Answers and Solutions

Static and Limiting Friction

1. (c)
2. (d) $\mu = \frac{F}{R} = \frac{F}{mg} = \frac{98}{100 \times 9.8} = \frac{1}{10} = 0.1$
3. (c) Here applied horizontal force F acts as normal reaction.

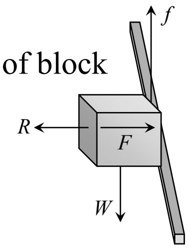
For holding the block

Force of friction = Weight of block

$f = W \Rightarrow \mu R = W \Rightarrow \mu F = W$

$\Rightarrow F = \frac{W}{\mu}$

As $\mu < 1 \therefore F > W$



4. (b)
5. (a)
6. (c)
7. (c) $F_l = \mu_s R = 0.4 \times mg = 0.4 \times 10 = 4\,N$ *i.e.* minimum 4N force is required to start the motion of a body. But applied force is only 3N. So the block will not move.
8. (a) For limiting condition $\mu = \frac{m_B}{m_A + m_C} \Rightarrow$

$0.2 = \frac{5}{10 + m_C}$

$\Rightarrow 2 + 0.2 m_C = 5 \Rightarrow m_C = 15 kg$

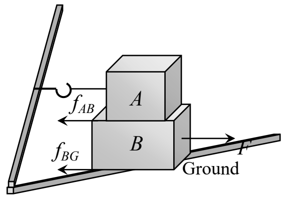
9. (a)
10. (d) Ball and bearing produce rolling motion for which force of friction is low. Lubrication and polishing reduce roughness of surface.
11. (c) For given condition we can apply direct formula

$l_1 = \left(\frac{\mu}{\mu + 1}\right) l$

12. (c) Sliding friction is greater than rolling friction.
13. (b) $F = \frac{W}{\mu} = \frac{1 \times 9.8}{0.2} = 49\,N$
14. (a) $l' = \left(\frac{\mu}{\mu + 1}\right) l = \left(\frac{0.25}{0.25 + 1}\right) l = \frac{l}{5} = 20\%$ of l.
15. (a) $\mu = \frac{m_B}{m_A} \Rightarrow 0.2 = \frac{m_B}{10} \Rightarrow m_B = 2 kg$
16. (d) Work done by friction can be positive, negative and zero depending upon the situation.
17. (c) $\mu = \frac{\text{Lenght of chain hanging from the table}}{\text{Lenght of chain lying on the table}} = \frac{l}{L - l}$
18. (b) Surfaces always slide over each other.
19. (a) Coefficient of friction $\mu_s = \frac{F_l}{R} = \frac{75}{mg} = \frac{75}{20 \times 9.8}$

$= 0.38$

20. (c)



$F = f_{AB} + f_{BG}$

$= \mu_{AB} m_a g + \mu_{BG}(m_A + m_B) g$

$= 0.2 \times 100 \times 10$

$+ 0.3(300) \times 10$

$= 200 + 900 = 1100\,N$

21. (c)
22. (b) $\mu = \tan$ (Angle of repose) $= \tan 60° = 1.732$
23. (a) Applied force $= 2.5\,N$

Limiting friction $= \mu mg = 0.4 \times 2 \times 9.8 = 7.84\,N$

For the given condition applied force is very smaller than limiting friction.

$\therefore$ Static friction on a body = Applied force = 2.5 N

24. (c) Sand is used to increase the friction.
25. (a) $F = \mu R = 0.3 \times 250 = 75\,N$
26. (b) For the given condition, Static friction

= Applied force = Weight of body

$= 2 \times 10 = 20\,N$

27. (a) $F = \frac{W}{\mu} \therefore W = \mu F = 0.2 \times 10 = 2N$
28. (d) $\mu_s = \frac{m_B}{m_A} \Rightarrow 0.2 = \frac{m_B}{2} \Rightarrow m_B = 0.4\,kg$

29. (a) $\mu_s = \frac{m_B}{m_A} \Rightarrow 0.2 = \frac{m_B}{10} \Rightarrow m_B = 2\ kg$

30. (d) $\mu_s = \frac{\text{Lenght of the chain hanging from the table}}{\text{Length of the chain lying on the table}}$

$= \frac{l/3}{l - l/3} = \frac{l/3}{2l/3} = \frac{1}{2}$

31. (d)

32. (a)

Kinetic Friction

1. (b)

2. (d) In the given condition the required centripetal force is provided by frictional force between the road and tyre.

