FINAL JEE–MAIN EXAMINATION – JULY, 2022

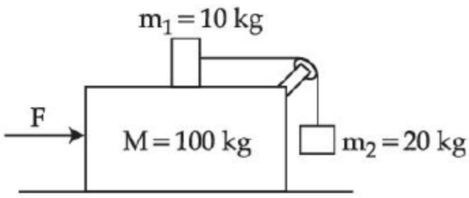
(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Three masses M = 100 kg, m_1 = 10 kg and m_2=20 kg are arranged in a system as shown in figure. All the surfaces are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force F is applied on the system so that the mass m_2 moves upward with an acceleration of 2 ms^{-2}. The value of F is :

(Take g = 10 ms^{-2})



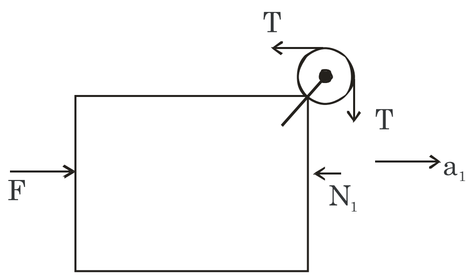
(A) 3360 N (B) 3380 N

(C) 3120 N (D) 3240 N

Official Ans. by NTA (C)

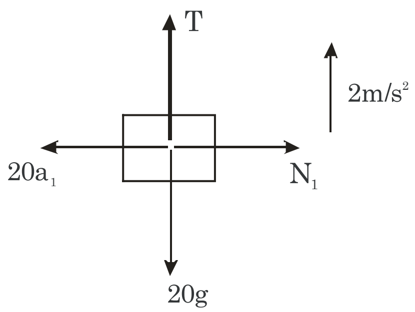
Sol. Let acceleration of 100 kg block = a_1

FBD of 100 kg block w.r.t ground



$F–T–N_1 = 100\ a_1$ ....(i)

FBD of 20 block wrt 100kg

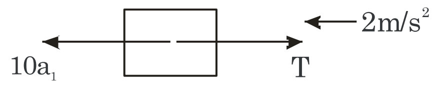


$T – 20g = 20(2)$

$T = 240$ ..... (ii)

$N_1 = 20a_1$ ..... (iii)

FBD of 10 kg block wrt 100 kg



$$10a_1 - 240 = 10(2)$$

$$a_1 = 26\,m/s^2$$

$$F - 240 - 20(26) = 100 \times 26$$

$\Rightarrow$ F = 3360 N

2. A radio can tune to any station in 6 MHz to 10 MHz band. The value of corresponding wavelength bandwidth will be :

(A)4 m (B) 20 m

(C) 30 m (D) 50 m

Official Ans. by NTA (B)

Sol. Given: Frequency f_1 = 6MHz

Frequency f_2 = 10MHz

$$\lambda_1 = \frac{c}{f_1}$$

$$\lambda_2 = \frac{c}{f_2}$$

Wavelength bandwidth = λ_2-λ_1 = 20 m

3. The disintegration rate of a certain radioactive sample at any instant is 4250 disintegrations per minute. 10 minutes later, the rate becomes 2250 disintegrations per minute. The approximate decay constant is :

(Take $\log_{10}1.88 = 0.274$)

(A)0.02 min^{-1} (B) 2.7 min^{-1}

(C) 0.063 min^{-1} (D) 6.3 min^{-1}

Official Ans. by NTA (C)

Sol. At t=0 disintegration rate = 4250 dpm

At t=10 disintegration rate = 2250 dpm

$A = A_o e^{-\lambda t}$

$2250 = 4250\, e^{-\lambda(10)}$

$\Rightarrow \lambda(10) = \ln\left(\frac{4250}{2250}\right)$

$\Rightarrow \lambda = 0.063\,\text{min}^{-1}$

4. A parallel beam of light of wavelength 900 nm and intensity 100 Wm^{-2} is incident on a surface perpendicular to the beam. Tire number of photons crossing 1 cm^2 area perpendicular to the beam in one second is :

(A) 3×10^{16} (B) 4.5×10^{16}

(C) 4.5×10^{17} (D) 4.5×10^{20}

Official Ans. by NTA (B)

Sol. Wavelength of incident beam $\lambda = 900 \times 10^{-9}$ m

Intensity of incident beam =I= 100 W/m^2

No. of photons crossing per unit sec

$= n = \frac{E_{net}}{E_{single\ photon}} = \frac{IA\lambda}{hc}$

$= \frac{(100)(1\times10^{-4})(900\times10^{-9})}{6.62\times10^{-34}\times3\times10^{8}} = 4.5\times10^{16}$

