Critical Thinking Questions

1	b	2	ab	3	a	4	b	5	a
6	c	7	bcd	8	b	9	d	10	b
11	c	12	c	13	acd	14	b	15	c
16	c	17	b	18	c				

Graphical Questions

1	c	2	c	3	c	4	b	5	d
6	c	7	c	8	d	9	c	10	c

Assertion and Reason

1	a	2	b	3	d	4	c	5	a
6	c	7	b	8	a	9	e	10	a
11	a	12	a	13	c	14	e	15	c
16	b	17	b	18	e	19	d	20	a
21	a	22	c	23	c	24	b	25	e
26	a	27	a	28	a	29	c	30	e
31	a	32	b	33	a				

Answers and Solutions

Newton's Law of Gravitation

1. (a)

2. (b) As it depends on the weight of the body.

3. (b) Due to inertia of direction.

4. (b)

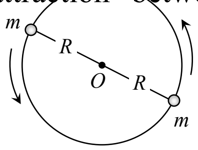
5. (a)

6. (d) $F \propto \frac{1}{r^2}$. If r becomes double then F reduces to $\frac{F}{4}$

7. (b)

8. (c) $F = G\frac{m_1 m_2}{r^2} = 6.675 \times \frac{1 \times 1}{1^2} \times 10^{-11} = 6.675 \times 10^{-11}\, N$

9. (c) Centripetal force provided by the gravitational force of attraction between two particles



i.e. $\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2}$

$\Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$

10. (d) $m = 6 \times 10^{24}\, kg$, $\omega = 2 \times 10^{-7}\, rad/s$, $R = 1.5 \times 10^{11}\, m$

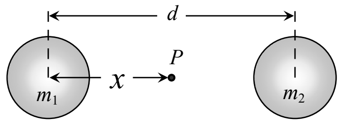
The force exerted by the sun on the earth $F = m\omega^2 R$

By substituting the value we can get, $F = 36 \times 10^{21}\, N$

11. (d)

12. (a) k represents gravitational constant which depends only on the system of units.

13. (d)



Force will be zero at the point of zero intensity

$$x = \frac{\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}}\, d = \frac{\sqrt{81M}}{\sqrt{81M} + \sqrt{M}}\, D = \frac{9}{10} D.$$

14. (a)

15. (d) $g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{(1.74 \times 10^6)^2} = 1.62\, N/kg$

16. (b) Actually gravitational force provides the centripetal force.

17. (c)

18. (a) $F \propto xm \times (1 - x)m = xm^2(1 - x)$

For maximum force $\frac{dF}{dx} = 0$

$\Rightarrow \frac{dF}{dx} = m^2 - 2xm^2 = 0 \Rightarrow x = 1/2$

19. (c)

20. (a)

21. (a) Gravitational force does not depend on the medium.

22. (a)

23. (e)

24. (c) $F = \frac{G \times m \times m}{(2R)^2} = \frac{G \times \left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2} = \frac{4}{9}\pi^2 \rho^2 R^4$

$\therefore\ F \propto R^4$

Acceleration Due to Gravity

1. (d)

2. (b) The value of g at the height h from the surface of earth

$g' = g\left(1 - \frac{2h}{R}\right)$

The value of g at depth x below the surface of earth

$g' = g\left(1 - \frac{x}{R}\right)$

These two are given equal, hence $\left(1 - \frac{2h}{R}\right) = \left(1 - \frac{x}{R}\right)$

On solving, we get $x = 2h$

3. (d) Time period of simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

In artificial satellite $g = 0$ $\therefore$ T = infinite.

4. (a) $g = \frac{4}{3}\pi\rho GR$. If ρ = constant then $\frac{g_1}{g_2} = \frac{R_1}{R_2}$

5. (b) Time of decent $t = \sqrt{\frac{2h}{g}}$. In vacuum no other force works except gravity so time period will be exactly equal.

6. (a)

7. (b) Because acceleration due to gravity increases

8. (d) Because acceleration due to gravity decreases

9. (b) We know that $g = \frac{GM}{R^2}$

On the planet $g_p = \frac{GM/7}{R^2/4} = \frac{4g}{7} = \frac{4}{7}g$

Hence weight on the planet $= 700 \times \frac{4}{7} = 400\ gmwt$

10. (b) In pendulum clock the time period depends on the value of g, while in spring watch, the time period is independent of the value of g.

11. (c) $g = \frac{GM}{R^2} = \frac{GM_0}{(D_0/2)^2} = \frac{4GM_0}{D_0^2}$

12. (a)

13. (b) $\frac{g'}{g} = \frac{M'}{M}\left(\frac{R}{R'}\right)^2 = \left(\frac{2M}{M}\right)\left(\frac{R}{2R}\right)^2 = \frac{1}{2}$

$\Rightarrow g' = \frac{g}{2} = \frac{9.8}{2} = 4.9\ m/s^2$

14. (c)

15. (a)

16. (c) For the condition of weightlessness at equator

$\omega = \sqrt{\frac{g}{R}}$ $\therefore$ $\omega = \sqrt{\frac{1}{640 \times 10^3}} = \frac{1}{800}\ rad/s$

17. (c) $g = \frac{GM}{r^2}$. Since M and r are constant, so $g = 9.8\ m/s^2$

18. (c) $g = \frac{GM}{R^2}$ and $M = \frac{4}{3}\pi R^3 \times \rho$

$\therefore g = \frac{4}{3}\frac{\pi R^3 \times G\rho}{R^2} \Rightarrow \rho = \frac{3g}{4\pi RG}$

19. (a) Because value of g decreases when we move either in coal mine or at the top of mountain.

20. (d) $g = \frac{4}{3}\pi\rho GR \therefore \frac{g_1}{g_2} = \frac{R_1\rho_1}{R_2\rho_2}$

21. (a) $g = \frac{GM}{R^2}$ (Given $M_e = 81M_m,\ R_e = 3.5R_m$)

Substituting the above values, $\frac{g_m}{g_e} = 0.15$

22. (c) Value of g decreases when we go from poles to equator.

23. (d)

24. (b) Because value of g decreases with increasing height.

25. (a) $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{6400}{6400+64}\right)^2 \Rightarrow g' = 960.40\ cm/s^2$

26. (d)

27. (b) $g' = g - \omega^2 R\cos^2\lambda$

Rotation of the earth results in the decreased weight apparently. This decrease in weight is not felt at the poles as the angle of latitude is 90°.

28. (b) $g = \frac{GM}{R^2}$. If radius shrinks to half of its present value then g will becomes four times.

29. (b) Using $g = \frac{GM}{R^2}$ we get $g_m = g/5$

30. (a) $g = g_p - R\omega^2\cos^2\lambda = g_p - \omega^2 R\cos^2 60° = g_p - \frac{1}{4}R\omega^2$

31. (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \left(\frac{n-1}{n}\right)R$

32. (a) $g \propto \frac{GM}{r^2}$ $\therefore g \propto \frac{1}{r^2}$ or $r \propto \frac{1}{\sqrt{g}}$

If g decrease by one percent then r should be increase by $\frac{1}{2}\%$ *i.e.* R = $\frac{1}{2 \times 100} \times 6400 = 32\ km$

33. (c) $g = \frac{4}{3} G\pi R\rho \Rightarrow \frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{2} \times \frac{4}{1} = \frac{2}{1}$

34. (b) $g = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{g}{4} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{2} = \frac{R}{R+h}$

$\Rightarrow R + h = 2R \therefore h = R$

35. (c) Acceleration due to gravity at poles is independent of the angular speed of earth.

