FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Consider the efficiency of Carnot's engine is given by $\eta = \frac{\alpha\beta}{\sin\theta}\log_e\frac{\beta x}{kT}$, where α and β are constants. If T is temperature, k is Boltzman constant, θ is angular displacement and x has the dimensions of length. Then, choose the **incorrect** option.

(A) Dimensions of β is same as that of force.

(B) Dimensions of α^{-1} x is same as that of energy.

(C) Dimensions of $\eta^{-1}\sin\theta$ is same as that of $\alpha\beta$

(D) Dimensions of α is same as that of β

Official Ans. by NTA (D)

Sol. $[\alpha\beta] = [\eta] = [\sin\theta] = \text{Dimensionless}$

$\left[\eta^{-1}\sin\theta\right] = [\alpha\beta] = \text{D.L.}$

2. At time t = 0 a particle starts travelling from a height $7\hat{z}$ cm in a plane keeping z coordinate constant. At any instant of time it's position along the x and y directions are defined as 3t and $5t^3$ respectively. At t = 1s acceleration of the particle will be

(A) $-30y$ (B) $30y$

(C) $3x+15y$ (D) $3x+15y+7\hat{z}$

Official Ans. by NTA (B)

Sol. $\vec{r} = 3t\hat{i} + 5t^3\hat{j} + 7k$

$\frac{d^2\vec{r}}{dt^2} = 30t\,\hat{j}$

At $t = 1 \Rightarrow \boxed{\frac{d^2\vec{r}}{dt^2} = 30\hat{j}}$

3. A pressure-pump has a horizontal tube of cross-sectional area 10 cm^2 for the outflow of water at a speed of 20 m/s. The force exerted on the vertical wall just in front of the tube which stops water horizontally flowing out of the tube, is: [given : density of water = 1000 kg/m^3]

(A) 300 N (B) 500 N

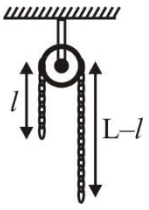
(C) 250 N (D) 400 N

Official Ans. by NTA (D)

Sol. $F = \rho a \upsilon^2 = 10^3 \times 10 \times 10^{-4} \times 20 \times 20$

$F = 400$

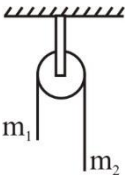
4. A uniform metal chain of mass m and length 'L' passes over a massless and frictionless pulley. It is released from rest with a part of its length 'l' is hanging on one side and rest of its length 'L – l' is hanging on the other side of the pulley. At a certain point of time, when $l = \frac{L}{x}$, the acceleration of the chain is $\frac{g}{2}$. The value of x is .....



(A) 6 (B) 2 (C) 1.5 (D) 4

Official Ans. by NTA (D)

Sol.

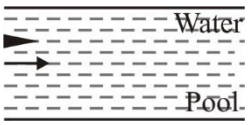


$a = \frac{(m_2 - m_1)}{(m_2 + m_1)}g$

$\frac{g}{2} = \frac{(\lambda(L-\ell) - \lambda\ell)g}{\lambda L} \Rightarrow L = \frac{L}{4} = \frac{L}{x}$

$x = 4$

5. A bullet of mass 200 g having initial kinetic energy 90 J is shot inside a long swimming pool as shown in the figure. If it's kinetic energy reduces to 40 J within 1s, the minimum length of the pool, the bullet has a to travel so that it completely comes to rest is



(A) 45 m (B) 90 m

(C) 125 m (D) 25 m

Official Ans. by NTA (A)

Sol. Using $mv = \sqrt{2mk}$

$$u = \frac{1}{0.2}\sqrt{2\times0.2\times90} = 30 \text{ m/s}$$

$$v = \frac{1}{0.2}\sqrt{2\times0.2\times40} = 20 \text{ m/s}$$

$$a = \frac{20-30}{1} = -10 \text{ m/s}^2$$

$$s = \frac{-u^2}{2a} = 45 \text{ m}$$

6. Assume there are two identical simple pendulum Clocks-1 is placed on the earth and Clock-2 is placed on a space station located at a height h above the earth surface. Clock-1 and Clock-2 operate at time periods 4s and 6s respectively. Then the value of h is –

(consider radius of earth R_E = 6400 km and g on earth 10 m/s^2)

(A) 1200 km

(B) 1600 km

(C) 3200 km

(D) 4800 km

Official Ans. by NTA (C)

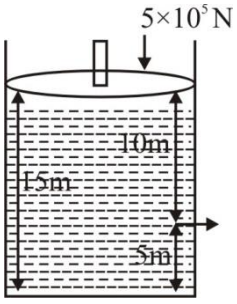
Sol. $t \propto \frac{1}{\sqrt{g}}$ and $g \propto \frac{1}{(R+h)^2}$

$$\frac{t_1}{t_2} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{R^2}{(R+h)^2}}$$

$$\frac{t_1}{t_2} = \frac{4}{6} = \frac{R}{(R+h)} \Rightarrow h = 3200\,\text{km}$$

7. Consider a cylindrical tank of radius 1m is filled with water. The top surface of water is at 15 m from the bottom of the cylinder. There is a hole on the wall of cylinder at a height of 5m from the bottom. A force of 5×10^5 N is applied an the top surface of water using a piston. The speed of efflux from the hole will be :

(given atmospheric pressure $P_A = 1.01 \times 10^5$ Pa, density of water $\rho_w = 1000$ kg/m^3 and gravitational acceleration g = 10 m/s^2)



(A) 11.6 m/s (B) 10.8 m/s

(C) 17.8 m/s (D) 14.4 m/s

Official Ans. by NTA (C)

Sol. Apply Bernoulli's theorem between Piston and hole $P_A + \rho gh = P_0 + \frac{1}{2}\rho v_e^2$

Assuming there is no atmospheric pressure on piston

$$\frac{5\times10^5}{\pi} + 10^3\times10\times10 = 1.01\times10^5 + \frac{1}{2}\times10^3\times v_e^2$$

v_e = 17.8 m/s

8. A vessel contains 14 g of nitrogen gas at a temperature of 27°C. The amount of heat to be transferred to the gap to double the r.m.s. speed of its molecules will be : (Take R = 8.32 J mol^{-1}k^{-1})

(A) 2229 J (B) 5616 J

(C) 9360 J (D) 13,104 J

Official Ans. by NTA (C)

Sol. $v_{rms} \propto \sqrt{T}$

$v_{rms} \propto \sqrt{300K}$, $v_{rms_f} = 2v_{rms_i}$

$v_{rms_f} \propto \sqrt{1200K}$

$T_f = 1200K$, $T_i = 300K$, $n = \frac{14}{28} = \frac{1}{2}$

$Q = nC_v\Delta T = \frac{1}{2} \times \frac{5R}{2} \times 900$

$Q = 9360$ J

9. A slab of dielectric constant K has the same cross-sectional area as the plates of a parallel plate capacitor and thickness $\frac{3}{4}d$, where d is the separation of the plates. The capacitance of the capacitor when the slab is inserted between the plates will be :

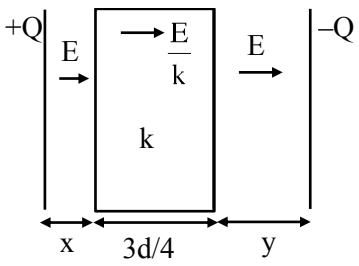
(Given C_o = capacitance of capacitor with air as medium between plates.)

(A) $\frac{4KC_0}{3+K}$ (B) $\frac{3KC_0}{3+K}$

(C) $\frac{3+K}{4KC_0}$ (D) $\frac{K}{4+K}$

Official Ans. by NTA (A)

Sol.



$x + y + \frac{3d}{4} = d$

$x + y = \frac{d}{4}$

$\frac{A\epsilon_0}{d} = C_0$

$\Delta V = Ex + \frac{E}{k} \times \frac{3d}{4} + Ey$

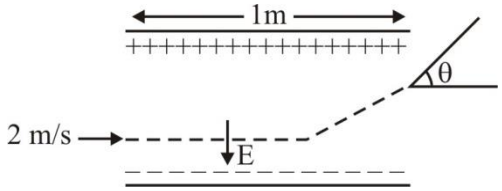
$= \frac{3Ed}{4k} + E(x+y)$

$\Delta V = E\left[\frac{3d}{4k} + \frac{d}{4}\right]$

$\Delta V = \frac{\sigma}{\epsilon_0}\left[\frac{3d+dk}{4k}\right] = \frac{Qd}{A\epsilon_0}\left[\frac{3+k}{4k}\right]$

$\frac{Q}{\Delta V} = C = \frac{A\epsilon_0}{d}\left[\frac{4k}{3+k}\right] = \frac{4kC_0}{k+3}$

10. A uniform electric field E = (8m/e) V/m is created between two parallel plates of length 1m as shown in figure, (where m = mass of electron and e = charge of electron). An electron enters the field symmetrically between the plates with a speed of 2m/s. The angle of the deviation (θ) of the path of the electron as it comes out of the field will be .....

