Critical Thinking Questions

1. (a) For dc, $R = \frac{V}{i} = \frac{100}{1} = 100\ \Omega$

For ac, $Z = \frac{V}{i} = \frac{100}{0.5} = 200\ \Omega$

$\because\ Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow 200 = \sqrt{(100)^2 + 4\pi^2 (50)^2 L^2}$

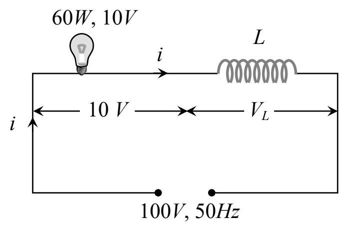
$\therefore\ L = 0.55\ H$

2. (c) $P = E_{rms} i_{rms} \cos\phi = \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \times \frac{R}{Z}$

$\Rightarrow \frac{E_0}{\sqrt{2}} \times \frac{E_0}{Z\sqrt{2}} \times \frac{R}{Z} \Rightarrow P = \frac{E_0^2 R}{2Z^2}$

Given $X_L = R$ so, $Z = \sqrt{2}R \Rightarrow P = \frac{E_0^2}{4R}$

3. (a) Current through the bulb $i = \frac{P}{V} = \frac{60}{10} = 6A$



$V = \sqrt{V_R^2 + V_L^2}$

$(100)^2 = (10)^2 + V_L^2 \Rightarrow V_L = 99.5\ Volt$

Also $V_L = iX_L = i \times (2\pi\nu L)$

$\Rightarrow 99.5 = 6 \times 2 \times 3.14 \times 50 \times L \Rightarrow L = 0.052\ H$

4. (c) $V^2 = V_R^2 + (V_L - V_C)^2$

Since $V_L = V_C$ hence $V = V_R = 200\ V$

5. (c) $V^2 = V_R^2 + (V_L - V_C)^2 \Rightarrow V_R = V = 220\ V$

Also $i = \frac{220}{100} = 2.2\ A$

6. (a) When a bulb and a capacitor are connected in series to an ac source, then on increasing the frequency the current in the circuit is increased, because the impedance of the circuit is decreased. So the bulb will give more intense light.

7. (d) The instantaneous values of emf and current in inductive circuit are given by $E = E_0 \sin\omega t$ and $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ respectively.

So, $P_{inst} = Ei = E_0 \sin\omega t \times i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

$= E_0 i_0 \sin\omega t\left(\sin\omega t \cos\frac{\pi}{2} - \cos\omega t \sin\frac{\pi}{2}\right)$

$= E_0 i_0 \sin\omega t \cos\omega t$

$= \frac{1}{2} E_0 i_0 \sin 2\omega t \qquad (\sin 2\omega t = 2\sin\omega t\cos\omega t)$

Hence, angular frequency of instantaneous power is 2ω.

8. (b) $V = 50 \times 2\sin 100\pi t \cos 100\pi t = 50 \sin 200\pi t$

$\Rightarrow V_0 = 50\ Volts$ and $\nu = 100 Hz$

9. (b) In RC series circuit voltage across the capacitor leads the voltage across the resistance by $\frac{\pi}{2}$

10. (d) The voltage V_L and V_C are equal and opposite so voltmeter reading will be zero.

Also $R = 30\Omega,\ X_L = X_C = 25\Omega$

So $i = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{R} = \frac{240}{30} = 8\ A$

11. (d) $V = 120 \sin 100\pi t \cos 100\pi t \Rightarrow V = 60 \sin 200\pi t$

$V_{max} = 60V$ and $\nu = 100 Hz$

12. (d) $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$;

$R = 10\Omega,\ X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\Omega$ i.e. $Z = 10\Omega$

Maximum current $i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2A$

Hence $i_{rms} = \frac{2}{\sqrt{2}} = 1.4\ A$

and $V_{rms} = 4 \times 1.41 = 5.64\ V$

13. (a) Capacitance of wire

$C = 0.014 \times 10^{-6} \times 200 = 2.8 \times 10^{-6} F = 2.8\mu F$

For impedance of the circuit to be minimum

$X_L = X_C \Rightarrow 2\pi \nu L = \frac{1}{2\pi \nu C}$

$\Rightarrow L = \frac{1}{4\pi^2 \nu^2 C} = \frac{1}{4(3.14)^2 \times (5\times 10^3)^2 \times 2.8 \times 10^{-6}}$