36. (a) Mass of the ball always remain constant. It does not depend upon the acceleration due to gravity

37. (d) $g_m = \frac{GM_m}{R_m^2}$ and $g_m = \frac{g_e}{6} = \frac{9.8}{6}\ m/s^2 = 1.63\ m/s^2$

Substituting $R_m = 1.768 \times 10^6\ m$, $g_m = 1.63\ m/s^2$ and $G = 6.67 \times 10^{-11}\ N\text{-}m^2/kg^2$ We get

$M_m = 7.65 \times 10^{22}\ kg$

38. (b) $g = g\left(\frac{R}{R+h}\right)^2 \Rightarrow$ when $h = R$ then $g = \frac{g}{4}$

So the weight of the body at this height will become one-fourth.

39. (c) $g = \frac{GM}{R^2}$ and $K = \frac{L^2}{2I}$

If mass of the earth and its angular momentum remains constant then $g \propto \frac{1}{R^2}$ and $K \propto \frac{1}{R^2}$

i.e. if radius of earth decreases by 2% then g and K both increases by 4%.

40. (b) Weight is least at the equator.

41. (c) $g \propto \frac{1}{R^2}$

Percentage change in g = 2(percentage change in R)

$= 2 \times 1.5 = -3\%$

42. (b) $g \propto \frac{1}{R^2}$. If radius of earth decreases by 2% then g will increase by 4% *i.e.* weight of the body at earth surface will increase by 4%

43. (c) Mass does not vary from place to place.

44. (b) $g = \frac{GM}{R^2} \Rightarrow R = \sqrt{\frac{GM}{g}}$

Substituting the above values we get $R = 1.87 \times 10^6\ m.$

45. (c) Weight of the body at equator = $\frac{3}{5}$ of initial weight

$\therefore g = \frac{3}{5} g$ (because mass remains constant)

$g' = g - \omega^2 R\cos^2\lambda \Rightarrow \frac{3}{5} g = g - \omega^2 R\cos^2(0°)$

$\Rightarrow \omega^2 = \frac{2g}{5R} \Rightarrow \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2 \times 10}{5 \times 6400 \times 10^3}}$

$= 7.8 \times 10^{-4} \frac{rad}{\sec}$

46. (b) $h = 32\ km$, $R = 6400\ km$, so $h << R$

$g' = g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{2 \times 32}{6400}\right) \Rightarrow$

$g' = \frac{99}{100} g = 0.99\ g$

47. (a) Same change in the value of g can be observed at a depth x and height $2x$

given $d = x = 10\ km \ \therefore h = 2x = 20\ km$

48. (a)

49. (b) $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{100} = \left(\frac{R}{R+h}\right)^2 \Rightarrow h = 9R$

50. (a) $g' = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R + \frac{R}{2}}\right)^2 = \frac{4}{9} g$

$\therefore W = \frac{4}{9} \times W = \frac{4}{9} \times 72 = 32\ N$

51. (a) $g' = g - \omega^2 R\cos^2\lambda \Rightarrow 0 = g - \omega^2 R\cos^2 60^o$

$0 = g - \frac{\omega^2 R}{4} \Rightarrow \omega = 2\sqrt{\frac{g}{R}} = \frac{1}{400} \frac{rad}{\sec} = 2.5 \times 10^{-3} \frac{rad}{sec}$

52. (a) $g = g\left(1 - \frac{d}{R}\right) = 9.8\left(1 - \frac{100}{6400}\right) = 9.66\ m/s^2$

53. (c) $g = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{4}$. By solving $h = R$

54. (a) $g = \frac{4}{3}\pi\rho GR \therefore g \propto r\rho \therefore \frac{g_e}{g_m} = \frac{R}{r} \times \frac{\rho_e}{\rho_m}$

55. (c) $g_p = g_e\left(\frac{M_p}{M_e}\right)\left(\frac{R_e}{R_p}\right)^2 = 9.8\left(\frac{1}{80}\right)(2)^2$

$= 9.8/20 = 0.49\ m/s^2$

56. (d) Range of projectile $R = \frac{u^2 \sin 2\theta}{g}$

if u and θ are constant then $R \propto \frac{1}{g}$

$\frac{R_m}{R_e} = \frac{g_e}{g_m} \Rightarrow \frac{R_m}{R_e} = \frac{1}{0.2} \Rightarrow R_m = \frac{R_e}{0.2} \Rightarrow R_m = 5R_e$

57. (a) For condition of weightlessness of equator

$\omega = \sqrt{\frac{g}{R}} = \frac{1}{800} = 1.25 \times 10^{-3} \frac{rad}{s}$

58. (d) $g = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{\sqrt{2}} = \frac{R}{R+h}$

$\Rightarrow R + h = \sqrt{2}\,R \Rightarrow h = (\sqrt{2}-1)R = 0.414\,R$

59. (b) $g \propto \rho R$

60. (c) $H = \frac{u^2}{2g} \Rightarrow H \propto \frac{1}{g} \Rightarrow \frac{H_B}{H_A} = \frac{g_A}{g_B}$

Now $g_B = \frac{g_A}{12}$ as $g \propto \rho R$

$\therefore \frac{H_B}{H_A} = \frac{g_A}{g_B} = 12 \Rightarrow H_B = 12 \times H_A = 12 \times 1.5 = 18m$

61. (b)

62. (a) $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

63. (c) $g = \frac{4}{3}\pi\rho GR \Rightarrow g \propto dR$ ($\rho = d$ given in the problem)

64. (a) Inside the earth $g = \frac{4}{3}\pi\rho Gr \therefore g \propto r$

65. (c) $g = g\left(\frac{R}{R+h}\right)^2 = \frac{4}{9}g \therefore W = \frac{4}{9}W$

66. (a) $g \propto \rho$

67. (d)

68. (d)

69. (a) $g = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{3R/2}\right)^2 = \frac{4}{9}g$

$\therefore W = \frac{4}{9} \times mg = \frac{4 \times 200 \times 9.8}{9} = 880\ N$

70. (a) $g = \frac{4}{3}\pi G\rho R \Rightarrow g \propto \rho R \Rightarrow \frac{g_e}{g_m} = \frac{\rho_e}{\rho_m} \times \frac{R_e}{R_m}$

$\Rightarrow \frac{6}{1} = \frac{5}{3} \times \frac{R_e}{R_m} \Rightarrow R_m = \frac{5}{18}R_e$

71. (a) $g = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+2R}\right)^2 = \frac{g}{9}$

72. (b) $g = g\left(1-\frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1-\frac{d}{R}\right) \Rightarrow d = \frac{3}{4}R$

73. (b) For height $\frac{\Delta g}{g} \times 100\% = \frac{2h}{R} = 1\%$;

For depth $\frac{\Delta g}{g} \times 100\% = \frac{d}{R} = \frac{h}{R} = \frac{1}{2}\% = 0.5\%$

74. (b) As $g = \frac{GM}{R^2}$ therefore 1% decrease in mass will decreases the value of g by 1%.

But 1% decrease in radius will increase the value of g by 2%.

As a whole value of g increase by 1%.