5. In young's double slit experiment, the fringe width is 12mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm)

(A) 16 (B) 9

(C) 48 (D) 12

Official Ans. by NTA (B)

Sol. For a given light wavelength corresponding a medium of refractive index μ

$\lambda_{med} = \frac{\lambda_{vacuum}}{\mu}$

and we know that fringe width $\beta = \frac{\lambda D}{d}$

Therefore, $\beta_{med} = \frac{\beta_{vacuum}}{\mu} = \frac{12}{\frac{4}{3}} = 9$ mm

6. The magnetic field of a plane electromagnetic wave is given by

$\vec{B} = 2\times10^{-8}\sin\left(0.5\times10^{3}x + 1.5\times10^{11}t\right)\hat{j}\,T$

The amplitude of the electric field would be

(A) 6Vm^{-1} along x-axis

(B) 3Vm^{-1} along z-axis

(C) 6Vm^{-1} along z-axis

(D) $2\times10^{-8}\text{Vm}^{-1}$ along z-axis

Official Ans. by NTA (C)

Sol. $c = \frac{E_0}{B_0} \Rightarrow E_0 = cB_0$

$E_0 = (3\times10^{8})(2\times10^{-8})$

$E_0 = 6\text{Vm}^{-1}$

As, $\vec{B} =$ along y-axis

$\vec{v} =$ along negative x-axis

hence $\vec{E}_0 =$ along z-axis

7. In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. The ratio $\frac{P_1}{P_2}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{\sqrt{2}}$ (D) 2 : 1

Official Ans. by NTA (B)

Sol. In case of L-R circuit

$Z=\sqrt{X_L^2+R^2}$ & power factor

$P_1=\cos\phi=\frac{R}{Z}$

As $X_L=R$

$\Rightarrow Z=\sqrt{2}R$

$\Rightarrow P_1=\frac{R}{\sqrt{2}R}\Rightarrow P_1=\frac{1}{\sqrt{2}}$

In case of L-C-R circuit

$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$

As $X_L=X_C$

$\Rightarrow Z=R$

$\Rightarrow P_2=\cos\phi=\frac{R}{R}=1$

$\Rightarrow \frac{P_1}{P_2}=\frac{1}{\sqrt{2}}$

8. A charge particle is moving in a uniform magnetic field $\left(2\hat{i}+3\hat{j}\right)T$. If it has an acceleration of $\left(\alpha\hat{i}-4\hat{j}\right)m/s^2$, then the value of α will be

(A) 3 (B) 6 (C) 12 (D) 2

Official Ans. by NTA (B)

Sol. **As** $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$

$\vec{a}=\frac{q}{m}\left(\vec{v}\times\vec{B}\right)$

So, $\vec{a}\,\&\,\vec{B}$ are $\perp$ to each other

Hence, $\vec{a}.\vec{B}=0$

$\left(\alpha\hat{i}-4\hat{j}\right).\left(2\hat{i}+3\hat{j}\right)=0$

$\alpha(2)+(-4)(3)=0$

$\alpha=\frac{12}{2}\Rightarrow\alpha=6$

9. B_X and B_Y are the magnetic field at the centre of two coils of two coils X and Y respectively, each carrying equal current. If coil X has 200 turns and 20 cm radius and coil Y has 400 turns and 20 cm radius, the ratio of B_X and B_Y is

(A)1 : 1 (B) 1 : 2

(C)2 : 1 (D) 4 : 1

Official Ans. by NTA (B)

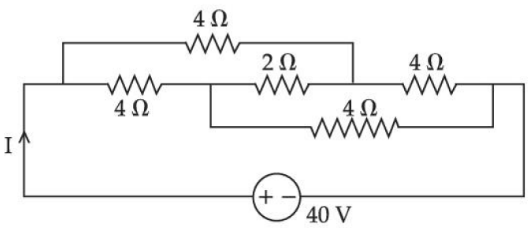
Sol. At centre $B=N\left(\frac{\mu_0 i}{2R}\right)$

$B_x=200\left(\frac{\mu_0 i}{2\times20cm}\right)$

$B_y=400\left(\frac{\mu_0 i}{2\times20cm}\right)$

$\frac{B_x}{B_y}=\frac{1}{2}$

10. The current I in the given circuit will be :

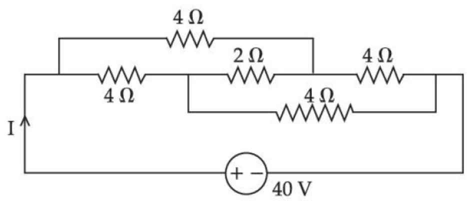


(A) 10A (B) 20 A

(C) 4A (D) 40A

Official Ans. by NTA (A)

Sol.