$= 0.35 \times 10^{-3} H = 0.35\, mH$

14. (c) $\overline{i^2} = \frac{\int i^2 dt}{\int dt} = \frac{\int_2^4 (4t) dt}{\int_2^4 dt} = \frac{4\int_2^4 t\, dt}{2}$

$= 2\left[\frac{t^2}{2}\right]_2^4 = \left[t^2\right]_2^4 = 12$

$\Rightarrow i_{rms} = \sqrt{\overline{i^2}} = \sqrt{12} = 2\sqrt{3}\ A$

15. (b) 1. *rms* value $= \frac{x_0}{\sqrt{2}}$

2. $x_0 \sin\omega t \cos\omega t = \frac{x_0}{2} \sin 2\omega t \Rightarrow rms\ \text{value} = \frac{x_0}{2\sqrt{2}}$

3. $x_0 \sin\omega t + x_0 \cos\omega t \Rightarrow rms\ \text{value} = \sqrt{\left(\frac{x_0}{\sqrt{2}}\right)^2 + \left(\frac{x_0}{\sqrt{2}}\right)^2}$

$= \sqrt{x_0^2} = x_0$

16. (c) Given $X_L = X_C = 5\Omega$, this is the condition of resonance. So $V_L = V_C$, so net voltage across *L* and *C* combination will be zero.

17. (a) At angular frequency ω, the current in *RC* circuit is given by

$i_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$ ......(i)

Also $\frac{i_{rms}}{2} = \frac{V_{rms}}{\sqrt{R^2 + \left(\frac{1}{\frac{\omega}{3} C}\right)^2}} = \frac{V_{rms}}{\sqrt{R^2 + \frac{9}{\omega^2 C^2}}}$ ......(ii)

From equation (i) and (ii) we get

$3R^2 = \frac{5}{\omega^2 C^2} \Rightarrow \frac{\frac{1}{\omega C}}{R} = \sqrt{\frac{3}{5}} \Rightarrow \frac{X_C}{R} = \sqrt{\frac{3}{5}}$

18. (d) $\tan\phi = \frac{X_L}{R} = \frac{X_C}{R} \Rightarrow \tan 60^o = \frac{X_L}{R} = \frac{X_C}{R}$

$\Rightarrow X_L = X_C = \sqrt{3}\ R$

i.e. $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

So average power $P = \frac{V^2}{R} = \frac{200 \times 200}{100} =$ 400 *W*

19. (b) $R = \frac{P}{i_{rms}^2} = \frac{240}{16} = 15\Omega$

$Z = \frac{V}{i} = \frac{100}{4} = 25\Omega$

Now $X_L = \sqrt{Z^2 - R^2} = \sqrt{(25)^2 - (15)^2} = 20\Omega$

$\therefore 2\pi \nu L = 20 \Rightarrow L = \frac{20}{2\pi \times 50} = \frac{1}{5\pi} Hz$

20. (b) $X_L = R,\ X_C = R/2$

$\therefore \tan\phi = \frac{X_L - X_C}{R} = \frac{R - \frac{R}{2}}{R} = \frac{1}{2}$

$\Rightarrow \phi = \tan^{-1}(1/2)$

Also $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \frac{R^2}{4}} = \frac{\sqrt{5}}{2} R$

21. (d) At resonance net voltage across *L* and *C* is zero.

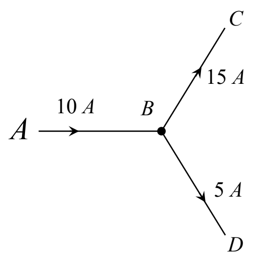
22. (c) $i_L = \frac{90}{30} = 3\ A,\quad i_C = \frac{90}{20} = 4.5\ A$

Net current through circuit $i = i_C - i_L = 1.5\ A$

$\therefore Z = \frac{V}{i} = \frac{90}{1.5} = 60\Omega$

23. (c) $i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \frac{T^2}{\sqrt{5}}$

24. (a) Yes, in *AC* if branch *AB* has *R*, *BC* has a capacitor *C*, and *BD* has a pure inductance *L*



25. (d) Current will be maximum in the condition of resonance so $i_{\max} = \frac{V}{R} = \frac{V}{10} A$

Energy stored in the coil $W_L = \frac{1}{2}Li_{max}^2$

$= \frac{1}{2}L\left(\frac{E}{10}\right)^2$

$= \frac{1}{2}\times 10^{-3}\left(\frac{E^2}{100}\right) = \frac{1}{2}\times 10^{-5}E^2\ joule$