75. (d) $g = \frac{4}{3}\pi\rho GR \Rightarrow \frac{R_p}{R_e} = \left(\frac{g_p}{g_e}\right)\left(\frac{\rho_e}{\rho_p}\right) = (1) \times \left(\frac{1}{2}\right)$

$\Rightarrow R_p = \frac{R_e}{2} = \frac{R}{2}$

76. (a) $\frac{g_1}{g_2} = \frac{\rho_1}{\rho_2} \times \frac{R_1}{R_2} = \frac{3}{2} \times \frac{2}{3} = 1$

77. (d) Because the body weighs zero in satellite

78. (a) Radius of earth $R = 6400\ km \therefore h = \frac{R}{4}$

Acceleration due to gravity at a height h

$g_h = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+\frac{R}{4}}\right)^2 = \frac{16}{25}g$

At depth 'd' value of acceleration due to gravity

$g_d = \frac{1}{2}g_h$ (According to problem)

$\Rightarrow g_d = \frac{1}{2}\left(\frac{16}{25}\right)g \Rightarrow g\left(1-\frac{d}{R}\right) = \frac{1}{2}\left(\frac{16}{25}\right)g$

By solving we get $d = 4.3 \times 10^6\ m$

79. (a) $g' = g - \omega^2 R\cos^2\lambda$

For weightlessness at equator $\lambda = 0°$ and $g' = 0$

$\therefore 0 = g - \omega^2 R \Rightarrow \omega = \sqrt{\frac{g}{R}} = \frac{1}{800} \frac{rad}{\sec}$

80. (b) Weight on surface of earth, $mg = 500\ N$

and weight below the surface of earth at $d = \frac{R}{2}$

$$mg' = mg\left(1 - \frac{d}{R}\right) = mg\left(1 - \frac{1}{2}\right) = \frac{mg}{2} = 250\ N$$

81. (a) $g = \frac{4}{3}\pi GR\rho$ and $g' = \frac{4}{3}\pi GR'\rho$

$\therefore \frac{g'}{g} = \frac{R'}{R} = 0.2 \Rightarrow g' = 0.2\, g$

82. (a)

83. (d) $\frac{g_m}{g_e} = \frac{M_m}{M_e} \times \left(\frac{R_e}{R_m}\right)^2 = \left(\frac{1}{9}\right)\left(\frac{2}{1}\right)^2 = \frac{4}{9} \Rightarrow g_m = \frac{4}{9} g_e$

$\therefore W_m = \frac{4}{9} \times W_e = \frac{4}{9} \times 90 = 40\ kg$

84. (a) $g' = g - \omega^2 R$, when ω increases g' decreases.

85. (b) $\frac{g'}{g} = \frac{M'}{M} \times \frac{R^2}{R'^2} = \frac{1}{2} \times \frac{4}{1} = \frac{2}{1}$

86. (b) Acceleration due to gravity at latitude λ is given by

$g' = g - R\omega^2 \cos^2 \lambda$

At 30^o, $g_{30^o} = g - R\omega^2 \cos^2 30^o = g - \frac{3}{4} R\omega^2$

$\therefore g - g_{30} = \frac{3}{4}\omega^2 R.$

87. (b) Acceleration due to gravity $g = \frac{GM}{R^2}$ $\therefore$ $\frac{g}{G} = \frac{M}{R^2}$

Gravitational Potential, Energy and Escape Velocity

1. (c) $\Delta U = \frac{mgh}{1 + h/R}$

Substituting $R = 5h$ we get $\Delta U = \frac{mgh}{1 + 1/5} = \frac{5}{6} mgh$

2. (a) $I = \frac{-dV}{dx}$

If $V = 0$ then gravitational field is necessarily zero.

3. (d) Gravitational potential $= \int I\, dx = \int_x^\infty \frac{K}{x^3} dx$

$$= K\left(\frac{x^{-3+1}}{-3+1}\right)_x^\infty = \left|\frac{-K}{2x^2}\right|_x^\infty = \frac{K}{2x^2}$$

4. (a) $U = -\frac{GMm}{r}$

$\Rightarrow 7.79 \times 10^{28} = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22} \times 6 \times 10^{24}}{r}$

$\Rightarrow r = 3.8 \times 10^8\ m$

5. (d) $\Delta U = \frac{mgh}{1 + \frac{h}{R}} = \frac{mgnR}{1 + \frac{nR}{R}} = \frac{nmgR}{n+1}$

6. (a) Gravitational potential at mid point

$V = \frac{-GM_1}{d/2} + \frac{-GM_2}{d/2}$

Now, $PE = m \times V = \frac{-2Gm}{d}(M_1 + M_2)$

[m = mass of particle]

So, for projecting particle from mid point to infinity

$KE = |PE|$

$\Rightarrow \frac{1}{2}mv^2 = \frac{2Gm}{d}(M_1 + M_2) \Rightarrow v = 2\sqrt{\frac{G(M_1 + M_2)}{d}}$

7. (b) Potential energy of the 1 kg mass which is placed at the earth surface = $-\frac{GM}{R}$

its potential energy at infinite = 0

$\therefore$ Work done = change in potential energy $= \frac{GM}{R}$

8. (c)

9. (c) $\frac{G \times 100}{x^2} = \frac{G \times 10000}{(1-x)^2} \Rightarrow \frac{10}{x} = \frac{100}{1-x} \Rightarrow x = \frac{1}{11} m$

10. (c)

11. (d) $\Delta U = U_2 - U_1 = \frac{mgh}{1 + \frac{h}{R_e}} = \frac{mgR_e}{1 + \frac{R_e}{R_e}} = \frac{mgR_e}{2}$

$\Rightarrow U_2 - (-mgR_e) = \frac{mgR_e}{2} \Rightarrow$ $U_2 = -\frac{1}{2} mgR_e$

12. (a) $v_e = \sqrt{\frac{2GM}{R}} = 100 \Rightarrow \frac{GM}{R} = 5000$

Potential energy $U = -\frac{GMm}{R} = -5000\ J$

13. (b) $\Delta U = \frac{mgh}{1 + \frac{h}{R}} = \frac{mg \times 3R}{1 + \frac{3R}{R}} = \frac{3}{4} mgR$

14. (b) Potential energy $U = \frac{-GMm}{r} = -\frac{GMm}{R+h}$

$U_{initial} = -\frac{GMm}{3R}$ and $U_{final} = -\frac{-GMm}{2R}$

Loss in PE = gain in $KE = \frac{GMm}{2R} - \frac{GMm}{3R} = \frac{GMm}{6R}$

15. (a) If body is projected with velocity v $(v < v_e)$ then

height up to which it will rise, $h = \frac{R}{\frac{v_e^2}{v^2} - 1}$

$v = \frac{v_e}{2}$ (given) $\therefore$ $h = \frac{R}{\left(\frac{v_e}{v_e/2}\right)^2 - 1} = \frac{R}{4-1} = \frac{R}{3}$

16. (d) Change in potential energy in displacing a body from r_1 to r_2 is given by

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = GMm\left(\frac{1}{2R} - \frac{1}{3R}\right) = \frac{GMm}{6R}$$

17. (c) $\frac{1}{2}mv_e^2 = \frac{1}{2}m\,2gR = mgR$

18. (a) $K.E. = \frac{GMm}{2R}$

19. (b) $I = \frac{-dV}{dr}$. If $I = 0$ then V = constant

20. (b) This should be equal to escape velocity *i.e.* $\sqrt{2gR}$

21. (c) $v_e = \sqrt{\frac{2GM}{R}}$ *i.e.* escape velocity depends upon the mass and radius of the planet.

22. (b) $v_e = \sqrt{\frac{2GM}{R}} = R\sqrt{\frac{8}{3}\pi G\rho}$

If mean density is constant then $v_e \propto R$

$\frac{v_e}{v_p} = \frac{R_e}{R_p} = \frac{1}{2} \Rightarrow v_e = \frac{v_p}{2}$

23. (a) Escape velocity does not depend on the mass of the projectile

24. (c) $\frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_p}{R_e}} = \sqrt{2 \times 2} = 2$

$\Rightarrow v_p = 2 \times v_e = 2 \times 11.2 = 22.4\ km/s$

25. (a) $v_e = \sqrt{\frac{2GM}{R}} = R\sqrt{\frac{8}{3}\pi G\rho}$ $\therefore v_e \propto R$ if ρ = constant

Since the planet having double radius in comparison to earth therefore the escape velocity becomes twice *i.e.* 22 *km/s.*

26. (b) If missile launched with escape velocity then it will escape from the gravitational field and at infinity its total energy becomes zero.