Given circuit is balanced wheat stone bridge

Hence 2Ω can be neglected

$R_{net}=4\Omega$

$I=\frac{40}{4}$

I = 10A

11. The total charge on the system of capacitance $C_1 = 1\mu F$, $C_2 = 2\mu F, C_3 = 4\mu F$ and $C_4 = 3\mu F$ connected in parallel is

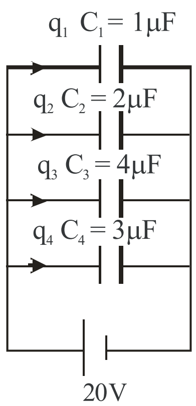
(Assume a battery of 20V is connected to the combination)

(A) $200\mu C$ (B) 200C

(C) $10\mu C$ (D) 10C

Official Ans. by NTA (A)

Sol.



Total charge = $q_1 + q_2 + q_2 + q_4$

$= 1\times 20 + 2\times 20 + 4\times 20 + 3\times 20 = 200\mu C$

12. When a particle executes simple Harmonic motion, the nature of graph of velocity as function of displacement will be :

(A)Circular (B)Ellipitical

(C)Sinusoidal (D) Straight line

Official Ans. by NTA (B)

Sol. For a particle in SHM, its speed depends on position as

$v = \omega\sqrt{A^2 - x^2}$

Where ω is angular frequency and A is amplitude

Now $v^2 = \omega^2 A^2 - \omega^2 x^2$

So, $\frac{v^2}{(\omega A)^2} + \frac{x^2}{(A)^2} = 1$

So graph between v and x is elliptical

13. 7 mole of certain monoatomic ideal gas undergoes a temperature increase of 40K at constant pressure. The increase in the internal energy of the gas in this process is

(Given $R = 8.3\ JK^{-1}mol^{-1}$)

(A) 5810 J (B) 3486 J

(C) 11620J (D) 6972 J

Official Ans. by NTA (B)

Sol. For a quasi-static process the change in internal energy of an ideal gas is

$\Delta U = nC_V\,\Delta T$

$= n\times\frac{3R}{2}\times\Delta T$

[molar heat capacity at constant volume for mono atomic gas = $\frac{3R}{2}$]

$\Delta U = 7\times\frac{3}{2}\times 8.3\times 40 = 3486J$

14. A monoatomic gas at pressure P and volume V is suddenly compressed to one eighth of its original volume. The final pressure at constant entropy will be:

(A) P (B) 8P (C) 32P (D) 64 P

Official Ans. by NTA (C)

Sol. Constant entropy means process is adiabatic

PV^γ = constant

$V_2 = \frac{V_1}{8}$

$P_1V_1^\gamma = P_2V_2^\gamma$

$P_1V_1^\gamma = P_2\left(\frac{V_1}{8}\right)^{5/3}$

$P_1V_1^{5/3} = \frac{P_2V_1^{5/3}}{32}$

$\mathbf{P_2} = 32P_1$

15. A water drop of radius 1cm is broken into 729 equal droplets. If surface tension of water is 75 dyne/cm, then the gain in surface energy upto first decimal place will be :

[Given $\pi = 3.14$]

(A) 8.5×10^{-4} J (B) 8.2×10^{-4} J

(C) 7.5×10^{-4} J (D) 5.3×10^{-4} J

Official Ans. by NTA (C)

Sol. Initial surface energy = TA

Where T is surface tension and A is surface area

$$U_i = \left(\frac{75\times10^{-5}}{10^{-2}}\frac{N}{m}\right)\times\left[4\pi\left(1\times10^{-2}\right)^2\right]$$

$$= 75\times10^{-3}\times4\pi\times10^{-4} = 942\times10^{-7}\ J$$

To get final radius of drops by volume conservation

$$\frac{4}{3}\pi R^3 = 729\left(\frac{4}{3}\pi r^3\right)$$

R = Initial radius

r = final radius

$$r = \frac{R}{(729)^{1/3}} = \frac{R}{9} = \frac{1}{9}\text{cm}$$

Final surface energy

$$U_f = 729[TA]$$

$$= 729\left[\frac{75\times10^{-5}}{10^{-2}}\frac{N}{m}\right]\times\left[4\pi\left(\frac{1}{9}\times10^{-2}\right)^2\right]$$