$\therefore$ Energy stored in the capacitor

$W_C = \frac{1}{2}CE^2 = \frac{1}{2}\times 2\times 10^{-6}E^2 = 10^{-6}E^2\ joule$

$\therefore \frac{W_C}{W_L} = \frac{1}{5}$

Graphical Questions

1. (c) $Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$

From above equation at $f = 0 \Rightarrow z = \infty$

When $f = \frac{1}{2\pi\sqrt{LC}}$ (resonant frequency)

$\Rightarrow Z = R$

For $f > \frac{1}{2\pi\sqrt{LC}} \Rightarrow Z$ starts increasing.

i.e., for frequency $0 - f_r$, Z decreases

and for f_r to ∞, Z increases. This is justified by graph *c*.

2. (b) At $t = 0$, phase of the voltage is zero, while phase of the current is $-\frac{\pi}{2}$ i.e., voltage leads by $\frac{\pi}{2}$

3. (c) At $A: X_C > X_L$

At $B: X_C = X_L$

At $C: X_C < X_L$

4. (c) I_L lags behind I_R by a phase of $\frac{\pi}{2}$, while I_C leads by a phase of $\frac{\pi}{2}$.

5. (d) As explained in solution (1) for frequency $0 - f_r$, Z decreases hence $(i = V/Z)$, increases and for frequency $f_r - \infty$, Z increases hence i decrees.

6. (a) $V_{rms} = \sqrt{\frac{1}{T}\int_0^T 10^2\,dt} = 10\ V$

7. (b) For capacitive circuits $X_C = \frac{1}{\omega C}$

$\therefore i = \frac{V}{X_C} = V\omega C \Rightarrow i \propto \omega$

8. (c) $I_{av} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin(\omega t)dt}{T/2}$

$= \frac{2I_0}{T}\left[\frac{-\cos\omega t}{\omega}\right]_0^{T/2} = \frac{2I_0}{T}\left[-\frac{\cos\left(\frac{\omega T}{2}\right)}{\omega} + \frac{\cos 0^o}{\omega}\right]$

$= \frac{2I_0}{\omega T}[-\cos\pi + \cos 0^o] = \frac{2I_0}{2\pi}[1+1] = \frac{2I_0}{\pi}$

9. (b) (1) For time interval $0 < t < T/2$

$I = kt$, where k is the slope

For inductor as we know, induced voltage

$V = -L\frac{di}{dt}$

$\Rightarrow V_1 = -KL$

(2) For time interval $\frac{T}{2} < t < T$

$I = -Kt \Rightarrow V_2 = KL$

10. (a) As the current i leads the voltage by $\frac{\pi}{4}$, it is an *RC* circuit, hence $\tan\phi = \frac{X_C}{R} \Rightarrow$

$\tan\frac{\pi}{4} = \frac{1}{\omega CR}$

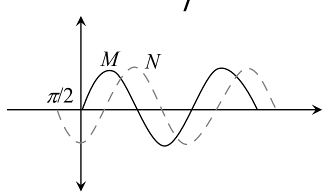
$\Rightarrow \omega CR = 1$ as $\omega = 100\ rad/sec$

$\Rightarrow CR = \frac{1}{100}sec^{-1}$.

From all the given options only option (a) is correct.

11. (b) From the graph shown below. It is clear that phase lead of N over M is $-\frac{\pi}{2}$. Since time period (*i.e.* taken to complete one cycle) = 0.4 *sec*.

Hence frequency $\nu = \frac{1}{T} = 2.5\ Hz$



12. (d) In purely inductive circuit voltage leads the current by 90°.

13. (c) $X_L = 2\pi fL \Rightarrow X_L \propto f \Rightarrow \frac{1}{X_L} \propto \frac{1}{f}$

i.e., graph between $\frac{1}{X_L}$ and f will be a hyperbola.

14. (c) From phasor diagram it is clear that current is lagging with respect to E_{rms}. This may be happen in *LCR* or *LR* circuit.

15. (c) At resonance $X_L = X_C$

16. (b) For anti-resonant circuit current is minimum at resonant frequency and at frequencies other than resonant frequency current rises with frequency.

17. (c) We have $X_C = \frac{1}{C \times 2\pi f}$ and $X_L = L \times 2\pi f$

18. (d) Reactance $X = X_L - X_C = 2\pi fL - \frac{1}{2\pi fC}$

19. (b) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$ *i.e.* $X_C \propto \frac{1}{f}$

Assertion and Reason

1. (a) At resonant frequency, $X_L = X_C \therefore Z = R$ (minimum) there for current in the circuit is maximum.