But if the velocity of projection is less than escape velocity then sum of energies will be negative. This shows that attractive force is working on the satellite.

27. (b)

28. (c)

29. (c) Because it does not depend on the mass of projectile

30. (b) $v_e = \sqrt{2}\,v_0$, *i.e.* if the orbital velocity of moon is increased by factor of $\sqrt{2}$ then it will escape out from the gravitational field of earth.

31. (a)

32. (b) $v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e \propto \sqrt{M}$ if R = constant

If the mass of the planet becomes four times then escape velocity will become 2 times.

33. (c) Escape velocity $v_e = \sqrt{\frac{2GM}{R}}$

$\therefore \frac{v_e}{v_m} = \sqrt{\frac{M_e R_m}{M_m R_e}} = \sqrt{\frac{81}{3.5}} = 4.81$

34. (a) $v_e = \sqrt{\frac{2GM}{R}}$ $\therefore$ $v_e \propto \sqrt{\frac{M}{R}}$

If mass and radius of the planet are three times than that of earth then escape velocity will be same.

35. (c) Potential energy of a body at the surface of earth

$PE = -\frac{GMm}{R} = -\frac{gR^2 m}{R} = -mgR$

$= -500 \times 9.8 \times 6.4 \times 10^6 = -3.1 \times 10^{10}\ J$

So if we give this amount of energy in the form of kinetic energy then body escape from the earth.

36. (d) Escape velocity $v = \sqrt{\frac{2GM}{R}} \Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$

$\Rightarrow v_p = 5v_e = 5 \times 11.2 = 56\ km/s$

37. (a) $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}} = \sqrt{6 \times \frac{1}{2}} = \sqrt{3}$ $\therefore$ $v_p = \sqrt{3}\, v_e$

38. (a) $\frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_p}{R_e}} = \sqrt{9 \times 4} = 6$ $\therefore$

$v_p = 6 \times v_e = 67.2 km/s$

39. (c) $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_p}{R_e}} = \sqrt{8 \times \frac{1}{2}} = 2$ $\therefore$

$v_p = 2 \times v_e = 22.4 km/s$

40. (c) $v_e = \sqrt{\frac{2GM}{R}}$ $\therefore$ $v_e \propto \sqrt{\frac{M}{R}}$

If *M* becomes double and *R* becomes half then escape velocity becomes two times.

41. (c) On earth $v_e = \sqrt{\frac{2GM}{R}} = 11.2\ km/s$

On moon $v_m = \sqrt{\frac{2GM \times 4}{81 \times R}} = \frac{2}{9}\sqrt{\frac{2GM}{R}}$

$= \frac{2}{9} \times 11.2 = 2.5\ km/s$

42. (c) Escape velocity $v = \sqrt{\frac{2GM}{R}}$

If star rotates with angular velocity ω

then $\omega = \frac{v}{R} = \frac{1}{R}\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^3}}$

43. (d) Escape velocity from surface of earth $v_e = \sqrt{2gR}$

$= \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 \times 10^3\ m/s$

44. (a)

45. (a) $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e}\frac{R_e}{R_p}} = \sqrt{(1000) \times \left(\frac{1}{10}\right)} = 10$

$v_p = 10 \times 11.2 = 112\ km/s$

46. (b) $v_e = R\sqrt{\frac{8}{3}G\pi\rho}$ $\therefore$ $v_e \propto R\sqrt{\rho}$

47. (d) $v_e = \sqrt{\frac{2GM}{(R+h)}}$

48. (a) $v = \sqrt{2gR}$. If acceleration due to gravity and radius of the planet, both are double that of earth then escape velocity will be two times. *i.e.* $v_p = 2v_e$

49. (a)

50. (b) $v = \sqrt{2gR} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{g_A}{g_B} \times \frac{R_A}{R_B}} = \sqrt{k_1 \times k_2} = \sqrt{k_1 k_2}$

51. (d) $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{R}} = 3 \times 10^8$

By solving $R = 9\ mm$

52. (a) $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}} = \sqrt{2 \times \frac{1}{3}} = \sqrt{\frac{2}{3}}$ $\therefore$ $v_p = \sqrt{\frac{2}{3}}v_e$

53. (c) $v_e \propto \frac{1}{\sqrt{R}}$. If *R* becomes $\frac{1}{4}$ then v_e will be 2 times.

54. (b)

55. (a) $v = \sqrt{2gR} \Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_e}{R_p}} = \sqrt{2 \times \frac{1}{4}} = \frac{1}{\sqrt{2}}$

$\therefore$ $v_p = \frac{v_e}{\sqrt{2}}$

56. (b) $v \propto R\sqrt{\rho}$ $\therefore$ $\frac{v_p}{v_e} = \frac{R_p}{R_e} \times \sqrt{\frac{\rho_p}{\rho_e}} = 4 \times \sqrt{9} = 12$

$\Rightarrow v_p = 12 v_e$

57. (d) Escape velocity does not depends upon the angle of projection.

58. (d)

59. (b) Escape velocity is independent of mass of object.

60. (b) $v = \sqrt{2gR} \Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_p}{R_e}} = \sqrt{1 \times 4} = 2$

$\therefore$ $v_p = 2v_e$

61. (c) $v = \sqrt{2gR} \Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_p}{R_e}} = \sqrt{1 \times 4} = 2$

$\Rightarrow v_p = 2 \times v_e = 2 \times 11.2 = 22.4\ km/s$

62. (b) $v = \sqrt{2gR}$. If *g* and *R* both are doubled then *v* will becomes two times *i.e.* 11.2 × 2 = 22.4 *km/s*

63. (c) $v = R\sqrt{\frac{8}{3}\pi\rho G} \Rightarrow \frac{v_p}{v_e} = \frac{R_p}{R_e}\sqrt{\frac{\rho_p}{\rho_e}} = 2\sqrt{\frac{1}{4}} = 1$

64. (c) Velocity of body in inter planetary space $v' = \sqrt{v^2 - v_{es}^2}$

where $v_{es} =$ escape velocity and

v = velocity of projection

$\therefore$ $v' = \sqrt{(2v_{es})^2 - v_{es}^2} = \sqrt{3v_{es}^2} \Rightarrow v' = \sqrt{3}\ v_{es}$

65. (b) Potential energy of system of two mass

$U = \frac{-GMm}{R} = \frac{-6.67 \times 10^{-11} \times 100 \times 10 \times 10^{-3}}{10 \times 10^{-2}}$

$U = -6.67 \times 10^{-10} J$

So, the amount of work done to take the particle up to infinite will be 6.67×10^{-10} J

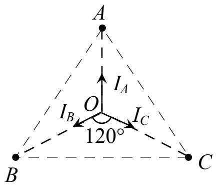
66. (b) For a moving satellite kinetic energy $= \frac{GMm}{2r}$

Potential energy $= \frac{-GMm}{r}$

$\therefore \frac{\text{Kinetic energy}}{\text{Potential energy}} = \frac{1}{2}$

67. (a) Due to three particles net intensity at the centre

$I = \vec{I}_A + \vec{I}_B + \vec{I}_C = 0$

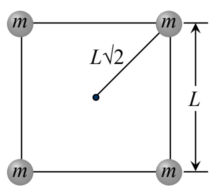


because out of these three intensities one equal in magnitude and the angle between each other is 120°.

68. (c) $v_0 = \frac{v_e}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\ km/s$

69. (a) Potential at the centre due to single mass = $\frac{-GM}{L/\sqrt{2}}$

Potential at the centre due to all four masses



$= -4\frac{GM}{L/\sqrt{2}} \quad -4\sqrt{2}\frac{GM}{L}$

$= -\sqrt{32} \times \frac{GM}{L}$.