$$= 729\left[75\times10^{-3}\times\frac{4\pi\times10^{-4}}{81}\right]$$

$$= 9\left[942\times10^{-7}\ J\right]$$

Gain in surface energy

$$\Delta U = 9\times942\times10^{-7} - 942\times10^{-7}$$

$$= 8\times942\times10^{-7}\ J = 7536\times10^{-7}\ J$$

$$= 7.5\times10^{-4}\ J$$

16. The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be :

(Radius of earth = 6400km)

(A) 1 % (B) 3%

(C) 4% (D) 0.5%

Official Ans. by NTA (A)

Sol. Acceleration due to gravity at a height h<< R is

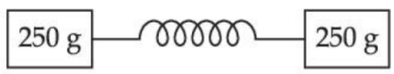
$$g' = g\left(1-\frac{2h}{R}\right)$$

$$\therefore \frac{\Delta g}{g} = \frac{2h}{R}$$

$$\Rightarrow \frac{\Delta g}{g}\times100 = \frac{2h}{R}\times100$$

$$= 2\times\frac{32}{6400}\times100 = 1\%$$

17. As per the given figure, two blocks each of mass 250g are connected to a spring of spring constant 2Nm^{-1}. If both are given velocity v in opposite directions, then maximum elongation of the spring is :

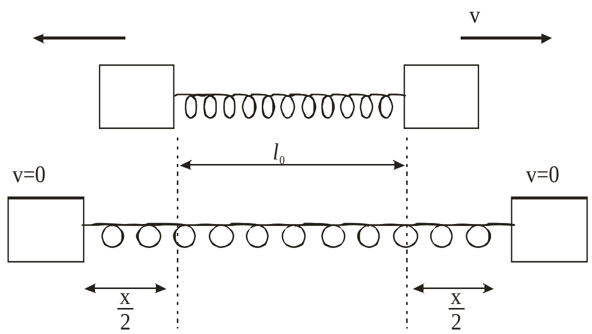


(A) $\frac{v}{2\sqrt{2}}$ (B) $\frac{v}{2}$

(C) $\frac{v}{4}$ (D) $\frac{v}{\sqrt{2}}$

Official Ans. by NTA (B)

Sol.



using energy conservation

$$\frac{1}{2}mv^2\times2 = \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{4}v^2 = \frac{1}{2}\times2\times x^2$$

$$\therefore x = \frac{v}{2}$$

18. A monkey of mass 50kg climbs on a rope which can withstand the tension (T) of 350N. If monkey initially climbs down with an acceleration of $4m/s^2$ and then climbs up with an acceleration of $5m/s^2$. Choose the correct option $\left(g=10m/s^2\right)$

(A) T = 700N while climbing upward

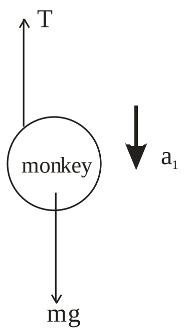
(B) T = 350 N while going downward

(C) Rope will break while climbing upward

(D) Rope will break while going downward

Official Ans. by NTA (C)

Sol. F.B.D of monkey while moving downward

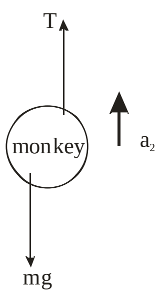


Using Newton's second law

$mg - T = ma_1$

$\therefore$ $500 - T = 50 \times 4 \Rightarrow T = 300$ N

F.B.D of monkey while moving up



Using Newton's second law of motion

$T - mg = ma_2$

$\Rightarrow$ $T - 500 = 50 \times 5$

$\Rightarrow$ $T = 750$ N

Breaking strength of string = 350 N

$\therefore$ String will break while monkey is moving upward

19. Two projectile thrown at 30^0 and 45^0 with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is

(A) $1:\sqrt{2}$ (B) 2 : 1

(C) $\sqrt{2}:1$ (D) $1:2$

Official Ans. by NTA (C)