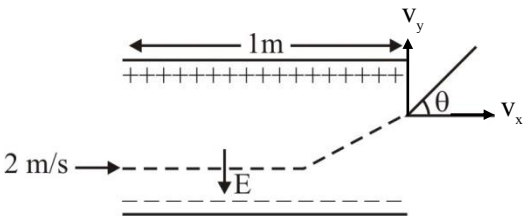


(A) $\tan^{-1}(4)$ (B) $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\frac{1}{3}\right)$ (D) $\tan^{-1}(3)$

Official Ans. by NTA (B)

Sol.



$a_y = \frac{F_y}{m} = \frac{e(E)}{m} = \frac{e\left(\frac{8m}{e}\right)}{m} = 8m/s^2$

$s_x = u_x t$

$1 = 2 \times t$

$t = \frac{1}{2}$ sec

$v_y = u_y + a_y t$

$v_y = 0 + 8 \times \frac{1}{2}$

$v_y = 4$ m/s

$\tan\theta = \frac{v_y}{v_x} = \frac{4}{2} = 2 \Rightarrow \theta = \tan^{-1}(2)$

11. Given below are two statements :

Statement I : A uniform wire of resistance 80Ω is cut into four equal parts. These parts are now connected in parallel. The equivalent resistance of the combination will be 5 Ω.

Statement II : Two resistance 2R and 3R are connected in parallel in a electric circuit. The value of thermal energy developed in 3R and 2R will be in the ratio 3 : 2.

In the light of the above statements, choose the most appropriate answer from the options given below

(A) Both statement I and statement II are correct

(B) Both statement I and statement II are incorrect

(C) Statement I is correct but statement II is incorrect

(D) Statement I is incorrect but statement II is correct.

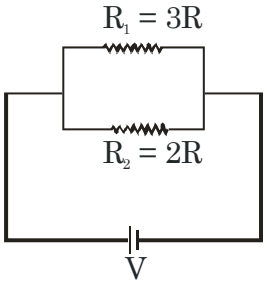
Official Ans. by NTA (C)

Sol. **Statement 1 -** $R = 80\Omega$

$R_1 = R_2 = R_3 = R_4 = 20\ \Omega$

In parallel $R_{eq} = \frac{20}{4} = 5\Omega$

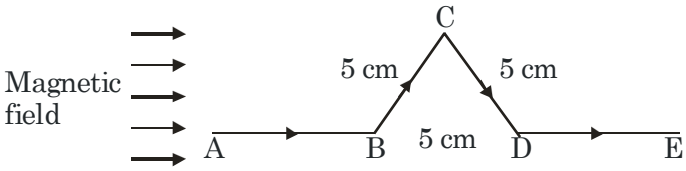
Statement 2 –



$P_{th} = \frac{v^2}{R}$

$\frac{P_1}{P_2} = \left(\frac{R_2}{R_1}\right) = \frac{2}{3}$ (where P is power)

12. A triangular shaped wire carrying 10A current is placed in a uniform magnetic field of 0.5T, as shown in figure. The magnetic force on segment CD is (Given BC = CD = BD = 5 cm).



(A) 0.126 N (B) 0.312 N

(C) 0.216 N (D) 0.245 N

Official Ans. by NTA (C)

Sol. $F_M(CD) = BI\ell_{eff}$

$= 0.5\times(10) \times (5\sin 60 \times 10^{-2})$

$= 0.216$ N

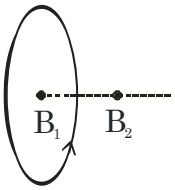
13. The magnetic field at the center of current carrying circular loop is B_1. The magnetic field at a distance of $\sqrt{3}$ times radius of the given circular loop from the center on its axis is B_2. The value of B_1/B_2 will be

(A) 9 : 4 (B) $12:\sqrt{5}$

(C) 8 : 1 (D) $5:\sqrt{3}$

Official Ans. by NTA (C)

Sol.



$B_1 = \frac{\mu_0 I}{2R}$

$B_2 = \frac{\mu_0 I R^2}{2\left(R^2+3R^2\right)^{3/2}} = \frac{1}{8}\left(\frac{\mu_0 I}{2R}\right) = \frac{B_1}{8}$

$\frac{B_1}{B_2} = \frac{8}{1}$

14. A transformer operating at primary voltage 8 kV and secondary voltage 160 V serves a load of 80 kW. Assuming the transformer to be ideal with purely resistive load and working on unity power factor, the loads in the primary and secondary circuit would be

(A) 800 Ω and 1.06 Ω (B) 10 Ω and 500 Ω

(C) 800 Ω and 0.32 Ω (D) 1.06 Ω and 500 Ω

Official Ans. by NTA (C)

Sol. $\frac{\left(8\times10^{3}\right)^{2}}{R_P}=80\times10^{3}$

$R_P=800\Omega$

$\frac{(160)^{2}}{R_S}=80\times10^{3}$

$R_S=0.32\Omega$

15. Sun light falls normally on a surface of area 36 cm^2 and exerts an average force of 7.2×10^{-9} N within a time period of 20 minutes. Considering a case of complete absorption, the energy flux of incident light is

(A) 25.92×10^{2} W/cm^2 (B) 8.64×10^{-6} W/cm^2

(C) 6.0 W/cm^2 (D) 0.06 W/cm^2

Official Ans. by NTA (D)

Sol. $\frac{I}{C}\times\text{area}=\text{force}$

$\frac{I}{C}\times36\times10^{-4}=7.2\times10^{-9}$

$I=\frac{7.2\times10^{-9}\times3\times10^{8}}{36\times10^{-9}\times10}$

$=\frac{6\times10^{-1}}{10^{-3}}$

$I=6\times10^{2}\frac{w}{m^2}$

$=0.06\frac{w}{cm^2}$

16. The power of a lens (biconvex) is 1.25 m^{-1} in particular medium. Refractive index of the lens is 1.5 and radii of curvature are 20 cm and 40 cm respectively. The refractive index of surrounding medium :

(A) 1.0 (B) $\frac{9}{7}$

(C) $\frac{3}{2}$ (D) $\frac{4}{3}$

Official Ans. by NTA (B)

Sol. $P=\frac{\mu_2}{f}=(\mu_1-\mu_2)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (For this formula refer to NCERT Part-2, Chapter-9, Page no. 328, solved example 8)

(μ_1 is refractive index of lens and μ_2 is of surrounding medium)

$1.25=(1.5-\mu_2)\left(\frac{1}{0.2}+\frac{1}{0.4}\right)$

$\frac{1.25\times0.08}{0.6}=(1.5-\mu_2)$

$\Rightarrow\mu_2=\frac{4}{3}$

17. Two streams of photons, possessing energies to five and ten times the work function of metal are incident on the metal surface successively. The ratio of the maximum velocities of the photoelectron emitted, in the two cases respectively, will be

(A) 1 : 2 (B) 1 : 3

(C) 2 : 3 (D) 3 : 2

Official Ans. by NTA (C)

Sol. $\frac{1}{2}mv_1^2 = 4\phi$

$\frac{1}{2}mv_2^2 = 9\phi$

$\frac{v_1}{v_2} = \frac{2}{3}$

18. A radioactive sample decays $\frac{7}{4}$ times its original quantity in 15 minutes. The half-life of the sample is

(A) 5 min (B) 7.5 min

(C) 15 min (D) 30 min

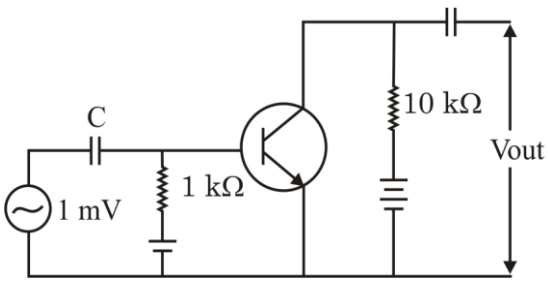
Official Ans. by NTA (A)

Sol. Remaining $= \frac{1}{8}$

$3t_{1/2} = 15\,\text{min}$

$t_{1/2} = 5\,\text{min}$

19. An n.p.n transistor with current gain β = 100 in common emitter configuration is shown in figure. The output voltage of the amplifier will be



(A) 0.1 V (B) 1.0 V

(C) 10 V (D) 100 V

Official Ans. by NTA (B)

Sol. $\frac{v_{out}}{v_{in}} = \beta\frac{R_{out}}{R_{in}}$

$v_{out} = \frac{100\times10\times10^3}{10^3}\times10^{-3}$

= 1 V

20. A FM Broad cast transmitter, using modulating signal of frequency 20 kHz has a deviation ratio of 10. The Bandwidth required for transmission is :

