surface of earth, then for the maximum range, the angle of projection should be slightly less than 45^o.

8. (a) Both body will take same time to reach the earth because vertical downward component of velocity for both the bodies will be zero and time of descent $t = \sqrt{\frac{2h}{g}}$.

Horizontal velocity has no effect on the vertical direction.

9. (c) $T \propto u$ and $R \propto u^2$

When velocity of projection of a body is made n times, then its time of flight becomes n times and range becomes n^2 times.

10. (c) Range will be maximum when $\theta = 45^\circ$ and in this condition $R = 4H \Rightarrow H = R/4$ (always) because $R = 4H\cot\theta$ and $\theta = 45^\circ$

So maximum height is 25% of maximum range.

It does not depends upon the velocity of projection.

11. (a) Range, $R = \frac{u^2 \sin 2\theta}{g}$

when $\theta = 45^o$, $R_{max} = \frac{u^2}{g}\sin 90^o = \frac{u^2}{g}$

when $\theta = 135^o$, $R_{max} = \frac{u^2}{g}\sin 270^o = \frac{-u^2}{g}$

Negative sign shows opposite direction.

12. (e) The man should point his rifle at a point higher than the target since the bullet suffers a vertically downward deflection $\left(y = \frac{1}{2}gt^2\right)$ due to gravity.

13. (b) In uniform circular motion, the magnitude of velocity and acceleration remains same, but due to change in direction of motion, the direction of velocity and acceleration changes. Also the centripetal acceleration is given by $a = \omega^2 r$.

14. (a) The body is able to move in a circular path due to centripetal force. The centripetal force in case of vehicle is provided by frictional force. Thus if the value of frictional force μmg is less than centripetal force, then it is not possible for a vehicle to take a turn and the body would overturn.

Thus condition for safe turning of vehicle is,

$\mu mg \geq \frac{mv^2}{r}$.

15. (c) In circular motion the frictional force acting towards the centre of the horizontal circular path provides the centripetal force and avoid overturning of vehicle. Due to the change in direction of motion, velocity changes in circular motion.

16. (b) On an unbanked road, friction provides the necessary centripetal force $\frac{mv^2}{r} = \mu mg$

$\therefore v = \sqrt{\mu rg}$.

Thus with increase in friction, safe velocity limit also increases.

When the road is banked with angle of θ then its limiting velocity is given by $v = \sqrt{\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$.

Thus limiting velocity increase with banking of road.

17. (d) If the speed of a body is constant, all curved paths are possible.

In uniform circular motion a body has constant speed, but its direction keeps on changing, due to which it has non-zero acceleration.

18. (a) We know that $W = Fs\cos\theta$

in the circular motion if $\theta = 90^o$ then $W = 0$

19. (d) While moving along a circle, the body has a constant tendency to regain its natural straight line path.

This tendency gives rise to a force called centrifugal force. The centrifugal force does not act on the body in motion, the only force acting on the body in motion is centripetal force. The centrifugal force acts on the source of centripetal force to displace it radially outward from centre of the path.

20. (c) Centripetal force is defined from formula

$F = \frac{mv^2}{r} \Rightarrow F \propto \frac{v^2}{r}$

If v and r both are doubled then F also gets doubled.

21. (b) When automobile moves in circular path then reaction on inner wheel and outer wheel will be different.

$R_{\text{inner}} = \frac{M}{2}\left[g - \frac{v^2 h}{ra}\right]$ and $R_{\text{outer}} = \frac{M}{2}\left[g + \frac{v^2 h}{ra}\right]$

In critical condition $v_{\text{safe}} = \sqrt{\frac{gra}{h}}$

If v is equal or more that thus critical value then reaction on inner wheel becomes zero. So it leaves the ground first.

22. (c) For safe turn $\tan\theta \geq \frac{v^2}{rg}$.

It is clear that for safe turn v should be small and r should be large. Also bending angle from the vertical would increase with increase in velocity.

23. (a) When roads are not properly banked, force of friction between tyres and road provides partially the necessary centripetal force. This cause wear and tear of tyres.

24. (d) When the milk is churned centrifugal force acts on it outward and due to which cream in milk is separated from it.

25. (e) Due to earth's axial rotation, the speed of the trains relative to earth will be different and hence the centripetal forces on them will be different. Thus their effective weights $mg - \frac{mv^2}{r}$ and $mg + \frac{mv^2}{r}$ will be different. So they exert different pressure on the rails.

26. (d) Within a certain speed of the turn table the frictional force between the coin and the turn table supplies the necessary centripetal force required for circular motion. On further increase of speed, the frictional force cannot supply the necessary centripetal force. Therefore the coin flies off tangentially.