2. (c) When ac flows through an inductor current lags behind the *emf.*, by phase of $\pi/2$, inductive reactance, $X_L = \omega L = \pi.2f.L,$ so when frequency increases correspondingly inductive reactance also increases.

3. (a) The capacitive reactance of capacitor is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

So this is infinite for dc ($f = 0$) and has a very small value for ac. Therefore a capacitor blocks dc.

4. (b) The phase angle for the *LCR* circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R}$$

Where X_L, X_C are inductive reactance and capacitive reactance respectively when $X_L > X_C$ then $\tan\phi$ is positive *i.e.* ϕ is positive (between 0 and $\pi/2$). Hence emf leads the current.

5. (a) If resistor is used in controlling ac supply, electrical energy will be wasted in the form of heat energy across the resistance wire. However, ac supply can be controlled with choke without any wastage of energy. This is because, power factor ($\cos\phi$) for resistance is unity and is zero for an inductance. [$P = EI\cos\phi$].

6. (a) When frequency of alternating current is increased, the effective resistance of the inductive coil increases. Current $(X_L = \omega L = 2\pi fL)$ in the circuit containing inductor is given by $I = \frac{V}{X_L} = \frac{V}{2\pi fL}$. As inductive resistance of the inductor increases, current in the circuit decreases.

7. (e) On introducing soft iron core, the bulb will glow dimmer. This is because on introducing soft iron core in the solenoid, its inductance L increases, the inductive reactance, $X_L = \omega L$ increases and hence the current through the bulb decreases.

8. (b) Like direct current, an alternating current also produces magnetic field. But the magnitude and direction of the field goes on changing continuously with time.

9. (c) Both ac and dc produce heat, which is proportional to square of the current. The reversal of direction of current in ac is immaterial so far as production of heat is concerned.

10. (a) The effect of ac on the body depends largely on the frequency. Low frequency currents of 50 to 60 *Hz* (*cycles/sec*), which are commonly used, are usually more dangerous than high frequency currents and are 3 to 5 times more dangerous than dc of same voltage and amperage (current). The usual frequency of 50 *cps* (or 60 *cps*) is extremely dangerous as it corresponds to the fibrillation frequency of the myocardium. This results in ventricular fibrillation and instant death.

11. (b) The mean average value of alternating current (or emf) during a half, cycle is given by $I_m = 0.636\, I_0$ (or $E_m = 0.636 E_0$)

During the next half cycle, the mean value of ac will be equal in magnitude but opposite in direction.

For this reason the average value of ac over a complete cycle is always zero. So the average value is always defined over a half cycle of ac.

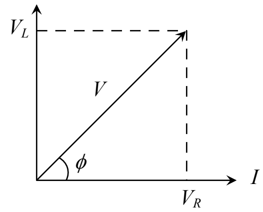
12. (d) An ac ammeter is constructed on the basics of heating effect of the electric current. Since heat produced varies as square of current $(H = I^2 R)$. Therefore the division marked on the scale of ac ammeter are not equally spaced.

13. (d) The power of a ac circuit is given by $P = EI\cos\phi$

where $\cos\phi$ is power factor and ϕ is phase angle. In case of circuit containing resistance only, phase angle is zero and power factor is equal to one. Therefore power is maximum in case of circuit containing resistor only.

14. (a) Capacitive reactance $X_C = \frac{1}{\omega C}$. When capacitance (C) increases, the capacitive reactance decreases. Due to decrease in its values, the current in the circuit will increases $\left(I = \frac{E}{\sqrt{R^2 + X_C^2}}\right)$ and hence brightness of source (or electric lamp) will also increases.

15. (b) As both the inductance and resistance are joined in series, hence current through both will be same. But in case of resistance, both the current and potential vary simultaneously, hence they are in same phase. In case of an inductance when current is zero, potential difference across it is maximum and when current reaches maximum (at $\omega t = \pi/2$), potential difference across it becomes zero *i.e.* potential difference leads the current by $\pi/2$ or current lags behind the potential difference by $\pi/2$, Phase angle in case of *LR* circuit is given as $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$.



16. (b) We can use a capacitor of suitable capacitance as a chock coil, because average power consumed per cycle in an ideal capacitor is zero. Therefore, like a choke coil, a condenser can reduce ac without power dissipation.