70. (a) $v = \sqrt{2gR} \therefore \frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2} \times \frac{R_1}{R_2}} = \sqrt{g \times K} = (Kg)^{1/2}$

Motion of Satellite

1. (b) $v_e = \sqrt{2gR}$ and $v_0 = \sqrt{gR} \therefore \sqrt{2}\, v_0 = v_e$

2. (d)

3. (d) $v_0 = \sqrt{\frac{GM}{r}}$

4. (d) $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{R+h}}$

5. (b) Centripetal acceleration works on it.

6. (b) $v = \sqrt{\frac{GM}{r}}$ if $r_1 > r_2$ then $v_1 < v_2$

Orbital speed of satellite does not depends upon the mass of the satellite

7. (b) $T \propto r^{3/2}$. If r becomes double then time period will becomes $(2)^{3/2}$ times.

So new time period will be $24 \times 2\sqrt{2}\ hr$ *i.e.* $T = 48\sqrt{2}$

8. (b) *K.E.* required for satellite to escape from earth's gravitational field

$\frac{1}{2}mv_e^2 = \frac{1}{2}m\left(\sqrt{\frac{2GM}{R}}\right)^2 = \frac{GMm}{R}$

K.E. required for satellite to move in circular orbit

$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{R}}\right)^2 = \frac{GMm}{2R}$

The ratio between these two energies = 2

9. (c) The velocity of the spoon will be equal to the orbital velocity when dropped out of the space-ship.

10. (c)

11. (b) $v_0 = \sqrt{gR}$

12. (d) Telecommunication satellites are geostationary satellite

13. (b) $v = \sqrt{\frac{GM}{R}} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$

$\therefore \frac{v_A}{v_B} = \frac{3V}{v_B} = \frac{1}{2} \therefore v_B = 6V$

14. (a)

15. (a)

16. (b)

17. (c) $v = \sqrt{\frac{GM}{R+h}}$

For first satellite $h = 0$, $v_1 = \sqrt{\frac{GM}{R}}$

For second satellite $h = \frac{R}{2}$, $v_2 = \sqrt{\frac{2GM}{3R}}$

$v_2 = \sqrt{\frac{2}{3}}v_1 = \sqrt{\frac{2}{3}}v$

18. (d) $v = \sqrt{\frac{GM}{r}} \quad \therefore K.E. \propto v^2 \propto \frac{1}{r}$ and $T^2 \propto r^3$

$\therefore K.E. \propto T^{-2/3}$

19. (d)

20. (d) $T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow r^3 = \frac{GMT^2}{4\pi^2} \Rightarrow r = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3}$

21. (d) $v \propto \frac{1}{\sqrt{r}}$. The speed of satellite decreases with an increase in the radius of its orbit.

22. (b) $v \propto \frac{1}{\sqrt{r}}$.

% increase in speed = $\frac{1}{2}$ (% decrease in radius)

$$= \frac{1}{2}(1\%) = 0.5\%$$

i.e. speed will increase by 0.5%

23. (b) $v = \sqrt{\frac{GM}{r}}$

24. (a)

25. (a) $v \propto \frac{1}{\sqrt{r}}$. If orbital radius becomes 4 times then orbital velocity will become half. *i.e.* $\frac{7}{2} = 3.5\ km/s$

26. (d) Orbital radius of satellites $r_1 = R + R = 2R$

$$r_2 = R + 7R = 8R$$

$U_1 = \frac{-GMm}{r_1}$ and $U_2 = \frac{-GMm}{r_2}$

$K_1 = \frac{GMm}{2r_1}$ and $K_2 = \frac{GMm}{2r_2}$

$E_1 = \frac{GMm}{2r_1}$ and $E_2 = \frac{GMm}{2r_2}$

$\therefore \frac{U_1}{U_2} = \frac{K_1}{K_2} = \frac{E_1}{E_2} = 4$

27. (a) Escape velocity is same for all angles of projection.

28. (d) Orbital velocity $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}}$ and $v_0 = r\omega$

This gives $r^3 = \frac{R^2 g}{\omega^2}$

29. (a)

30. (a) Due to inertia it will continue to move along the original path of the space craft.

31. (b)

32. (d)

33. (a)

34. (c) (i) $T_{st} = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$ [As $h << R$ and $GM = gR^2$]

(ii) $T_{ma} = 2\pi\sqrt{\frac{R}{g}}$

(iii) $T_{sp} = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}} = 2\pi\sqrt{\frac{R}{2g}}$

[As $l = R$]

(iv) $T_{is} = 2\pi\sqrt{\frac{R}{g}}$ [As $l = \infty$]

35. (d)

36. (c) $v \propto \frac{1}{\sqrt{r}}$, If $r = R$ then $v = V_0$

If $r = R + h = R + 3R = 4R$ then $v = \frac{V_0}{2} = 0.5\,V_0$

37. (d)

38. (b) $6R$ from the surface of earth and $7R$ from the centre.

39. (c) $T^2 = \frac{4\pi^2}{GM}r^3$. If G is variable then time period, angular velocity and orbital radius also changes accordingly.

40. (b) $T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{(2R)^3}{gR^2}} = 4\sqrt{2}\pi\sqrt{\frac{R}{g}}$

41. (c) $T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T^2 = \frac{4\pi^2}{GM}(R+h)^3$

$$\Rightarrow R + h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} \Rightarrow h = \left[\frac{GMT^2}{4\pi^2}\right]^{\frac{1}{3}} - R$$

42. (d) Distances of the satellite from the centre are $7R$ and $3.5R$ respectively.

$$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} \Rightarrow T_2 = 24\left(\frac{3.5R}{7R}\right)^{3/2} = 6\sqrt{2}\ hr$$

43. (a) $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{384000\times10^3}} = 1\,km/s$

44. (d) % change in $T = \frac{3}{2}$ (% change in R)

$$= \frac{3}{2}\times(2)\% = 3\%$$

45. (a)

46. (c)

47. (c) $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{10\times(64\times10^5)^2}{8000\times10^3}}$

$= 71.5\times10^2\ m/s = 7.15\,km/s$

48. (d) Total mechanical energy of satellite

$E = \frac{-GMm}{2r} \Rightarrow \frac{E_A}{E_B} = \frac{m_A}{m_B} \times \frac{r_B}{r_A} \Rightarrow \frac{3}{1} \times \frac{4r}{r} = \frac{12}{1}$

49. (b)

50. (b) Gravitational force provides the required centripetal force for orbiting the satellite

$\frac{mv^2}{R} = \frac{K}{R}$ because $\left(F \propto \frac{1}{R}\right)$

$\therefore\ v \propto R^0$

51. (a) Total energy = – (kinetic energy) = $-\frac{1}{2}Mv^2$

52. (a) Binding energy = – kinetic energy

And if this amount of energy (E_k) given to satellite then it will escape into outer space

53. (a) Potential energy = $\frac{-GMm}{r} = \frac{GMm}{R_e + h} = \frac{-GMm}{2R_e}$

$= -\frac{gR_e^2 m}{2R_e} = -\frac{1}{2}mgR_e = -0.5mgR_e$

54. (c) $B.E. = -\frac{GMm}{r}$. If *B.E.* decreases then *r* also decreases and *v* increases as $v \propto \frac{1}{\sqrt{r}}$

55. (b) Time period of communication satellite $T_c = 1$ day

Time period of another satellite = T_s

$\frac{T_s}{T_c} = \left(\frac{r_s}{r_c}\right)^{3/2} = (4)^{3/2} \Rightarrow T_s = T_c \times (4)^{3/2} = 8$ days.

56. (b) Angular momentum is conserved in central field.