$\frac{mv^2}{R} = \mu mg \quad \therefore \quad v = \sqrt{\mu Rg}$

3. (a) Retarding force $F = ma = \mu R = \mu mg \therefore a = \mu g$

Now from equation of motion $v^2 = u^2 - 2as$

$\Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a} = \frac{u^2}{2\mu g} \quad \therefore \quad = \frac{v_0^2}{2\mu g}$

4. (d) Net force = Applied force – Friction force

$ma = 24 - \mu mg = 24 - 0.4 \times 5 \times 9.8 = 24 - 19.6$

$\Rightarrow a = \frac{4.4}{5} = 0.88\ m/s^2$

5. (a) Work done = Force × Displacement $= \mu mg \times (v \times t)$

$W = (0.2) \times 2 \times 9.8 \times 2 \times 5$ *joule*

Heat generated $Q = \frac{W}{J} = \frac{0.2 \times 2 \times 9.8 \times 2 \times 5}{4.2}$

$= 9.33\ cal$

6. (c) For given condition $s \propto \frac{1}{m^2} \therefore$

$\frac{s_2}{s_1} = \left(\frac{m_1}{m_2}\right)^2 = \left(\frac{200}{300}\right)^2$

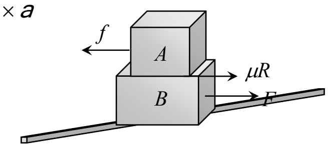
$\Rightarrow \quad s_2 = s_1 \times \frac{4}{9} = 36 \times \frac{4}{9} = 16\ m$

7. (a) There is no friction between the body B and surface of the table. If the body B is pulled with force F then

$F = (m_A + m_B)a$

Due to this force upper body A will feel the pseudo force in a backward direction.

$f = m_A \times a$



But due to friction between A and B, body will not move. The body A will start moving when pseudo force is more than friction force.

i.e. for slipping, $m_A a = \mu m_A g \therefore a = \mu g$

8. (d) Limiting friction $= \mu_s R = \mu_s mg = 0.5 \times 60 \times 10 = 300\ N$

Kinetic friction $= \mu_k R = \mu_k mg = 0.4 \times 60 \times 10 = 240\ N$

Force applied on the body = 300 N and if the body is moving then, Net accelerating force

=Applied force – Kinetic friction

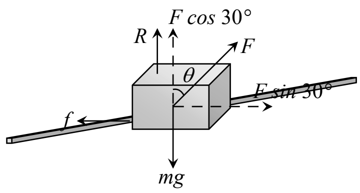
$\Rightarrow ma = 300 - 240 = 60 \quad \therefore a = \frac{60}{60} = 1\ m/s^2$

9. (a) $v = \sqrt{\mu g r} \Rightarrow r = \frac{v^2}{\mu g} = \frac{100}{0.5 \times 10} = 20$

10. (b)

11. (b) $S = \frac{u^2}{2\mu g} = \frac{(10)^2}{2 \times 0.2 \times 10} = 25\ m$

12. (d)



For limiting condition $f = \mu R$

$F\sin 30° = \mu(mg - F\cos 30°)$, By solving $F = 294.3\ N$

13. (c) Net force on the body = Applied force – Friction

$ma = F - \mu_k mg \Rightarrow \mu_k = \frac{F - ma}{mg} = \frac{129.4 - 10 \times 10}{10 \times 9.8} = 0.3$

14. (c) $v = \sqrt{\mu gr} = \sqrt{0.5 \times 9.8 \times 40} = \sqrt{196} = 14\ m/s$

15. (b) $s = \frac{u^2}{2\mu g} = \frac{(20)^2}{2 \times 0.5 \times 10} = 40\ m$

16. (d) Net force in forward direction = Accelerating force + Friction

$= ma + \mu mg = m(a + \mu g) = (1500 + 500)(1 + 0.2 \times 10)$

$= 2000 \times 3 = 6000\ N$

17. (b) $v = \sqrt{\mu rg} = \sqrt{0.4 \times 30 \times 9.8} = 10.84\ m/s$

18. (a) $W = \mu mgS = 0.2 \times 50 \times 9.8 \times 1 = 98\ J$

19. (a) $F_l = \mu mg = 0.6 \times 1 \times 9.8 = 5.88\ N$

Pseudo force on the block $= ma = 1 \times 5 = 5\ N$

Pseudo is less then limiting friction hence static force of friction = 5 *N*.