Sol. **Time taken to reach maximum height**

$$t=\frac{u\sin\theta}{g}$$

$$\therefore \frac{u_1\sin\theta_1}{g}=\frac{u_2\sin\theta_2}{g}$$

$$\Rightarrow u_1\sin 30=u_2\sin 45$$

$$\Rightarrow \frac{u_1}{u_2}=\frac{1/\sqrt{2}}{1/2}=\frac{\sqrt{2}}{1}$$

20. A screw gauge of pitch 0.5mm is used to measure the diameter of uniform wire of length 6.8cm, the main scale reading is 1.5 mm and circular scale reading is 7. The calculated curved surface area of wire to appropriate significant figures is :

[Screw gauge has 50 divisions on the circular scale]

(A) $6.8cm^2$ (B) $3.4cm^2$

(C) $3.9cm^2$ (D) $2.4cm^2$

Official Ans. by NTA (B)

Sol. $L.C.=\frac{P}{N}=\frac{0.5mm}{50}=0.01$ mm

Length of wire = 6.8 cm

Diameter of wire = 1.5 mm + 7 × L.C

= 1.5 mm + 7 × .01 = 1.57 mm

Curved surface area = $\pi D\ell$

$= 3.14 \times 6.8 \times 1.57 \times 10^{-1}\ cm^2$

$= 3.352\ cm^2 = 3.4\ cm^2$

SECTION-B

1. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1-x)$. The y component vector of the initial velocity is _____ $\hat{j}$

(Take $g = 10 m/s^2$)

Official Ans. by NTA (5)

Sol. $u_x = 1$

$y = 5x(1-x)$

$$\frac{dy}{dt} = 5\frac{dx}{dt} - 10x\frac{dx}{dt}$$

For initial y-component of velocity

$$u_y = \left(\frac{dy}{dt}\right)_{x=0} \Rightarrow 5(1) = 5$$

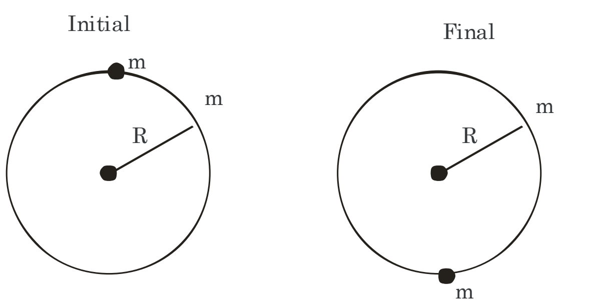
$$\vec{u}_y = 5\hat{j}$$

2. A disc of mass 1 kg and radius R is free of rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc is fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, its angular speed will be $4\sqrt{\frac{x}{3R}}\ rad\,s^{-1}$ where x = _______

$(g = 10ms^{-2})$

Official Ans. by NTA (5)

Sol.



using conservation of mechanical energy

$$mg2R = \frac{1}{2}I_{disc}\ \omega^2 + \frac{1}{2}I_{particle}\omega^2$$

$$mg2R = \frac{\omega^2}{2}\left[\frac{mR^2}{2} + mR^2\right]$$

$$mg2R = \frac{\omega^2}{2}\frac{3}{2}mR^2$$

$$\frac{3}{4}\omega^2 = \frac{2g}{R}$$

$$\omega^2 = \frac{8g}{3R}$$

$$\omega = \sqrt{\frac{80}{3R}}$$

Given $\omega = 4\sqrt{\frac{x}{3R}}$

$$16\frac{x}{3R} = \frac{80}{3R}$$

$x = 5$

3. In an experiment of determine the Young's modulus of wire of a length exactly 1m, the extension in the length of the wire is measured as 0.4mm with an uncertainty of ± 0.02 mm when a load of 1kg is applied. The diameter of the wire is measured as 0.4mm with an uncertainty of ± 0.01 mm. The error in the measurement of Young's modulus (ΔY) is found to be $x \times 10^{10} Nm^{-2}$. The value of x is _____

[Take $g = 10m/s^2$]

Official Ans. by NTA (2)

Sol. $L = 1m$

$\Delta L = 0.4 \times 10^{-3}$ m

$m = 1kg$

$d = 0.4 \times 10^{-3}m$

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$Y = \frac{FL}{A\Delta L} = \frac{(mg).(1)}{\left(\frac{\pi d^2}{4}\right)0.4\times10^{-3}}$$

$$\Rightarrow \frac{10\times4}{\pi\left(0.4\times10^{-3}\right)^2\times0.4\times10^{-3}}$$

$$Y = \frac{40}{\pi\left(0.4\times10^{-3}\right)^3}$$

$$Y = \frac{40\times7}{22\times64\times10^{-3}\times10^{-9}}$$

$Y = 0.199 \times 10^{-12}\ N/m^2$

$$\frac{\Delta Y}{Y} = \frac{\Delta F}{F} + \frac{\Delta L}{L} + \frac{\Delta A}{A} + \frac{\Delta(\Delta L)}{(\Delta L)}$$