57. (b) $T^2 \propto r^3$

58. (d) $T^2 \propto R^3\ \therefore\ \frac{T^2}{R^3} =$ constant

59. (d) $T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = T_1(4)^{3/2} = 8T_1 = 40hr$

60. (b) Given that, $T_1 = 1$ day and $T_2 = 8$ days

$\therefore\ \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \Rightarrow \frac{r_2}{r_1} = \left(\frac{T_2}{T_1}\right)^{2/3} = \left(\frac{8}{1}\right)^{2/3} = 4$

$\Rightarrow r_2 = 4r_1 = 4R$

61. (c)

62. (c) $\frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{4R}{R}} = 2$

$\Rightarrow v_B = 2 \times v_A = 2 \times 3v = 6v$

63. (b) Gravitational force provides the required centripetal force

$m\omega^2 R = \frac{GMm}{R^3} \Rightarrow \frac{4\pi^2}{T^2} = \frac{GM}{R^4} \Rightarrow T \propto R^2$

64. (a)

65. (b) $v_e = \sqrt{2}v_0 = 1.414\ v_0$

Fractional increase in orbital velocity $\left(\frac{\Delta v}{v}\right)$

$= \frac{v_e - v_0}{v_0} = 0.414$

∴ Percentage increase = 41.4%

66. (c) Time period of revolution of moon around the earth

= 1 lunar month.

$\frac{T_s}{T_m} = \left(\frac{r_s}{r_m}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2} \Rightarrow T_s = 2^{-3/2}$ lunar month.

67. (d)

Kepler's Laws of Planetary Motion

1. (c) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = (1000)^{1/2} = 10\sqrt{10}$

2. (c) Areal velocity of the planet remains constant. If the areas *A* and *B* are equal then $t_1 = t_2$.

3. (c) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} \Rightarrow T_2 = 8T_1$

4. (b)

5. (c)

6. (b) Since $T^2 \propto r^3\ \therefore \left(\frac{T'}{T}\right)^2 = \left(\frac{1}{4}\right)^3 \Rightarrow T' = \frac{1}{8}T$

7. (b) Kinetic and potential energies varies with position of earth *w.r.t.* sun. Angular momentum remains constant every where.

8. (c)

9. (c) Angular momentum remains constant

$mv_1d_1 = mv_2d_2 \Rightarrow v_2 = \frac{v_1d_1}{d_2}$

10. (b) Orbital radius of Jupiter > Orbital radius of Earth

$\frac{v_J}{v_e} = \frac{r_e}{r_J}$. As $r_J > r_e$ therefore $v_J < v_e$

11. (d) $T^2 \propto r^3 \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

12. (b) $\frac{dA}{dt} = \frac{L}{2m} =$ constant

13. (c) The earth moves around the sun is elliptical path. so by using the properties of ellipse

$r_1 = (1+e)a$ and $r_2 = (1-e)a$

$\Rightarrow a = \frac{r_1 + r_2}{2}$ and $r_1 r_2 = (1-e^2)a^2$

where a = semi major axis

b = semi minor axis

e = eccentricity

Now required distance = semi latusrectum $= \frac{b^2}{a}$

$= \frac{a^2(1-e^2)}{a} = \frac{(r_1 r_2)}{(r_1 + r_2)/2} = \frac{2r_1 r_2}{r_1 + r_2}$

14. (c) For first satellite $r_1 = R$ and $T_1 = 83$ *minute*

For second satellite $r_2 = 4R$

$T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = T_1(4)^{3/2} = 8T_1 = 8 \times 83 = 664$ *minutes*

15. (a) Angular momentum = Mass ×Orbital velocity ×Radius

$= m \times \left(\sqrt{\frac{GM}{R_0}}\right) \times R_0 = m\sqrt{GMR_0}$

16. (b) $\frac{T^2}{r^3} =$ constant $\Rightarrow T^2 r^{-3} =$ constant

17. (d) $\frac{T_{plant}}{T_{earth}} = \left(\frac{r_{plant}}{r_{earth}}\right)^{3/2} = (1.588)^{3/2} = 2$ $\therefore$

$T_{planet} = 2$ *year*

18. (d) Mass of the satellite does not effects on time period

$\frac{T_A}{T_B} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \left(\frac{1}{8}\right)^{1/2} = \frac{1}{2\sqrt{2}}$

19. (b) Speed of the earth will be maximum when its distance from the sun is minimum because $mvr =$ constant

20. (c) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2} \Rightarrow 8 = \left(\frac{r_A}{r_B}\right)^{3/2} \Rightarrow r_A = (8)^{2/3} r_B = 4r_B$.

21. (c) $T^2 \propto r^3$. If r made half then T will become $\frac{T}{8}$.

22. (c)

23. (b) Mass of satellite does not affects on orbital radius.

24. (b) $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = (2)^{3/2} = 2\sqrt{2} \Rightarrow T_2 = 2\sqrt{2}$ *years.*

25. (c)

26. (a)

27. (a) By conservation of angular momentum mvr = constant

$v_{min} \times r_{max} = v_{max} \times r_{min}$

$\therefore v_{min} = \frac{60 \times 1.6 \times 10^{12}}{8 \times 10^{12}} = \frac{60}{5} = 12\, m/s$

28. (b) $\omega_{body} = 27\omega_{earth}$

$T^2 \propto r^3 \Rightarrow \omega^2 \propto \frac{1}{r^3} \Rightarrow \omega \propto \frac{1}{r^{3/2}} \therefore r \propto \frac{1}{\omega^{2/3}}$

$\Rightarrow \frac{r_{body}}{r_{earth}} = \left(\frac{\omega_{earth}}{\omega_{body}}\right)^{2/3} = \left(\frac{1}{27}\right)^{2/3} = \frac{1}{9}$

29. (d) Time period does not depends upon the mass of satellite.

30. (b) $\frac{T^2}{R^3} = \frac{T^2}{d^3} = \frac{1}{n^2 d^3} =$ constant

$\therefore n_1^2 d_1^3 = n_2^2 d_2^3$ [where n = frequency]

31. (a)

32. (a)

33. (a) During path *DAB* planet is nearer to sun as comparison with path *BCD*. So time taken in travelling *DAB* is less than that for *BCD* because velocity of planet will be more in region *DAB*.

34. (c) Because distance of point C is maximum from the sun.

35. (b) $T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 1 \times (2)^{3/2} = 2.8$ *year*

36. (d) $L = mvr = m\sqrt{\frac{GM}{r}}\, r = m\sqrt{GMr}$ $\therefore L \propto \sqrt{r}$

37. (a)

38. (c) $\frac{dA}{dt} = \frac{L}{2m} = \frac{dA}{dt} \propto vr \propto \omega r^2$

39. (b)

40. (c)

41. (d)

42. (a) $T^2 \propto R^3 \Rightarrow \left(\frac{T_P}{T_E}\right)^2 = \left(\frac{R_P}{R_E}\right)^3 = \left(\frac{2R_E}{R_E}\right)^3$

$\Rightarrow \frac{T_P}{T_E} = (2)^{3/2} = 2\sqrt{2}$

$\Rightarrow T_P = 2\sqrt{2} \times 365 = 1032.37 = 1032$ days

43. (d)

44. (d) For central force, torque is zero.

$\because \tau = \frac{dL}{dt} = 0 \Rightarrow L =$ constant

i.e. Angular momentum is constant.