(A) 220 kHz

(B) 180 kHz

(C) 360 kHz

(D) 440 kHz

Official Ans. by NTA (D)

Sol. Given

FM broadcast

Modulating frequency = 20 k Hz = f

$\text{Deviation ratio} = \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta f}{f}$

$\Rightarrow$ frequency deviation – $\Delta f = f \times 10$

= 20 kHz × 10 = 200 kHz

$\Rightarrow$ Bandwidth = 2(f + Δf)

= 2 (20 + 200) kHz

= 440 kHz

SECTION-B

1. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)$m. The value of k is ….. (use g = 9.8 m/s^2)

Official Ans. by NTA (392)

Sol. $t_a = \frac{u}{g} = \frac{19.6}{9.8} = 2s$

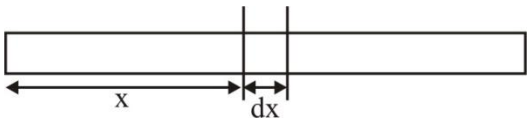
$t_d = 6 - 2s = \sqrt{\frac{2h_{max}}{g}}$

$\Rightarrow h_{max} = \frac{16 \times 9.8}{2} = \frac{392}{5}$

2. The distance of centre of mass from end A of a one dimensional rod (AB) having mass density $\rho = \rho_0\left(1 - \frac{x^2}{L^2}\right)$ kg/m and length L (in meter) is $\frac{3L}{\alpha}$m. The value of α is ……. (where x is the distance form end A)

Official Ans. by NTA (8)

Sol.



$dm = \lambda \cdot dx = \lambda_0\left(1 - \frac{x^2}{\ell^2}\right)$

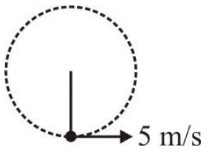
$X_{cm} = \frac{\int x dm}{\int dm_\ell}$

$= \frac{\lambda_0 \int_0^\ell \left(x - \frac{x^3}{\ell^2}\right)dx}{\int_0^\ell \lambda_0\left(1 - \frac{x^2}{\ell^2}\right)dx} = \frac{\frac{\ell^2}{2} - \frac{\ell^4}{4\ell^2}}{\ell - \frac{\ell^3}{3\ell^2}} = \frac{3\ell}{8}$

3. A string of area of cross-section 4 mm^2 and length 0.5 is connected with a rigid body of mass 2 kg. The body is rotated in a vertical circular path of radius 0.5 m. The body acquires a speed of 5 m/s at the bottom of the circular path. Strain produced in the string when the body is at the bottom of the circle is …… × 10^{-5}. (Use Young's modulus 10^{11} N/m^2 and g = 10 m/s^2)

Official Ans. by NTA (30)

Sol.



Strain = F/AY

$= \frac{mg + \frac{mv^2}{R}}{AY}$

$= \frac{20 + \frac{2(5)^2}{0.5}}{3 \times 10^{-6} \times 10^{11}} = 30 \times 10^{-5}$

4. At a certain temperature, the degrees of freedom per molecule for gas is 8. The gas performs 150 J of work when it expands under constant pressure. The amount of heat absorbed by the gas will be …….. J.

Official Ans. by NTA (750)

Sol. $W = nR\Delta T = 150$ J

$\mathbf{Q} = \left(\frac{f}{2} + 1\right)nR\Delta T = \left(\frac{8}{2} + 1\right)150 = 750\,J$

5. The potential energy of a particle of mass 4 kg in motion along the x-axis is given by U = 4(1 – cos 4x) J. The time period of the particle for small oscillation $(\sin\theta \simeq \theta)$ is $\left(\frac{\pi}{K}\right)$s. The value of K is ……..

Official Ans. by NTA (2)

Sol. $U = 4(1 - \cos 4x)$

$F = -\frac{dU}{dx} = -4(+\sin 4x)4 = -16\sin(4x)$

For small θ

$\sin\theta \approx \theta$

$F = -64x$

$a = -64x/m = -16x$

$\omega^2 = 16$

$T = \frac{2\pi}{\omega} = \frac{\pi}{2}$

6. An electrical bulb rated 220 V, 100 W, is connected in series with another bulb rated 220 V, 60 W. If the voltage across combination is 220 V, the power consumed by the 100 W bulb will be about ……. W.

Official Ans. by NTA (14)

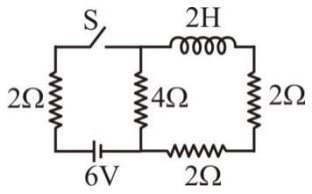
Sol. $R_1 = \frac{V^2}{P} = \frac{220^2}{100} = 484$

$R_2 = \frac{V^2}{P} = \frac{220^2}{60} = 484\left(\frac{10}{6}\right)$

$I = \frac{220}{484 + 484 \times \frac{10}{6}}$

$P_1 = I^2R_1 = 14.06\,W$

7. For the given circuit the current through battery of 6 V just after closing the switch 'S' will be …..…… A.

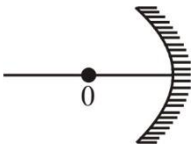


Official Ans. by NTA (1)

Sol. Just after closing the switch S, inductor behaves like an open circuit.

$I = \frac{6}{2+4} = 1A$

8. An object 'o' is placed at a distance of 100 cm in front of a concave mirror of radius of curvature 200 cm as shown in the figure. The object starts moving towards the mirror at a speed 2 cm/s. The position of the image from the mirror after 10s will be at ……. cm.



Official Ans. by NTA (400)

Sol. After 10 sec.

u = –80 cm

f = –100 cm

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

v = 400 cm

9. In an experiment with a convex lens. The plot of the image distance (v') against the object distance (μ') measured from the focus gives a curve v' μ' = 225. If all the distances are measured in cm. The magnitude of the focal length of the lens is …….. cm.

Official Ans. by NTA (15)

Sol. $vu = f^2$ (by Newton's formula)

$f^2 = 225$

$f = 15$ cm

10. In an experiment to find acceleration due to gravity (g) using simple pendulum, time period of 0.5 s is measured from time of 100 oscillation with a watch of 1s resolution. If measured value of length is 10 cm known to 1mm accuracy. The accuracy in the determination of g is found to be x %. The value of x is

Official Ans. by NTA (5)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$g = \frac{1}{4\pi^2}\frac{T^2}{\ell}$$

$$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta g}{g} = 2\cdot\frac{1}{100\times 0.5} + \frac{1mm}{10cm}$$

$$\frac{\Delta g}{g} = \frac{5}{100}$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Zero orbital overlap is an out of phase overlap.

Reason : It results due to different orientation/ direction of approach of orbitals.

In the light of the above statements. Choose the ***correct*** answer from the options given below

(A) Both A and R are true and R is the correct explanation of A

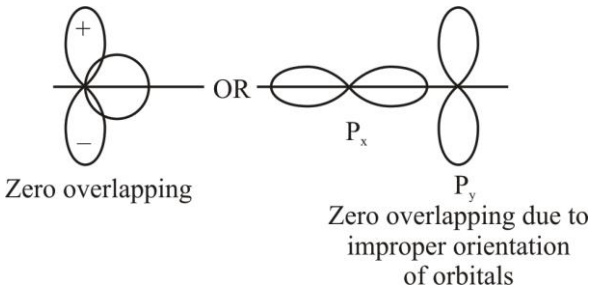
(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

Sol.



Zero overlapping

Zero overlapping due to improper orientation of orbitals

2. The correct decreasing order for metallic character is

(A) Na > Mg > Be > Si > P

(B) P > Si > Be > Mg > Na

(C) Si > P > Be > Na > Mg

(D) Be > Na > Mg > Si > P

Official Ans. by NTA (A)

Sol. Across a period metallic character decreases

3. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : The reduction of a metal oxide is easier if the metal formed is in liquid state than solid state.

Reason R : The value of $\Delta G^{\ominus}$ becomes more on negative side as entropy is higher in liquid state than solid state.

In the light of the above statements. Choose the most appropriate answer from the options given below

(A) Both A and R are correct and R is the correct explanation of A

(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol. $\Delta G = \Delta H - T\Delta S$

$\because$ Entropy of liquid is more than solid

$\therefore$ on melting the entropy increases and ΔG becomes more negative and hence it becomes easier to reduce metal

4. The products obtained during treatment of hard water using Clark's method are:

(A) $CaCO_3$ and $MgCO_3$

(B) $Ca(OH)_2$ and $Mg(OH)_2$

(C) $CaCO_3$ and $Mg(OH)_2$

(D) $Ca(OH)_2$ and $MgCO_3$

Official Ans. by NTA (C)

Sol. In Clark's method lime water is used

$$Ca(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + 2H_2O$$

$$Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + Mg(OH)_2 + 2H_2O$$

5. **Statement I:** An alloy of lithium and magnesium is used to make aircraft plates.