$$= \frac{0.02}{0.4} + 2\frac{\Delta d}{d} = \frac{0.2}{4} + 2\times\frac{0.01}{0.4}$$

$$= \frac{0.1}{2} + \frac{0.1}{2} = 0.1$$

$\Rightarrow \Delta Y = 0.1\times Y$

$= 0.199\times10^{11} = 1.99\times10^{10}$

4. When a car is approaching the observer, the frequency of horn is 100Hz. After passing the observer, it is 50Hz. If the observer moves with the car, the frequency will be $\frac{x}{3}$ Hz where x = ___

Official Ans. by NTA (200)

Sol. $f_1 = 100 = f_0\left(\frac{C}{C - V_s}\right)$

C = speed of sound

V_S = speed of source

$$f_2 = 50 = f_0\left(\frac{C}{C + V_s}\right)$$

$$\frac{f_1}{f_2} = 2 = \frac{C + V_s}{C - V_s}$$

$2C - 2V_s = C + V_s$

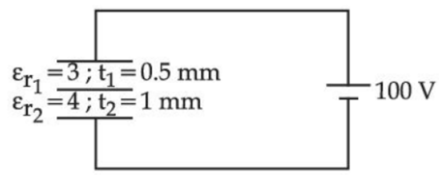
$3V_s = C$

$$V_S = \frac{C}{3}$$

$$100 = f_0\frac{C}{\frac{2C}{3}} = \frac{3}{2}f_0$$

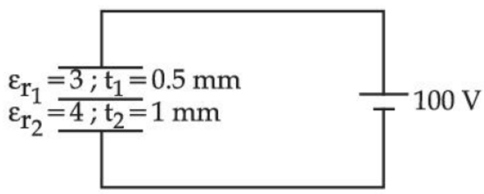
$$f_0 = \frac{200}{3}$$

5. A composite parallel plate capacitor is made up of two different dielectric materials with different thickness $(t_1 \text{ and } t_2)$ as shown in figure. The two different dielectric material are separated by a conducting foil F. The voltage of the conducting foil is ____V.



Official Ans. by NTA (60)

Sol.



Capacitance of each capacitor

$$C_1 = \frac{A3\epsilon_0}{\frac{1}{2}} = 6A\epsilon_0$$

$C_2 = A4\epsilon_0 = 4A\epsilon_0$

Equivalent capacitance

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} \Rightarrow \frac{24}{10}A\epsilon_0$$

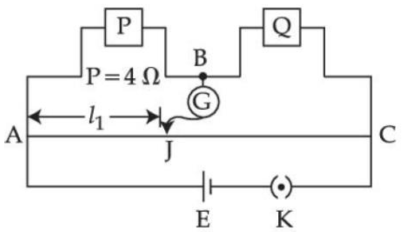
$q_{net} = C_{eq}(\Delta V) \Rightarrow 240\,A\epsilon_0$

$$\Delta V_2 = \frac{240\ A\epsilon_0}{4A\epsilon_0} = 60V$$

$(\Delta V_2 = \text{Potential drop across } C_2)$

$V_{foil} = 60V$

6. Resistance are connected in a meter bridge circuit as shown in the figure. The balancing length l_1 is 40cm. Now an unknown resistance x is connected in series with P and new balancing length is found to be 80cm measured from the same end. Then the value of x will be _____ Ω



Official Ans. by NTA (20)

Sol. Initially, $\frac{P}{Q} = \frac{40cm}{60cm} = \frac{2}{3}$ ...(1)

Finally, $\frac{P+x}{Q} = \frac{80cm}{20cm} = \frac{4}{1}$ ...(2)