20. (d) $S = \dfrac{u^2}{2\mu g} = \dfrac{m^2u^2}{2\mu g m^2} = \dfrac{P^2}{2\mu m^2 g}$

21. (d) Weight of the body = 64*N*

so mass of the body $m = 6.4\ kg$, $\mu_s = 0.6$, $\mu_k = 0.4$

Net acceleration $= \dfrac{\text{Applied force - Kinetic friction}}{\text{Mass of the body}}$

$= \dfrac{\mu_s mg - \mu_k mg}{m} = (\mu_s - \mu_k)g = (0.6 - 0.4)g = 0.2\,g$

22. (b)

23. (b) $a = \dfrac{\text{Applied force - Kinetic friction}}{\text{mass}}$

$= \dfrac{100 - 0.5 \times 10 \times 10}{10} = 5 m/s^2$

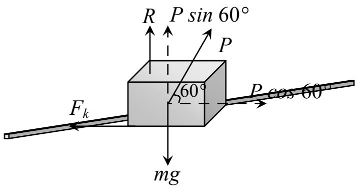
24. (b)

25. (d) $v = u - at \Rightarrow u - \mu gt = 0 \ \therefore\ \mu = \dfrac{u}{gt} = \dfrac{6}{10 \times 10} = 0.06$

26. (b) From the relation $F - \mu mg = ma$

$a = \dfrac{F - \mu mg}{m} = \dfrac{129.4 - 0.3 \times 10 \times 9.8}{10} = 10\ m/s^2$

27. (b) Let body is dragged with force *P*, making an angle 60° with the horizontal.



$F_k =$ Kinetic friction in the motion $= \mu_k R$

From the figure $F_k = P\cos 60°$ and $R = mg - P\sin 60°$

$\therefore\ P\cos 60° = \mu_k(mg - P\sin 60°)$

$\Rightarrow \dfrac{P}{2} = 0.5\left(60 \times 10 - \dfrac{P\sqrt{3}}{2}\right) \Rightarrow P = 315.1\ N$

$\therefore\ F_k = P\cos 60° = \dfrac{315.1}{2} N$

Work done $= F_k \times s = \dfrac{315.1}{2} \times 2 = 315\ Joule$

28. (d) $v = u - at \Rightarrow t = \dfrac{u}{a}$ [As $v = 0$]

$t = \dfrac{u \times m}{F} = \dfrac{30 \times 1000}{5000} = 6$ sec

29. (c)

30. (b) Kinetic energy acquired by body

= (Total work done on the body) – (work against friction)

$= F \times S - \mu mgS = 25 \times 10 - 0.2 \times 5 \times 10 \times 10$

$= 250 - 100 = 150\ Joule$

31. (a) $v = \sqrt{\mu rg} = \sqrt{0.5 \times 500 \times 10} = 50\ m/s$

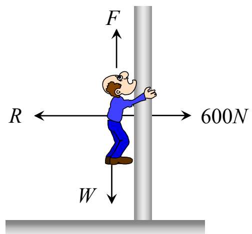
32. (d) Net downward acceleration

$= \dfrac{\text{Weight - Friction force}}{\text{Mass}}$

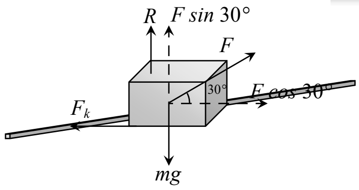
$= \dfrac{(mg - \mu R)}{m}$

$= \dfrac{60 \times 10 - 0.5 \times 600}{60}$

$= \dfrac{300}{60} = 5\ m/s^2$



33. (a)



Kinetic friction $= \mu_k R = 0.2(mg - F\sin 30°)$

$= 0.2\left(5 \times 10 - 40 \times \dfrac{1}{2}\right) = 0.2(50 - 20) = 6\ N$

Acceleration of the block $= \dfrac{F\cos 30° - \text{Kinetic friction}}{\text{Mass}}$

$= \dfrac{40 \times \dfrac{\sqrt{3}}{2} - 6}{5} = 5.73\ m/s^2$

34. (b) We know $s = \dfrac{u^2}{2\mu g}$ $\therefore$ $\mu = \dfrac{u^2}{2gs} = \dfrac{(6)^2}{2 \times 10 \times 9} = 0.2$

35. (d) $s = \dfrac{u^2}{2\mu g} = \dfrac{(100)^2}{2 \times 0.5 \times 10} = 1000\ m$

36. (d) Kinetic energy of the cylinder will go against friction

$\therefore$ $\dfrac{1}{2}mv^2 = \mu\, mgs \Rightarrow$

$s = \dfrac{u^2}{2\mu g} = \dfrac{(10)^2}{2 \times (0.5) \times 10} = 10\,m$

Motion on Inclined Surface

1. (b) When the body is at rest then static friction works on it, which is less than limiting friction (μR).