Statement II : The magnesium ions are important for cell-membrane integrity.

In the light the above statements, choose the *correct* answer from the options given below

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (B)

Sol. Alloy of Li and Mg is used to make armour plates and not aircraft plates.

Calcium plays important roles in neuromuscular function, interneuronal transmission and cell membrane integrity

6. White phosphorus reacts with thionyl chloride to give

(A) PCl_5, SO_2 and S_2Cl_2 (B) PCl_3. SO_2 and S_2Cl_2

(C) PCl_3, SO_2 and Cl_2 (D) PCl_5, SO_2 and Cl_2

Official Ans. by NTA (B)

Sol. $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

7. Concentrated HNO_3 reacts with Iodine to give

(A) HI, NO_2 and H_2O (B) HIO_2, N_2O and H_2O

(C) HIO_3, NO_2 and H_2O (D) HIO_4, N_2O and H_2O

Official Ans. by NTA (C)

Sol. $I_2 + 10HNO_{3(conc)} \Rightarrow 2HIO_3 + 10NO_2 + 4H_2O$

8. Which of the following pair is not isoelectronic species?

(At. no. Sm, 62; Er, 68: Yb, 70: Lu, 71; Eu, 63: Tb, 65; Tm, 69)

(A) Sm^{2+} and Er^{3+} (B) Yb^{2+} and Lu^{3+}

(C) Eu^{2+} and Tb^{4+} (D) Tb^{2+} and Tm^{4+}

Official Ans. by NTA (D)

Sol. $Sm^{2+} \rightarrow$ electron = 60

$Er^{3+} \rightarrow$ electron = 65

$Tb^{2+} \rightarrow$ electron = 63

$Tm^{4+} \rightarrow$ electron = 65

(not isoelectronic)

9. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Permanganate titrations are not performed in presence of hydrochloric acid.

Reason R : Chlorine is formed as a consequence of oxidation of hydrochloric acid.

In the light of the above statements, choose the ***correct*** answer from the options given below

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

Sol. $2KMnO_4 + 16HCl \rightarrow 2MnCl_2 + 2KCl + 8H_2O + Cl_2$

HCl gets oxidised by $KMnO_4$ into Cl_2

10. Match **List I** with **List II**

	List I (Complex)		**List II (Hybridization)**
A	$Ni(CO)_4$	I	sp^3
B	$[Ni(CN)_4]^{2-}$	II	sp^3d^2
C	$[Co(CN)_6]^{3-}$	III	d^2sp^3
D	$[CoF_6]^{3-}$	IV	dsp^2

Choose the correct answer from the options given below:

(A) A-IV, B-I, C-III, D-II

(B) A-I. B-IV, C-III, D-II

(C) A-I. B-IV, C-II, D-III

(D) A-IV, B-I, C-II. D-III

Official Ans. by NTA (B)

Sol. $Ni(CO)_4$ Hybridisation sp^3

$[Ni(CN)_4]^{2-}$ Hybridisation dsp^2

$[Co(CN)_6]^{3-}$ Hybridisation d^2sp^3

$[Co(F)_6]^{3-}$ Hybridisation sp^3d^2

11. Dinitrogen and dioxygen. the main constituents of air do not react with each other in atmosphere to form oxides of nitrogen because

(A) N_2 is unreactive in the condition of atmosphere.

(B) Oxides of nitrogen are unstable.

(C) Reaction between them can occur in the presence of a catalyst.

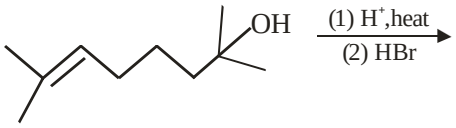
(D) The reaction is endothermic and require very high temperature.

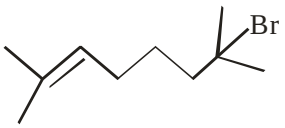
Official Ans. by NTA (D)

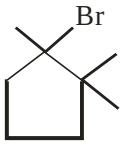
Sol. $N_2 + O_2 \underset{}{\overset{(1483-2000\ K)}{\rightleftharpoons}} 2NO$

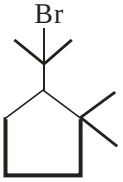
(Endothermic and feasible at high temperature)

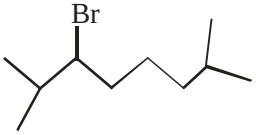
12. The major product in the given reaction is



(A) 

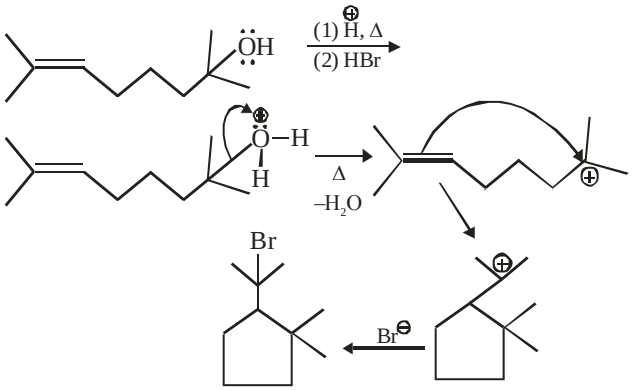
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

Sol.



13. Arrange the following in increasing order of reactivity towards nitration

A. p-xylene

B. bromobenzene

C. mesitylene

D, nitrobenzene

E. benzene

Choose the correct answer from the options given below

(A) C < D < E < A < B

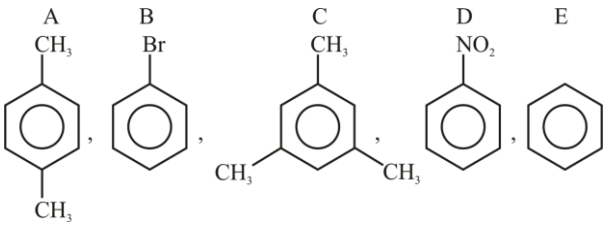
(B) D < B < E < A < C

(C) D < C < E < A < B

(D) C < D < E < B < A

Official Ans. by NTA (B)

Sol.



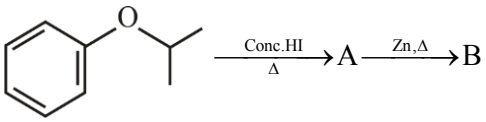
$-NO_2$ is strongly deactivating

–Br – deactivating

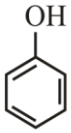
$-CH_3$–activating group

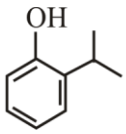
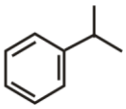
D < B < E < A < C

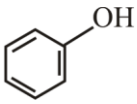
14. Compound I is heated with Conc. HI to give a hydroxy compound A which is further heated with Zn dust to give compound B. Identify A and B.



(A) A =  , B = 

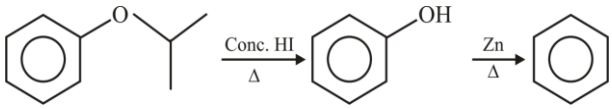
(B) A =  , B = 

(C) A =  , B = 

(D) A =  , B = 

Official Ans. by NTA (D)

Sol.



15. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Aniline on nitration yields ortho, meta & para nitro derivatives of aniline.

Reason R: Nitrating mixture is a strong acidic mixture.

In the light of the above statements, choose the ***correct*** answer from the options given below

(A) Both A and R are true and R is the correct explanation of A

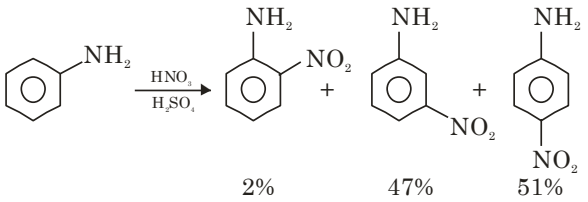
(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

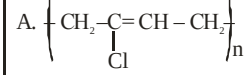
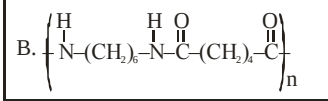
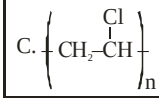
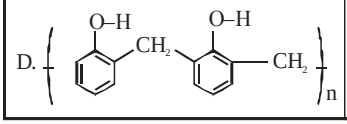
Official Ans. by NTA (A)

Sol.



Due to formation of anilinium ion in acidic medium meta product is also obtained in significant amount

16. Match **List I** with **List II**

List (Polymer)	List II(Nature)
A. 	I. Thermosetting polymer
B. 	II. Fibers
C. 	III. Elastomer
D. 	IV. Thermoplastic polymer

Choose the correct answer from the options given below:

(A) A-II, B-III, C-IV, D-I

(B) A-III, B-II, C-IV, D-I

(C) A-III, B-I, C-IV, D-II

(D) A-I. B-III, C-IV, D-II

Official Ans. by NTA (B)

Sol. Neoprene is elastomer

Nylon-6, 6 is fiber

PVC is thermoplastic

Novolac is thermosetting

17. Two statements in respect of drug-enzyme interaction are given below

Statement I : Action of an enzyme can be blocked only when an inhibitor blocks the active site of the enzyme.