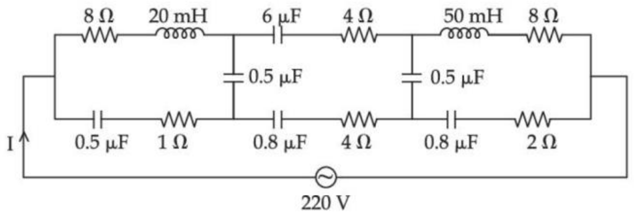
Divide (2) by (1)

$$\frac{P+x}{P} = 4 \times \frac{3}{2} = 6$$

$$\Rightarrow \quad 1 + \frac{x}{P} = 6 \Rightarrow \frac{x}{P} = 5$$

$\therefore$ $x = 5P = 5 \times 4 = 20\Omega$

7. The effective current I in the given circuit at very high frequencies will be ____A



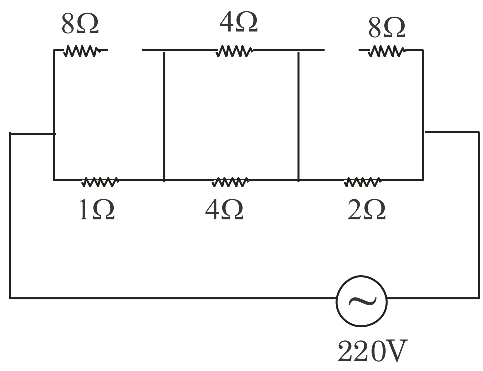
Official Ans. by NTA (44)

Sol. At very high frequencies,

$$X_C = \frac{1}{\omega C} \approx 0$$

Also $X_L = \omega L \approx \infty$

Thus, equivalent circuit can be redrawn as

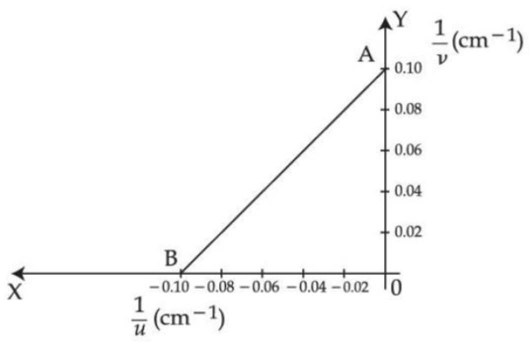


$Z = 1 + 2 + 2 = 5\Omega$

$$I = \frac{220V}{5\Omega} = 44A$$

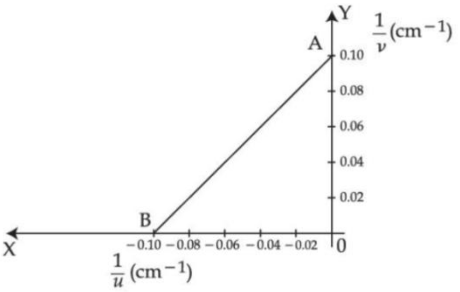
8. The graph between $\frac{1}{u}$ and $\frac{1}{v}$ for a thin convex lens in order to determine its focal length is plotted as shown in the figure. The refractive index of length is 1.5 and its both the surfaces have same radius of curvatures R. The value of R will be ____cm.

(Where u = object distance , v = image distance)



Official Ans. by NTA (10)

Sol.



For point B, $\frac{1}{u} = -0.10\,\text{cm}^{-1}, \frac{1}{v} = 0$

$\therefore$ Thus, u = –10cm, v = ∞

i.e. f = 10 cm

$$\Rightarrow \quad \frac{1}{10\text{cm}} = (1.5-1)\left(\frac{2}{R}\right) = \frac{1}{R} \Rightarrow R = 10\text{cm}$$

9. In a hydrogen spectrum ,λ be the wavelength of first transition line of Lyman series. The wavelength difference will be "aλ" between the wavelength of 3rd transition line of Paschen series and that of 2nd transition line of Balmer Series where a = ________

Official Ans. by NTA (5)

Sol. For first line of Lyman

$$\frac{1}{\lambda} = R\left(1-\frac{1}{4}\right) = R\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \lambda = \frac{4}{3R} \quad \ldots(1)$$

3rd line(Paschen)

$$\frac{1}{\lambda_3} = R\left(\frac{1}{3^2} - \frac{1}{6^2}\right) = \frac{R}{9} \times \frac{3}{4}$$

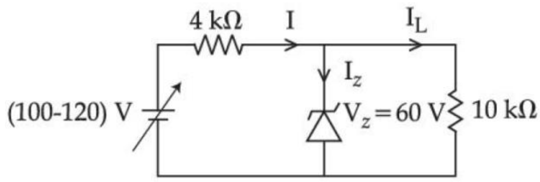
2nd line(Balmer)

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{R}{4} \times \frac{3}{4}$$

Thus $a\lambda = \lambda_3 - \lambda_2 = \frac{12}{R} - \frac{16}{3R} = \frac{20}{3R}$ putting (1)