45. (b) $\frac{v_{max}}{v_{min}} = \frac{1+e}{1-e} = \frac{1+0.0167}{1-0.0167} = 1.033$

46. (a) $m\omega^2 R = \frac{GMm}{R^2} \Rightarrow \left(\frac{2\pi}{T}\right)^2 R = \frac{GM}{R^2} \Rightarrow M = \frac{4\pi^2 R^3}{GT^2}$

47. (b) $v = \sqrt{\frac{GM}{R}} = G^{1/2} M^{1/2} R^{-1/2}$

48. (b)

49. (c) $v = \sqrt{\frac{GM}{R}}$ if $r_1 > r_2$ then $v_1 < v_2$

50. (c) Escape velocity for that body $v_e = \sqrt{\frac{2Gm}{r}}$

v_e should be more than or equal to speed of light

i.e. $\sqrt{\frac{2Gm}{r}} \geq c$

51. (c) $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2} = \frac{1}{8}$

Critical Thinking Questions

1. (b) For revolution of planet centripetal force is provided by gravitational force of attraction

$m\omega^2 R \propto R^{-5/2} \Rightarrow \frac{1}{T^2} \propto R^{-7/2} \Rightarrow T^2 \propto R^{7/2}$

2. (a, b) $g = \frac{4}{3}\pi\rho G r \quad \therefore \; g \propto r$ if $r < R$

$g = \frac{GM}{r^2} \quad \therefore \; g \propto \frac{1}{r^2}$ if $r > R$

If $r_1 < R$ and $r_2 < R$ then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \frac{r_1}{r_2}$

If $r_1 > R$ and $r_2 > R$ then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2$

3. (a)

4. (b) $F = \frac{Gm(M-m)}{r^2}$

For maximum force $\frac{dF}{dm} = 0$

$\Rightarrow \frac{d}{dm}\left(\frac{GmM}{r^2} - \frac{Gm^2}{r^2}\right) = 0$

$\Rightarrow M - 2m = 0 \Rightarrow \frac{m}{M} = \frac{1}{2}$

5. (a) $m\omega^2 R \propto \frac{1}{R^n} \Rightarrow m\left(\frac{4\pi^2}{T^2}\right) R \propto \frac{1}{R^n} \Rightarrow T^2 \propto R^{n+1}$

$\therefore T \propto R^{\left(\frac{n+1}{2}\right)}$

6. (c) $g = \frac{GM}{R^2}$. If mass remains constant then $g \propto \frac{1}{R^2}$

% increase in g = 2(% decrease in R) = 2 × 1% = 2%.

7. (b, c, d) $g = \frac{GM}{R^2}, v_e = \sqrt{\frac{2GM}{R}}$ and $U = \frac{-GMm}{R}$

$\therefore \; g \propto \frac{M}{R^2}, v_e \propto \sqrt{\frac{M}{R}}$ and $U \propto \frac{M}{R}$

If both mass and radius are increased by 0.5% then v_e and U remains unchanged where as g decrease by 0.5%.

8. (b) $g' = g - \omega^2 R\cos^2\lambda$

For weightlessness at equator $\lambda = 0$ and $g' = 0$

$\therefore \; 0 = g - \omega^2 R \Rightarrow \omega = \sqrt{\frac{g}{R}} = \frac{1}{800} \frac{rad}{s}$

9. (d) If acceleration due to gravity is g at the surface of earth then at height R it value becomes $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{4}$

$T_1 = 2\pi\sqrt{\frac{l}{g}}$ and $T_2 = 2\pi\sqrt{\frac{l}{g/4}} \; \therefore \frac{T_2}{T_1} = 2$

10. (b) $\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{1}{2} mgR \; (\because h = R)$

11. (c) Potential energy = 2 × (Total energy) = $2E_0$

Because we know = $U = \frac{-GMm}{r}$ and $E_0 = \frac{-GMm}{2r}$

12. (c) $\Delta K.E. = \Delta U$

$\Rightarrow \frac{1}{2}MV^2 = GM_eM\left(\frac{1}{R} - \frac{1}{R+h}\right)$ …(i)

Also $g = \frac{GM_e}{R^2}$ …(ii)

On solving (i) and (ii) $h = \frac{R}{\left(\frac{2gR}{V^2} - 1\right)}$

13. (a, c, d) Since cavities are symmetrical *w.r.t.* *O*. So the gravitational force at the centre is zero.

The radius of the circle $z^2 + y^2 = 36$ is 6.

For all points for $r \geq 6$, the body behaves as if whole of the mass is concentrated at the centre. So the gravitational potential is same.

Above is true for $z^2 + y^2 = 4$ as well.

14. (b) Let velocities of these masses at *r* distance from each other be v_1 and v_2 respectively.

By conservation of momentum

$m_1v_1 - m_2v_2 = 0$

$\Rightarrow m_1v_1 = m_2v_2$ … (i)

By conservation of energy

change in *P.E.*=change in *K.E.*

$\frac{Gm_1m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\Rightarrow \frac{m_1^2v_1^2}{m_1} + \frac{m_2^2v_2^2}{m_2} = \frac{2\,Gm_1m_2}{r}$ …(ii)

On solving equation (i) and (ii)

$v_1 = \sqrt{\frac{2\,Gm_2^2}{r(m_1+m_2)}}$ and $v_2 = \sqrt{\frac{2\,Gm_1^2}{r(m_1+m_2)}}$

$\therefore v_{app} = |v_1| + |v_2| = \sqrt{\frac{2G}{r}(m_1+m_2)}$

15. (c) Kinetic energy = Potential energy

$\frac{1}{2}m(kv_e)^2 = \frac{mgh}{1+\frac{h}{R}} \Rightarrow \frac{1}{2}mk^2 2gR = \frac{mgh}{1+\frac{h}{R}}$

$\Rightarrow h = \frac{Rk^2}{1-k^2}$

Height of Projectile from the earth's surface = *h*

Height from the centre $r = R + h = R + \frac{Rk^2}{1-k^2}$

By solving $r = \frac{R}{1-k^2}$

16. (c) In the problem orbital radius is increased by 1%.

Time period of satellite $T \propto r^{3/2}$

Percentage change in time period

$= \frac{3}{2}$(% change in orbital radius) $= \frac{3}{2}(1\%) = 1.5\%$

17. (b) According to Kepler's third law, the ratio of the squares of the periods of any two planets revolving about the sun is equal to the ratio of the cubes of their average distances from the sun *i.e.*

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left[\frac{r_1}{\frac{1}{2}r_1}\right]^3 = 8 \Rightarrow \frac{T_1}{T_2} = 2\sqrt{2}$

$\therefore T_2 = \frac{T_1}{2\sqrt{2}} = \frac{365\ days}{2\sqrt{2}} = 129\ days$

18. (c) $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \Rightarrow T_2 = 24\left(\frac{6400}{36000}\right)^{3/2} \cong 2\ hour$

Graphical Questions

1. (c) $g \propto r$ (if $r < R$) and $g \propto \frac{1}{r^2}$ (if $r > R$)

2. (c) $V_{in} = \frac{-Gm}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]$, $V_{surface} = \frac{-GM}{R}$, $V_{out} = \frac{-GM}{r}$

3. (c) For hollow sphere

$V_{in} = \frac{-GM}{R}$, $V_{surface} = \frac{-GM}{R}$, $V_{out} = \frac{-GM}{r}$

i.e. potential remain constant inside the sphere and it is equal to potential at the surface and increase when the point moves away from the surface of sphere.

4. (b) $F = 0$ when $0 \leq r \leq R_1$

because intensity is zero inside the cavity.

F increase when $R_1 \leq r \leq R_2$

$F \propto \frac{1}{r^2}$ when $r > R_2$

5. (d) Intensity will be zero inside the spherical shell.

I = 0 upto *r* = *a* and $I \propto \frac{1}{r^2}$ when $r > a$

6. (c) Initially the weight of the passenger = 60 × 10 = 600 N

Finally the weight of the passenger = 60 × 4 = 240 N

and during the flight in between some where its weight will be zero because at that point gravitational pull of earth and mars will be equal.

7. (c) Kepler's law $T^2 \propto R^3$

8. (d) The system will be bound at points where total energy is negative. In the given curve at point A, B and C the P.E. is more than K.E.