Statement II : An inhibitor can form a strong covalent bond with the enzyme.

In the light of the above statements. Choose the ***correct*** answer from the options given below

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (D)

Sol. Some drugs do not bind to active sites. These bind to different site of enzyme called allosteric sites.

18. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Thin layer chromatography is an adsorption chromatography.

Reason : A thin layer of silica gel is spread over a glass plate of suitable size in thin layer chromatography which acts as an adsorbent.

In the light of the above statements, choose the ***correct*** answer from the options given below

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

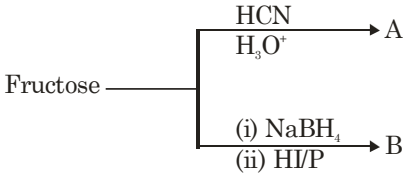
Official Ans. by NTA (A)

Sol. Theory based

Thin layer chromoatography (TLC) is another type of adsorption chromatography, which involve sepration of substance of a mixture ovel a thin layer of an adsorbent coated on glass plate.

A thin layer (about 0.2 mm thick) of an adsorbent (silica gel) or (Alumina) in spread overa glass plate of suitable size. Hence Assertion (A) is correct and Reason (R) is correct explanation of (A)

19. The formulas of A and B for the following reaction sequence are



(A) A = $C_7H_{14}O_8$, B = C_6H_{14}

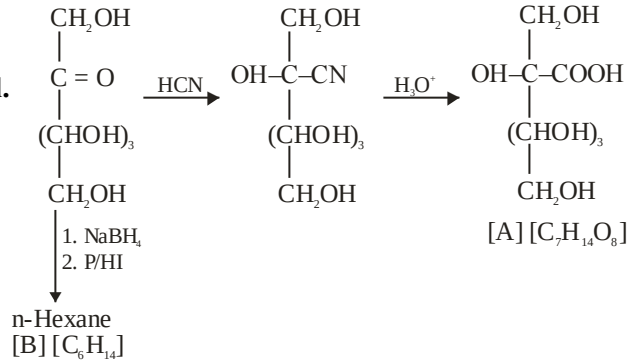
(B) A = $C_7H_{13}O_7$, B = $C_7H_{14}O$

(C) A = $C_7H_{12}O_8$, B = C_6H_{14}

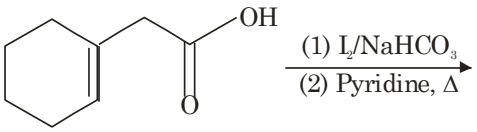
(D) A = $C_7H_{14}O_8$, B = $C_6H_{14}O_6$

Official Ans. by NTA (A)

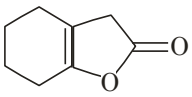
Sol.

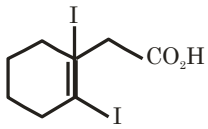


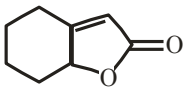
20.

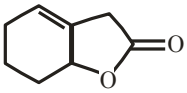


Find out the major product for the above reaction.

(A) 

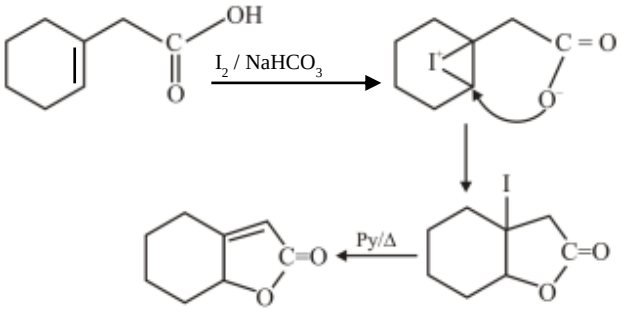
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

Sol.



SECTION-B

1. 2L of 0.2 M H_2SO_4 is reacted with 2L of 0.1 M NaOH solution, the molarity of the resulting product Na_2SO_4 in the solution is ____ millimolar. (Nearest integer).

Official Ans. by NTA (25)

Sol. $H_2SO_4 + 2NaOH \rightarrow Na_2SO_4 + 2H_2O$

0.4 mol	0.2 mol	-
0.3 mol	-	0.1 mol

Molarity of Na_2SO_4 is $\frac{0.1}{4} = 0.025M$

= 25 mM.

2. Metal M crystallizes into a FCC lattice with the edge length of 4.0×10^{-8} cm. The atomic mass of the metal is _____ g/mol. (Nearest integer).

(Use : $N_A = 6.02\times10^{23}$ mol^{-1}, density of metal, M = 9.03 g cm^{-3})

Official Ans. by NTA (87)

Sol. $a = 4 \times 10^{-8}$ cm

d = 9.03g/ml

$$d = \frac{ZM}{N_A a^3}$$

$$M = \frac{9.03\times6.02\times10^{23}\times64\times10^{-24}}{4} \quad 86.97$$

3. If the wavelength for an electron emitted from H-atom is 3.3×10^{-10}m, then energy absorbed by the electron in its ground state compared to minimum energy required for its escape from the atom, is _____ times. (Nearest integer).

[Given : h = 6.626×10^{-34} Js,

Mass of electron = 9.1×10^{-31}**]**

Official Ans. by NTA (2)

Sol. $$\lambda = \frac{h}{\sqrt{2mK}}$$

$$K = \frac{h^2}{2m\lambda^2}$$

$$K = \frac{h^2}{2m\lambda^2} = \frac{43.9\times10^{-68}}{2\times9.1\times10^{-31}\times10.89\times10^{-20}}$$

$K = 2.215 \times 10^{-18}$

$E_{abs} = E_{req} + K$

$$\frac{E_{abs}}{E_{req}} = 1 + \frac{K}{E_{req}} = 1 + \frac{2.215\times10^{-18}}{13.6\times1.602\times10^{-19}} = 2.0166$$

4. A gaseous mixture of two substances A and B, under a total pressure of 0.8 atm is in equilibrium with an ideal liquid solution. The mole fraction of substance A is 0.5 in the vapour phase and 0.2 in the liquid phase. The vapour pressure of pure liquid A is _____ atm. (Nearest integer)

Official Ans. by NTA (2)

Sol. **$Y_A = 0.5 \Rightarrow Y_B = 0.5$**

$P_A = P_B = 0.4$ atm

$$P_A = P_A^0 X_A$$

$$P_A^0 = 2$$

5. At 600K, 2 mol of NO are mixed with 1 mol of O_2.

$$2NO_{(g)} + O_2(g) \rightleftharpoons 2NO_2(g)$$

The reaction occurring as above comes to equilibrium under a total pressure of 1 atom. Analysis of the system shows that 0.6 mol of oxygen are present at equilibrium. The equilibrium constant for the reaction is _____. (Nearest integer).

Official Ans. by NTA (2)

Sol.

$2NO +$	$O_2 \rightarrow$	$2NO$
2	1	-
2-2x	1-x	2x
1.2	0.6	0.8

$$K_p = \frac{\left(\frac{0.8}{2.6}\right)^2}{\left(\frac{1.2}{2.6}\right)^2\left(\frac{0.6}{2.6}\right)} = 1.925$$

6. A sample of 0.125 g of an organic compound when analysed by Duma's method yields 22.78 mL of nitrogen gas collected over KOH solution at 280K and 759 mm Hg. The percentage of nitrogen in the given organic compound is ____. (Nearest integer).

(a) The vapour pressure of water at 280 K is 14.2 mm Hg

(b) R = 0.082 L atm K^{-1} mol^{-1}

Official Ans. by NTA (22)

Sol. V = 22.78 ml, T = 280 K

P_{total} = 759 mmHg

$$P_{N_2} = 759 - 14.2 = 744.8 mmHg$$

$$n_{N_2} = \frac{744.8 \times 22.78}{760 \times 1000 \times 0.082 \times 280} = 0.00097$$

$$W_{Nitrogen} = 0.02716$$

$$\%N = \frac{0.02716}{0.125} \times 1000 = 21.728$$

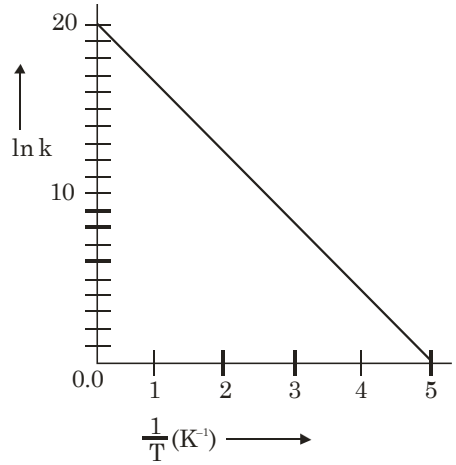
7. On reaction with stronger oxidizing agent like KIO_4, hydrogen peroxide oxidizes with the evolution of O_2. The oxidation number of I in KIO_4 changes to _____.