2. (b)

3. (c) Coefficient of friction = Tangent of angle of repose

$\therefore \; \mu = \tan\theta$

4. (a) $\mu = \tan\theta\left(1 - \frac{1}{n^2}\right) = 1 - \frac{1}{n^2}$ [As $\theta = 45^\circ$]

5. (a) Retardation in upward motion $= g(\sin\theta + \mu\cos\theta)$

$\therefore$ Force required just to move up $F_{up} = mg(\sin\theta + \mu\cos\theta)$

Similarly for down ward motion $a = g(\sin\theta - \mu\cos\theta)$

$\therefore$ Force required just to prevent the body sliding down

$F_{dn} = mg(\sin\theta - \mu\cos\theta)$

According to problem $F_{up} = 2F_{dn}$

$\Rightarrow mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$

$\Rightarrow \sin\theta + \mu\cos\theta = 2\sin\theta - 2\mu\cos\theta$

$\Rightarrow 3\mu\cos\theta = \sin\theta \Rightarrow \tan\theta = 3\mu$

$\Rightarrow \theta = \tan^{-1}(3\mu) = \tan^{-1}(3 \times 0.25) = \tan^{-1}(0.75)$

$= 36.8^\circ$

6. (c) $\mu = \tan\theta\left(1 - \frac{1}{n^2}\right)$

$\theta = 45^\circ$ and $n = 2$ (Given)

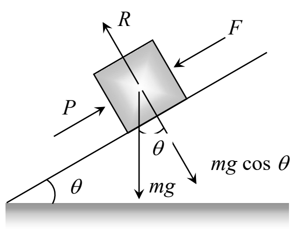
$\therefore \; \mu = \tan 45^\circ\left(1 - \frac{1}{2^2}\right) = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$

7. (a) $a = g(\sin\theta - \mu\cos\theta) = 9.8(\sin 45^o - 0.5\cos 45^o)$

$= \frac{4.9}{\sqrt{2}} \; m/\sec^2$

8. (d) Because if the angle of inclination is equal to or more than angle of repose then box will automatically slides down the plane.

9. (d)



Net force along the plane

$= P - mg\sin\theta = 750 - 500 = 250 \; N$

Limiting friction $= F_l = \mu_s R = \mu_s mg\cos\theta$

$= 0.4 \times 102 \times 9.8 \times \cos 30 = 346 \; N$

As net external force is less than limiting friction therefore friction on the body will be 250 N.

10. (c) $a = g(\sin\theta - \mu\cos\theta) = 10(\sin 60^\circ - 0.25\cos 60^\circ)$

$a = 7.4 \; m/s^2$

11. (b) $F_k = \mu_k R = \mu_k mg\cos\theta$

$F_k = 1.7 \times 0.1 \times 10 \times \cos 30^\circ = 1.7 \times \frac{\sqrt{3}}{2} \; N$

12. (a) $\mu = \tan\theta\left(1 - \frac{1}{n^2}\right) = \tan 30\left(1 - \frac{1}{2^2}\right) = \frac{\sqrt{3}}{4}$

13. (a) For angle of repose,

Friction =Component of weight along the plane

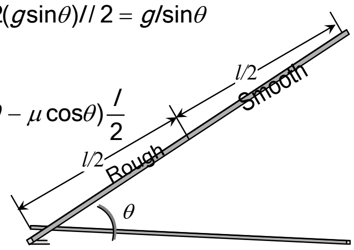
$= mg\sin\theta = 2 \times 9.8 \times \sin 45^o = 19.6 \sin 45^o$

14. (d) For upper half

$v^2 = u^2 + 2al/2 = 2(g\sin\theta)l/2 = gl\sin\theta$

For lower half

$\Rightarrow 0 = u^2 + 2g(\sin\theta - \mu\cos\theta)\frac{l}{2}$



$\Rightarrow \; -gl\sin\theta = gl(\sin\theta - \mu\cos\theta)$

$\Rightarrow \mu\cos\theta = 2\sin\theta \Rightarrow \mu = 2\tan\theta$

15. (c) Resultant downward force along the incline $= mg(\sin\theta - \mu\cos\theta)$

Normal reaction $= mg\cos\theta$

Given : $mg\cos\theta = 2mg(\sin\theta - \mu\cos\theta)$

By solving $\theta = 45^o$.