9. (c) $U = \frac{-GMm}{r}, K = \frac{GMm}{2r}$ and $E = \frac{-GMm}{2r}$

For a satellite U,K and E varies with r and also U and E remains negative whereas K remain always positive.

10. (c) Gravitational P.E. = m × gravitational potential

$U = mV$ So the graph of U will be same as that of V for a spherical shell.

Assertion and Reason

1. (a) According to Kepler's third law $T^2 \propto r^3$

If r is small then T will also be small.

2. (b) For two electron $\frac{F_g}{F_e} = 10^{-43}$ *i.e.* gravitational force is negligible in comparison to electrostatic force of attraction.

3. (d) The universal gravitational constant G is totally different from g.

$$G = \frac{FR^2}{Mm}$$

The constant G is scalar and posses the dimensions $[M^{-1}L^3T^{-2}]$.

$$g = \frac{GM}{R^2}$$

g is a vector and has got the dimensions $[M^0LT^{-2}]$.

It is not a universal constant.

4. (c) Acceleration due to gravity is given by $g = \frac{GM}{R^2}$. Thus it does not depend on mass of body on which it is acting. Also it is not a constant quantity it changes with change in value of both M and R (distance between two bodies).

5. (a) If a pendulum is suspended in a lift and lift is moving downward with some acceleration a, then time period of pendulum is given by, $T = 2\pi\sqrt{\frac{l}{g-a}}$.

In the case of free fall, $a = g$ then $T = \infty$

i.e., the time period of pendulum becomes infinite.

6. (c) The value of g at any place is given by the relation, $g' = g - \omega^2 R_e \cos^2\lambda$

When λ is angle of latitude and ω is the angular velocity of earth. If earth suddenly stops rotating, then $\omega = 0$

$\therefore$ $g' = g$ *i.e.*, the value of g will be same at all places.

7. (b) Acceleration due to gravity,

$g' = g - R\omega^2\cos^2\lambda$

At equator, $\lambda = 0°$ *i.e.* $\cos 0° = 1$ $\therefore$ $g_e = g = R\omega^2$

At poles, $\lambda = 90°$ *i.e.* $\cos 90° = 0$ $\therefore$ $g_p = g$

Thus, $g_p - g_e = g - g + R\omega^2 = R\omega^2$

Also, the value of g is maximum at poles and minimum at equators.

8. (a) As the rotation of earth takes place about polar axis therefore body placed at poles will not feel any centrifugal force and its weight or acceleration due to gravity remains unaffected.

9. (e) Earth revolves around the sun in circular path and required centripetal force is provided by gravitational force between earth and sun but the work done by this centripetal force is zero.

10. (a) Inertial mass and gravitational mass are equivalent. Both are scalar quantities and measured in the same unit. They are quite different in the method of their measurement. Also gravitational mass of a body is affected by the presence of other bodies near it where as internal mass remain unaffected.

11. (a) Because gravitational force is always attractive in nature and every body is bound by this gravitational force of attraction of earth.

12. (a) As no torque is acting on the planet, its angular momentum must remain constant in magnitude as well as direction. Therefore, plane of rotation must pass through the centre of earth.

13. (c) According to Kepler's law of planetary motion, a planet revolves around the sun in such a way that its areal velocity is constant. *i.e.*, it move faster, when it is closer the sun and vice-versa.

14. (e) Escape velocity = $\sqrt{2} \times$ orbital velocity.

15. (c) Due to resistance force of atmosphere, the satellite revolving around the earth losses kinetic energy. Therefore in a particular orbit the gravitational attraction of earth on satellite becomes greater than that required for circular orbit there. Therefore satellite moves down to a lower orbit. In the lower orbit as the potential energy $(U = -GMm/r)$ becomes more negative, Hence kinetic energy $(E_K = GMm/2r)$ increases, and hence speed of satellite increases.

16. (b) If root mean square velocity of the gas molecules is less than escape velocity from that planet (or satellite) then atmosphere will remain attached with that planet and if $v_{rms} > v_{\text{escape}}$ then there will be no atmosphere on the planet. This is the reason for no atmosphere at moon.

17. (b) As the geostationary satellite is established in an orbit in the plane of the equator at a particular place, so it move in the same sense as the earth and hence its time period of revolution is equal to 24 hours, which is equal to time period of revolution of earth about its axis.

18. (e) The total gravitational force on one particle due to number of particles is the resultant force of attraction (or gravitational force) exerted on the given particle due to individual particles. *i.e.*, $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + ...$ It means the principle of superposition is valid.

19. (d) As, escape velocity $= \sqrt{\frac{2GM}{R}}$, so its value depends on mass of planet and radius of the planet. The two different planets have same escape velocity, when these quantities (mass and radius) are equal.

20. (a) According to kepler's law $T^2 \propto r^3 \propto (R+h)^3$

i.e. if distance of satellite is more then its time period will be more.

21. (a) According to Newton's law of gravitation, $F = \frac{Gm_1m_2}{r^2}$. When m_1, m_2 and r_2 all are doubled,

$F = \frac{G(2m_1)(2m_2)}{(2r)^2} = \frac{Gm_1m_2}{r^2}$, *i.e.* F remains the same.

22. (c) Upto ordinary heights, the change in the distance of a projectile from the centre of earth is negligible compared to the radius of earth. Hence the projectile moves under a nearly uniform gravitational force and the path is parabolic. But for the projectiles moving to a large height, the gravitational force decreases quite decreasing variable force, the path of the projectile becomes elliptical.

23. (c) As the distance from centre of earth decreases, acceleration due to gravity decreases and at the centre of earth it becomes zero.

$g = g\left(1 - \frac{d}{R}\right)$. If $d = R$ then $g = 0$

24. (b) We know that earth revolves from west to east about its polar axis. Therefore, all the particles on the earth have velocity from west to east.

This velocity is maximum in the equatorial line, as $v = R\omega$, where R is the radius of earth and ω is the angular velocity of revolution of earth about its polar axis.

When a rocket is launched from west to east in equatorial plane, the maximum linear velocity is added to the launching velocity of the rocket, due to which launching becomes easier.

25. (e) Binding energy $= \frac{GMm}{R} = -$ [Total energy of satellite]

and it is clear that it depends upon the mass of the satellite.

26. (a) According to Newton's law of gravitation, every body in this universe attracts every other body with a force which is inversely proportional to the square of the distance between them. When we move our finger, the distance of the objects with respect to finger changes, hence the force of attraction changes, disturbing the entire universe, including stars.

27. (a) Intensity inside a hollow sphere is zero, so force is also equal to zero. $\vec{F} = m\vec{E}$

28. (a) The time period of satellite which is very near to earth is given by

$$T = 2\pi\sqrt{\frac{R}{g}} \cong 84\ min = 1 hr.\ 24\ min$$

29. (c) A person feels his weight only when the surface on which he is standing exerts a reactionary force on him. Because the acceleration of the person and that of the satellite revolving round the earth are equal (= g), hence acceleration of the person with respect to the satellite is zero.

Therefore person feels weightless on satellite, although the gravitational force is acting on a satellite.

30. (e) If the orbital path of a satellite is circular, then its speed is constant and if the orbital path of a satellite is elliptical, then its speed in its orbit is not constant. In that case its areal velocity is constant.

31. (a) $v_0 = R_e\sqrt{\frac{g}{R_e + h}}$

For satellite revolving very near to earth $R_e + h = R_e$

As ($h << R$)

$v_o = \sqrt{R_e g} \simeq \sqrt{64 \times 10^5 \times 10} = 8 \times 10^3\ m/s = 8\ kms^{-1}$

Which is independent of height of a satellite.

32. (b)

33. (a) The torque on a body is given by $\vec{\tau} = \frac{d\vec{L}}{dt}$

In case of planet orbiting around sun no torque is acting on it. $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} =$ constant.