Official Ans. by NTA (5)

Sol. $IO_4^- + H_2O_2 \rightarrow IO_3^- + O_2$

8. For a reaction, given below is the graph of ln k vs $\frac{1}{T}$. The activation energy for the reaction is equal to _____ cal mol^{-1}. (Nearest integer).

(Given : R = 2 cal K^{-1} mol^{-1}**)**



Official Ans. by NTA (8)

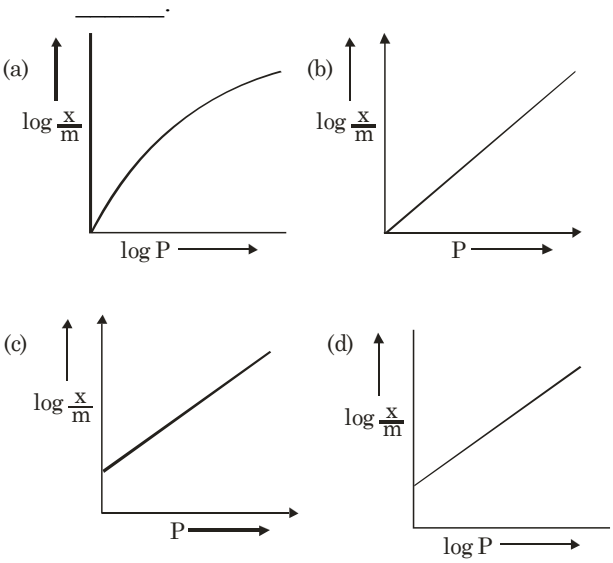
Sol. $K = Ae^{-Ea/RT}$

$$\ln k = \frac{-Ea}{RT} + \ln A$$

$$Slope = \frac{Ea}{R} = \frac{20}{5}$$

E_a = 4R = 8 Cal/mol

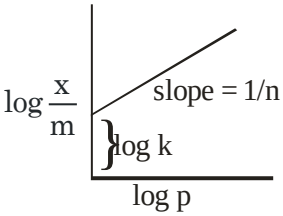
9. Among the following the number of curves not in accordance with Freundlich adsorption isotherm is _____.



Official Ans. by NTA (3)

Sol. $\frac{X}{m} = KP^{\frac{1}{n}}$

$$\log \frac{x}{m} = \frac{1}{n} \log p + \log k$$



10. Among the following the number of state variable is _____.

Internal energy (U)

Volume (V)

Heat (q)

Enthalpy (H)

Official Ans. by NTA (3)

Sol. Internal energy, volume enthalpy are state variable

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $S=\left\{x\in[-6,3]-\{-2,2\}:\frac{|x+3|-1}{|x|-2}\geq 0\right\}$ and $T=\{x\in Z:x^2-7|x|+9\leq 0\}$. Then the number of elements in $S\cap T$ is

(A) 7 (B) 5

(C) 4 (D) 3

Official Ans. by NTA (D)

Sol. $S\cap T=\{-5,-4,3\}$

2. Let α, β be the roots of the equation $x^2-\sqrt{2}x+\sqrt{6}=0$ and $\frac{1}{\alpha^2}+1, \frac{1}{\beta^2}+1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :

(A) non-real complex numbers

(B) real and both negative

(C) real and both positive

(D) real and exactly one of them is positive

Official Ans. by NTA (B)

Sol. $a=\frac{-1}{\alpha^2}-\frac{1}{\beta^2}-2$

$b=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+1+\frac{1}{\alpha^2\beta^2}$

$a+b=\frac{1}{(\alpha\beta)^2}-1=\frac{1}{6}-1=-\frac{5}{6}$

$x^2-\left(-\frac{5}{6}-2\right)x+\left(2-\frac{5}{6}\right)=0$

$6x^2 + 17x + 7 = 0$

$x=-\frac{7}{3}$, $x=-\frac{1}{2}$ are the roots

Both roots are real and negative.

3. Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then which of the following is **NOT** true?

(A) $A^4 – B^4$ is a symmetric matrix

(B) AB – BA is a symmetric matrix

(C) $B^5 – A^5$ is a skew-symmetric matrix

(D) AB + BA is a skew-symmetric matrix

Official Ans. by NTA (C)

Sol. Given that $A^T = A$, $B^T = –B$

(A) $C = A^4 – B^4$

$C^T = (A^4 – B^4) = (A^4)^T – (B^4)^T = A^4 – B^4 = C$

(B) $C = AB – BA$

$C^T = (AB – BA)^T = (AB)^T – (BA)^T$

$= B^TA^T – A^TB^T = –BA + AB = C$

(C) $C = B^5 – A^5$

$C^T = (B^5 – A^5)^T = (B^5)^T – (A^5)^T = –B^5 – A^5$

(D) $C = AB + BA$

$C^T = (AB + BA)^T = (AB)^T + (BA)^T$

$= –BA – AB = –C$

∴ Option C is not true.

4. Let $f(x) = ax^2 + bx + c$ be such that $f(1) = 3$, $f(– 2) = \lambda$ and $f(3) = 4$. If $f(0) + f(1) + f(–2) + f(3) = 14$, then λ is equal to

(A) – 4 (B) $\frac{13}{2}$

(C) $\frac{23}{2}$ (D) 4

Official Ans. by NTA (D)

Sol. $f(0) + 3 + \lambda + 4 = 14$

∴ $f(0) = 7 – \lambda = c$

$f(1) = a + b + c = 3$ …(i)

$f(3) = 9a + 3b + c = 4$ …(ii)

$f(–2) = 4a – 2b + c = \lambda$ …(iii)

(ii) – (iii)

$a + b = \frac{4-\lambda}{5}$ put in equation (i)

$\frac{4-\lambda}{5}+7-\lambda=3$

$6\lambda = 24$; $\lambda = 4$

5. The function $f : R \to R$ defined by

$$f(x) = \lim_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$$ is

continuous for all x in

(A) $R - \{-1\}$ (B) $R - \{-1, 1\}$

(C) $R - \{1\}$ (D) $R - \{0\}$

Official Ans. by NTA (B)

Note : n should be given as a natural number.

Sol. $$f(x) = \begin{cases} \frac{-\sin(x-1)}{x-1} & x < -1 \\ -(\sin 2+1) & x = -1 \\ \cos 2\pi x & -1 < x < 1 \\ 1 & x = 1 \\ \frac{-\sin(x-1)}{x-1} & x > 1 \end{cases}$$

f(x) is discontinuous at x = –1 and x = 1

6. The function $f(x) = xe^{x(1-x)}, x \in R,$ is

(A) increasing in $\left(-\frac{1}{2}, 1\right)$

(B) decreasing in $\left(\frac{1}{2}, 2\right)$

(C) increasing in $\left(-1, -\frac{1}{2}\right)$

(D) decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Official Ans. by NTA (A)

Sol. $f(x) = x\ e^{x(1-x)}$

$f'(x) = -e^{x(1-x)}\ (2x + 1)\ (x - 1)$

f(x) is increasing in $\left(-\frac{1}{2}, 1\right)$

7. The sum of the absolute maximum and absolute minimum values of the function

$f(x) = \tan^{-1}(\sin x - \cos x)$ in the interval $[0, \pi]$ is

(A) 0 (B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{4}$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$ (D) $\frac{-\pi}{12}$

Official Ans. by NTA (C)

Sol. $f(x) = \tan^{-1}(\sin x - \cos x)$

$$f'(x) = \frac{\cos x + \sin x}{(\sin x - \cos x)^2 + 1} = 0$$

$\therefore\ x = \frac{3\pi}{4}$

x	0	$\frac{3\pi}{4}$	π
f(x)	$-\frac{\pi}{4}$	$\tan^{-1}\sqrt{2}$	$\frac{\pi}{4}$

$$\therefore \begin{matrix} (f(x))_{max} = \tan^{-1}\sqrt{2} \\ (f(x))_{min} = -\frac{\pi}{4} \end{matrix}$$

$\text{sum} = \tan^{-1}\sqrt{2} - \frac{\pi}{4}$

$= \cos^{-1}\frac{1}{\sqrt{3}} - \frac{\pi}{4}$

8. Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t \in \left(0, \frac{\pi}{2}\right)$. Then

$\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to

(A) $\frac{-2\sqrt{2}}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{-2}{3}$

Official Ans. by NTA (D)