16. (b) $F = mg(\sin\theta + \mu\cos\theta)$

$= 10 \times 9.8(\sin 30^\circ + 0.5\cos 30^\circ) = 91.4 \; N$.

17. (c) $W = \mu mg\cos\theta \, S = 0.5 \times 1 \times 9.8 \times \frac{1}{2} \times 1 = 2.45 \; J$

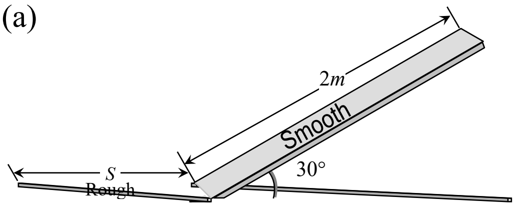
18. (d) $F = mg\sin 30^\circ = 50 \, N = 5 \, kg\text{-}wt$.

19. (a) $\mu = \tan 30^\circ = \frac{1}{\sqrt{3}}$.

20. (a) Work done against gravity $= mgh$ $= 2\times10\times10 = 200\ J$

Work done against friction = (Total work done – work done against gravity) $= 300 - 200 = 100\ J$

21. (a)



$v^2 = u^2 + 2as = 0 + 2\times g\sin 30 \times 2 \Rightarrow v = \sqrt{20}$

Let it travel distance 'S' before coming to rest

$S = \frac{v^2}{2\mu g} = \frac{20}{2\times0.25\times10} = 4m$

22. (a) Angle of repose $\alpha = \tan^{-1}(\mu) = \tan^{-1}(0.8) = 38.6°$

Angle of inclined plane is given $\theta = 30°$. It means block is at rest therefore,

Static friction = component of weight in downward direction $= mg\sin\theta = 10\ N$ $\therefore$

$m = \frac{10}{9\times\sin30°} = 2\ kg$

23. (a) $\mu = \tan\theta\left(1 - \frac{1}{n^2}\right) = \tan\theta\left(1 - \frac{1}{2^2}\right) = \frac{3}{4}\tan\theta$

24. (b) Acceleration (a) $= g(\sin\theta - \mu\cos\theta)$ and $s = l$

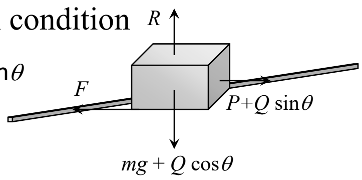
$v = \sqrt{2as} = \sqrt{2gl(\sin\theta - \mu\cos\theta)}$

Critical Thinking Questions

1. (a) By drawing the free body diagram of the block for critical condition

$F = \mu R \Rightarrow P + Q\sin\theta$

$= \mu(mg + Q\cos\theta)$



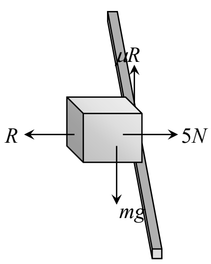
$\therefore\ \mu = \frac{P + Q\sin\theta}{mg + Q\cos\theta}$

2. (c)

3. (b) Limiting friction

$F_l = \mu_s R = 0.5\times(5) = 2.5\ N$

Since downward force is less than limiting friction therefore block is at rest so the static force of friction will work on it.



F_s = downward force = Weight

$= 0.1\times9.8 = 0.98\ N$

4. (c) Maximum force by surface when friction works

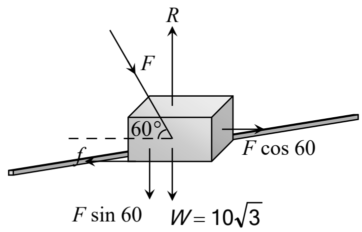
$F = \sqrt{f^2 + R^2} = \sqrt{(\mu R)^2 + R^2} = R\sqrt{\mu^2 + 1}$

Minimum force $= R$ when there is no friction

Hence ranging from R to $R\sqrt{\mu^2+1}$

We get, $Mg \le F \le Mg\sqrt{\mu^2+1}$

5. (a)



$f = \mu R$

$F\cos60° = \mu(W + F\sin60°)$

Substituting $\mu = \frac{1}{2\sqrt{3}}$ & $W = 10\sqrt{3}$ we get $F = 20\ N$

6. (b) When two blocks performs simple harmonic motion together then at the extreme position (at amplitude $=A$)