Sol. $x = 2\sqrt{2}\cos t\sqrt{\sin 2t}$

$\frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}$

$y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$

$\frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$

$\frac{dy}{dx} = \tan 3t$

$\frac{dy}{dx} = -1$ at $t = \frac{\pi}{4}$

$\frac{d^2y}{dx^2} = \frac{3}{2\sqrt{2}}\sec^3 3t \cdot \sqrt{\sin 2t} = -3$ at $t = \frac{\pi}{4}$

$$\therefore \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} = \frac{1+1}{-3} = -\frac{2}{3}$$

9. Let $I_n(x)=\int_0^x \frac{1}{(t^2+5)^n}dt$, n = 1, 2, 3, …. Then

(A) $50I_6-9I_5=xI_5'$ (B) $50I_6-11I_5=xI_5'$

(C) $50I_6-9I_5=I_5'$ (D) $50I_6-11I_5=I_5'$

Official Ans. by NTA (A)

Sol. $I_n(x)=\int_0^x \frac{dt}{(t^2+5)^n}$

Applying integral by parts

$$I_n(x)=\left[\frac{t}{(t^2+5)^n}\right]_0^x-\int_0^x n(t^2+5)^{-n-1}\cdot 2t^2$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\int_0^x\frac{t^2}{(t^2+5)^{n+1}}dt$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\int_0^x\frac{(t^2+5)-5}{(t^2+5)^{n+1}}dt$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\,I_n(x)-10n\,I_{n+1}(x)$$

$$10n\,I_{n+1}(x)+(1-2n)I_n(x)=\frac{x}{(x^2+5)^n}$$

Put n = 5

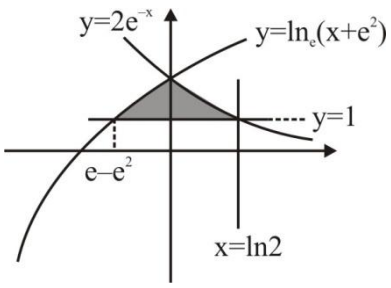
10. The area enclosed by the curves $y=\log_e(x+e^2)$, $x=\log_e\left(\frac{2}{y}\right)$ and $x=\log_e 2$, above the line y = 1 is

(A) $2+e-\log_e 2$ (B) $1+e-\log_e 2$

(C) $e-\log_e 2$ (D) $1+\log_e 2$

Official Ans. by NTA (B)

Sol.



Required area is

$$=\int_{e-e^2}^{0}\ell n\left(x+e^2\right)-1dx+\int_0^{\ell n2}2e^{-x}-1dx=1+e-\ell n2$$

11. Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$, x > 1 passing through the point $\left(2,\sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}y(8)$ is equal to

(A) $11+6\log_e 3$ (B) 19

(C) $12-2\log_e 3$ (D) $19-6\log_e 3$

Official Ans. by NTA (D)

Sol. $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$,

$$\frac{dy}{dx}+Py=Q$$

$$\text{I.F.}=e^{\int Pdx}=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$

$$y\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}=\int\left(\frac{x-1}{x+1}\right)^{1}dx$$

$$=x-2\log_e|x+1|+C$$

Curve passes through $\left(2,\frac{1}{\sqrt{3}}\right)$

$$\Rightarrow C=2\log_e 3-\frac{5}{3}$$

at x = 8,

$$\sqrt{7}y(8)=19-6\log_e 3$$

12. The differential equation of the family of circles passing through the points (0, 2) and (0, – 2) is

(A) $2xy\frac{dy}{dx}+\left(x^2-y^2+4\right)=0$

(B) $2xy\frac{dy}{dx}+\left(x^2+y^2-4\right)=0$

(C) $2xy\frac{dy}{dx}+\left(y^2-x^2+4\right)=0$

(D) $2xy\frac{dy}{dx}-\left(x^2-y^2+4\right)=0$

Official Ans. by NTA (A)

Sol. Equation of circle passing through (0, –2) and (0, 2) is

$$x^2+\left(y^2-4\right)+\lambda x=0,\ (\lambda\in R)$$

Divided by x we get

$$\frac{x^2+\left(y^2-4\right)}{x}+\lambda=0$$

Differentiating with respect to x

$$\frac{x\left[2x+2y\cdot\frac{dy}{dx}\right]-\left[x^2+y^2-4\right]\cdot1}{x^2}=0$$

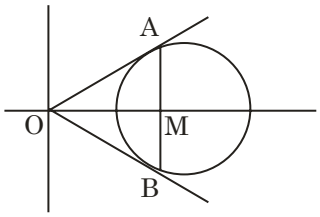
$$\Rightarrow 2xy\cdot\frac{dy}{dx}+\left(x^2-y^2+4\right)=0$$

13. Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at origin O (0, 0). Then the area of the triangle of OAB is

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{4}$

(C) $\frac{3}{2\sqrt{3}}$ (D) $\frac{3}{4\sqrt{3}}$

Official Ans. by NTA (B)



Sol. C : $(x-2)^2 + y^2 = 1$

Equation of chord AB : 2x = 3

OA = OB = $\sqrt{3}$

$$AM=\frac{\sqrt{3}}{2}$$

$$\text{Area of triangle OAB}=\frac{1}{2}(2AM)(OM)$$

$$=\frac{3\sqrt{3}}{4}\text{ sq. units}$$

14. Let the hyperbola H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ pass through the point $\left(2\sqrt{2},-2\sqrt{2}\right)$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola?

(A) $\left(2\sqrt{3},3\sqrt{2}\right)$ (B) $\left(3\sqrt{3},-6\sqrt{2}\right)$

(C) $\left(\sqrt{3},-\sqrt{6}\right)$ (D) $\left(3\sqrt{6},6\sqrt{2}\right)$

Official Ans. by NTA (B)

Sol. H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Foci : S (ae, 0), S' (–ae, 0)

Foot of directrix of parabola is (–ae, 0)

Focus of parabola is (ae, 0)

Now, semi latus rectum of parabola $=|SS'|=2ae$

Given, $4ae=e\left(\frac{2b^2}{a}\right)$

$\Rightarrow b^2=2a^2$ ..... (1)

Given, $\left(2\sqrt{2},-2\sqrt{2}\right)$ lies on H

$\Rightarrow \frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{8}$ ..... (2)

From (1) and (2)

$a^2 = 4, b^2 = 8$

$\because b^2=a^2\left(e^2-1\right)$

$\therefore e=\sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2=8\sqrt{3}x$

15. Let the lines $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does **NOT** lies on P?

(A) (0, – 2, –2) (B) (– 5, 0, –1)

(C) (3, –1, 0) (D) (0, 4, 5)

Official Ans. by NTA (D)

Sol. Given, $L_1:\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$

and $L_2:\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$

are coplanar

$$\Rightarrow\begin{vmatrix}27 & 20 & 31\\ \lambda & 1 & 2\\ -2 & 3 & \lambda\end{vmatrix}=0$$

$\Rightarrow\lambda=3$

Now, normal of plane P, which contains L_1 and L_2

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ -2 & 3 & 3\end{vmatrix}$$

$=-3\hat{i}-13\hat{j}+11\hat{k}$

$\Rightarrow$ Equation of required plane P :

$3x + 13y – 11z + 4 = 0$

(0, 4, 5) does not lie on plane P.

16. A plane P is parallel to two lines whose direction ratios are –2, 1, –3, and –1, 2, –2 and it contains the point (2, 2, –2). Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron OABC, where O is the origin and $p = \alpha+\beta+\gamma$, then the ordered pair (V, p) is equal to

(A) (48, –13) (B) (24, –13)

(C) (48, 11) (D) (24, –5)

Official Ans. by NTA (B)

Sol. Normal of plane P :

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 1 & -3\\ -1 & 2 & -2\end{vmatrix}=4\hat{i}-\hat{j}-3\hat{k}$$

Equation of plane P which passes through (2, 2,–2) is $4x – y – 3z – 12 = 0$

Now, A (3, 0, 0), B (0, – 12, 0), C (0, 0, –4)

$\Rightarrow\alpha=3,\beta=-12,\gamma=-4$

$\Rightarrow p=\alpha+\beta+\gamma=-13$

Now, volume of tetrahedron OABC

$$V=\left|\frac{1}{6}\overrightarrow{OA}\cdot\left(\overrightarrow{OB}\times\overrightarrow{OC}\right)\right|=24$$

(V, p) = (24, –13)

17. Let S be the set of all $a \in R$ for which the angle between the vectors $\vec{u}=a\left(\log_e b\right)\hat{i}-6\hat{j}+3\hat{k}$ and $\vec{v}=\left(\log_e b\right)\hat{i}+2\hat{j}+2a\left(\log_e b\right)\hat{k},(b>1)$ is acute. Then S is equal to

(A) $\left(-\infty,-\frac{4}{3}\right)$ (B) Φ

(C) $\left(-\frac{4}{3},0\right)$ (D) $\left(\frac{12}{7},\infty\right)$

Official Ans. by NTA (C)

Sol. For angle to be acute

$\vec{u}\cdot\vec{v}>0$

$\Rightarrow a\left(\log_e b\right)^2-12+6a\left(\log_e b\right)>0$

$\forall\ b>1$

let $\log_e b=t\Rightarrow t>0$ as $b>1$

$y=at^2+6at-12\ \&\ y>0,\ \forall\ t>0$

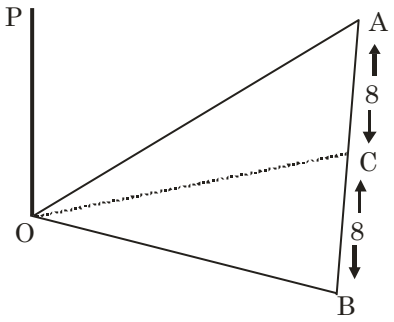
$\Rightarrow a\in\phi$

18. A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the midpoint of AB. Then $(OP)^2$ is equal to

(A) $\frac{32}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (B) $\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$

(C) $\frac{16}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (D) $\frac{16}{\sqrt{3}}\left(2-\sqrt{3}\right)$

Official Ans. by NTA (B)



Sol.

$$\frac{OP}{OA}=\tan 15^\circ$$

$$\Rightarrow OA=OP\cot 15^\circ$$

$$\frac{OP}{OC}=\tan 45^\circ \Rightarrow OP=OC$$

Now, $OP=\sqrt{OA^2-8^2}$

$$\Rightarrow OP^2=(OP)^2\cot^2 15^\circ-64$$

$$\Rightarrow OP^2=\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$$

19. Let A and B be two events such that $P(B|A)=\frac{2}{5}$, $P(A|B)=\frac{1}{7}$ and $P(A\cap B)=\frac{1}{9}$. Consider

$(S1)\ P(A'\cup B)=\frac{5}{6}$,

$(S2)\ P(A'\cap B')=\frac{1}{18}$. Then

(A) Both (S1) and (S2) are true

(B) Both (S1) and (S2) are false

(C) Only (S1) is true

(D) Only (S2) is true

Official Ans. by NTA (A)

Sol. $P(A|B)=\frac{1}{7}\Rightarrow\frac{P(A\cap B)}{P(B)}=\frac{1}{7}$

$$\Rightarrow P(B)=\frac{7}{9}$$

$$P(B|A)=\frac{2}{5}\Rightarrow\frac{P(A\cap B)}{P(A)}=\frac{2}{5}$$

$$\Rightarrow P(A)=\frac{5}{18}$$

Now, $P(A'\cup B)=1-P(A\cup B)+P(B)$

$$=1-P(A)+P(A\cap B)\ =\frac{5}{6}$$

$$P(A'\cap B')=1-P(A\cup B)$$

$$=1-P(A)-P(B)+P(A\cap B)=\frac{1}{18}$$

$\Rightarrow$ Both (S1) and (S2) are true.

20. Let

p : Ramesh listens to music.

q : Ramesh is out of his village

r : It is Sunday

s : It is Saturday

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as

(A) $((\sim q)\wedge(r\vee s))\Rightarrow p$

(B) $(q\wedge(r\vee s))\Rightarrow p$

(C) $p\Rightarrow(q\wedge(r\vee s))$

(D) $p\Rightarrow((\sim q)\wedge(r\vee s))$

Official Ans. by NTA (D)

Sol. $p\equiv$ Ramesh listens to music

$\sim q\equiv$ He is in village.

$r\vee s\equiv$ Saturday or sunday

$p\Rightarrow((\sim q)\wedge(r\vee s))$

SECTION-B

1. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4$, $(1-3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6$, $\beta>0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P., then $50-\frac{2d}{\beta^2}$ is equal to _____

Official Ans. by NTA (57)

Sol. ${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P

$$\beta^2 - \frac{5}{2}\beta^3 = -12\beta$$

$$\beta = \frac{12}{5} \text{ or } \beta = -2 \ \therefore \beta = \frac{12}{5}$$

$$d = -\frac{72}{5} - \frac{144}{25} = -\frac{504}{25}$$

$$\therefore 50 - \frac{2d}{\beta^2} = 57$$

2. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then b + 3 g is equal to

Official Ans. by NTA (17)

Sol. ${}^bC_3 \times {}^gC_2 = 168$

b(b – 1)(b – 2) (g)(g – 1) = 8 × 7 × 6 × 3 × 2

b + 3 g = 17

3. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ meet at the point $R(\sqrt{2}, 2\sqrt{2} - 2)$. If S is the focus of the ellipse on its negative major axis, then $SP^2 + SQ^2$ is equal to

Official Ans. by NTA (13)

Sol. Ellipse is

$$\frac{x^2}{2} + \frac{y^2}{4} = 1; \ e = \frac{1}{\sqrt{2}}; \ S \equiv (0, -\sqrt{2})$$

Chord of contact is

$$\frac{x}{\sqrt{2}} + \frac{(2\sqrt{2} - 2)y}{4} = 1$$

$$\Rightarrow \frac{x}{\sqrt{2}} = 1 - \frac{(\sqrt{2} - 1)y}{2} \text{ solving with ellipse}$$

$$\Rightarrow y = 0, \sqrt{2} \ \therefore x = \sqrt{2}, 1$$

$$P \equiv (1, \sqrt{2}) \ Q \equiv (\sqrt{2}, 0)$$

$$\therefore (SP)^2 + (SQ)^2 = 13$$

4. If $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \ldots + {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \ldots + {}^{50}C_{50})$ is equal to $2^n.m$, where m is odd, then n + m is equal to _____

Official Ans. by NTA (99)

Sol. $1 + (1 + 2^{49})(2^{49} - 1) = 2^{98}$

$m = 1, \ n = 98$

m + n = 99

5. Two tangent lines l_1 and l_2 are drawn from the point (2, 0) to the parabola $2y^2 = -x$. If the lines l_1 and l_2 are also tangent to the circle $(x - 5)^2 + y^2 = r$, then 17r is equal to

Official Ans. by NTA (9)

Sol. $y^2 = -\frac{x}{2}$

$$y = mx - \frac{1}{8m}$$

this tangent pass through (2, 0)

$m = \pm\frac{1}{4}$ i.e., one tangent is x – 4y – 2 = 0

17r = 9

6. If $\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \ldots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then m.n is equal to _____

Official Ans. by NTA (12)

Sol. $\frac{6}{3^{12}} + 10\left(\frac{1}{3^{11}} + \frac{2}{3^{10}} + \frac{2^2}{3^9} + \frac{2^3}{3^8} + \ldots + \frac{2^{10}}{3}\right)$

$$\frac{6}{3^{12}} + \frac{10}{3^{11}}\left(\frac{6^{11} - 1}{6 - 1}\right)$$

$= 2^{12} \cdot 1; \ m.n = 12$

7. Let $S=\left[-\pi,\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right\}$. Then the number of elements in the set

$A=\left\{\theta\in S:\tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$

is _____

Official Ans. by NTA (5)

Sol. $\tan\theta+\sqrt{5}\tan2\theta\tan\theta=\sqrt{5}-\tan2\theta$

$\tan3\theta=\sqrt{5}$

$\theta=\frac{n\pi}{3}+\frac{\alpha}{3};\quad\tan\alpha=\sqrt{5}$

Five solution

8. Let z = a + ib, b ≠ 0 be complex numbers satisfying $z^2=\overline{z}\cdot2^{1-|z|}$. Then the least value of n ∈ N, such that $z^n=(z+1)^n$, is equal to _____

Official Ans. by NTA (6)

Sol. $|z^2|=|\overline{z}|\cdot2^{1-|z|}\Rightarrow|z|=1$

$z^2=\overline{z}\Rightarrow z^3=1\therefore z=\omega\text{ or }\omega^2$

$\omega^n=(1+\omega)^n=(-\omega^2)^n$

Least natural value of n is 6.

9. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then 100 σ^2 is equal to

Official Ans. by NTA (56)

Sol.

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$\sigma^2=\sum X^2P(X)-\left(\sum XP(X)\right)^2=\frac{56}{100}$

$100\ \sigma^2=56$

10. The value of the integral $\int_0^{\frac{\pi}{2}}60\frac{\sin(6x)}{\sin x}dx$ is equal to

Official Ans. by NTA (104)

Sol.

$I=60\int_0^{\pi/2}\left(\frac{\sin6x-\sin4x}{\sin x}+\frac{\sin4x-\sin2x}{\sin x}+\frac{\sin2x}{\sin x}\right)dx$

$I=60\int_0^{\pi/2}(2\cos5x+2\cos3x+2\cos x)dx$

$I=60\left(\frac{2}{5}\sin5x+\frac{2}{3}\sin3x+2\sin x\right)\Big|_0^{\pi/2}=104$