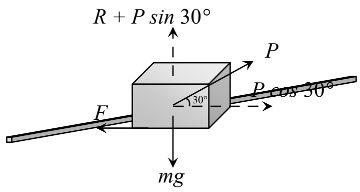
Restoring force $F = KA = 2ma \Rightarrow a = \frac{KA}{2m}$

There will be no relative motion between P and Q if pseudo force on block P is less than or just equal to limiting friction between P and Q.

i.e. $m\left(\frac{KA}{2m}\right) =$ Limiting friction

$\therefore$ Maximum friction $= \frac{KA}{2}$

7. (c) Normal reaction $R = mg - P\sin30^o = mg - \frac{P}{2}$



$\therefore$ Limiting friction between body and surface is given by, $F = \mu R = \mu\left(mg - \frac{P}{2}\right)$.

8. (a) Limiting friction between block and slab $= \mu_s m_A g$

$$= 0.6 \times 10 \times 9.8 = 58.8\,N$$

But applied force on block A is 100 N. So the block will slip over a slab.

Now kinetic friction works between block and slab $F_k = \mu_k m_A g = 0.4 \times 10 \times 9.8 = 39.2\ N$

This kinetic friction helps to move the slab

$\therefore$ Acceleration of slab $= \frac{39.2}{m_B} = \frac{39.2}{40} = 0.98\ m/s^2$

9. (a) Limiting friction $F_l = \mu\ mg\cos\theta$

$F_l = 0.7 \times 2 \times 10 \times \cos 30° = 12\ N$ (approximately)

But when the block is lying on the inclined plane then component of weight down the plane $= mg\sin\theta$

$$= 2 \times 9.8 \times \sin 30° = 9.8\ N$$

It means the body is stationary, so static friction will work on it

$\therefore$ Static friction = Applied force = 9.8 N

10. (a,c) In cycling, the rear wheel moves by the force communicated to it by pedalling while front wheel moves by it self. So, while pedalling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moving by itself experience force of friction in backward direction (like rolling of a ball). [However, if pedalling is stopped both wheels move by themselves and so experience force of friction in backward direction].

11. (a)

Assertion & Reason

1. (a) On a rainy day, the roads are wet. Wetting of roads lowers the coefficient of friction between the tyres and the road. Therefore, grip of car on the road reduces and thus chances of skidding increases.

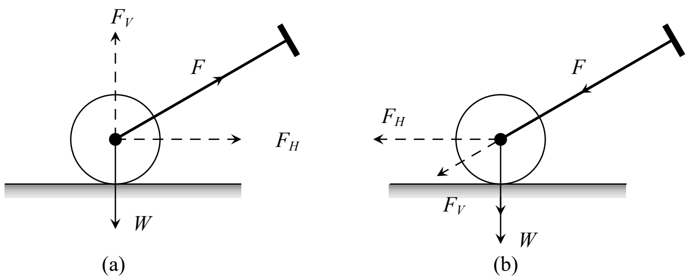
2. (e) When a bicycle is in motion, two cases may arise :

(i) When the bicycle is being pedalled. In this case, the applied force has been communicated to rear wheel. Due to which the rear wheel pushes the earth backwards. Now the force of friction acts in the forward direction on the rear wheel but front wheel move forward due to inertia, so force of friction works on it in backward direction

(ii) When the bicycle is not being pedalled :

In this case both the wheels move in forward direction, due to inertia. Hence force of friction on both the wheels acts in backward direction.

3. (a) Suppose the roller is pushed as in figure (b). The force F is resolved into two components, horizontal component F_H which helps the roller to move forward, and the vertical component acting downwards adds to the weight. Thus weight is increased. But in the case of pull [fig (a)] the vertical component is opposite to its weight. Thus weight is reduced. So pulling is easier than pushing the lawn roller.



4. (b)

5. (d) The force acting on the body of mass M are its weight Mg acting vertically downwards and air resistance F acting vertically upward.

$\therefore$ Acceleration of the body, $a = \frac{Mg - F}{M} = g - \frac{F}{M}$

Now, $M > m$, therefore, the body with larger mass will have greater acceleration and it will reach the ground first.

6. (d) Only static friction is a self adjusting force. This is because force of static friction is equal and opposite to applied force (so long as actual motion does not start). Frictional force = μmg *i.e.* friction depends on mass.

7. (a)

8. (d) Acceleration down a rough inclined plane $a = g(\sin\theta - \mu\cos\theta)$ and this is less than g.