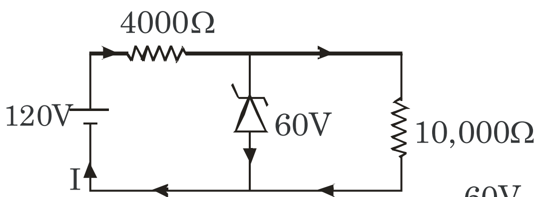
$$a\left(\frac{4}{3R}\right) = \frac{20}{3R} \Rightarrow a = 5$$

10. In the circuit shown below, maximum zener diode current will be ____mA



Official Ans. by NTA (9)

Sol. Consider input 120V



$$I_L = \frac{60V}{10{,}000\Omega} = 0.006A$$

$$I = \frac{(120-60)\,V}{4000\Omega} = 0.015A$$

Thus $\quad I_2 = I - I_L$

= 0.015 – 0.006 = 0.009 A = 9 mA

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. Match **List - I** with **List - II**.

List - I (Compound)	**List – II (Shape)**
(A) BrF_5	(I) bent
(B) $[CrF_6]^{3-}$	(II) square pyramidal
(C) O_3	(III) trigonal bipyramidal
(D) PCl_5	(IV) octahedral

Choose the **correct** answer from the options given below :

(A) (A) – (I), (B) - (II), (C) - (III), (D) - (IV)

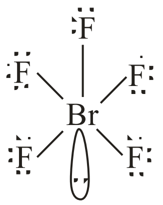
(B) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

(C) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(D) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

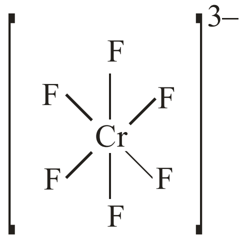
Official Ans. by NTA (C)

Sol.



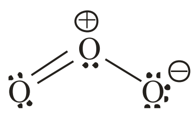
(Square pyramidal)

$[CrF_6]^{3-}$:



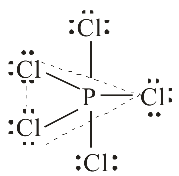
(Octahedral)

O_3 **:**



(Bent)

PCl_5 **:**



(Trigonal bipyramidal)

2. Match **List - I** with **List - II.**

List –I (Processes/Reactions)	**List – II (Catalyst)**
(A) $2SO_2(g)+O_2(g)\rightarrow 2SO_3(g)$	(I) Fe(s)
(B) $4NH_3(g)+5O_2(g)\rightarrow 4NO(g)+6H_2O(g)$	(II) Pt(s)-Rh(s)
(C) $N_2(g)+3H_2(g)\rightarrow 2NH_3(g)$	(III) V_2O_5
(D) Vegetable oil(l)+$H_2\rightarrow$Vegetable ghee(s)	(IV) Ni(s)

Choose the correct answer from the options given below :

(A) (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

(B) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)

(C) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(D) (A) - (IV), (B) - (II), (C) - (III), (D) - (I)

Official Ans. by NTA (B)

Sol. $2SO_2(g)+O_2(g)\xrightarrow{V_2O_5}2SO_3(g)$:

contact process

$4NH_3(g)+5O_2(g)\xrightarrow{Pt(s)-Rh(s)}4NO(g)+6H_2O(g)$:

Ostwald's process

$N_2(g)+3H_2(g)\xrightarrow{Fe(s)}2NH_3(g)$; Haber's process

Vegetable oil $(l)+H_2(g)\xrightarrow{Ni(s)}$ vegetable ghee

: Hydrogenation

3. Given two statements below :

Statement I : In Cl_2 molecule the covalent radius is double of the atomic radius of chlorine.

Statement II : Radius of anionic species is always greater than their parent atomic radius.

Choose the **most appropriate** answer from options given below :

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (D)

Sol. In Cl_2 molecule, the covalent radius is half of the internuclear distance, so statement(I) is false.

For the same element, anion has lower effective nuclear charge than atom $\Rightarrow$ so anion is larger than atom. $\Rightarrow$ statement (II) is correct.

4. Refining using liquation method is the most suitable for metals with :

(A) Low melting point

(B) High boiling point

(C) High electrical conductivity

(D) Less tendency to be soluble in melts than impurities

Official Ans. by NTA (A)

Sol. Liquation is used to purify metals having lower melting point than impurities present in them.

5. Which of the following can be used to prevent the decomposition of H_2O_2?

(A) Urea

(B) Formaldehyde

(C) Formic acid

(D) Ethanol

Official Ans. by NTA (A)

Sol. Urea acts as stabiliser for H_2O_2.

6. Reaction of $BeCl_2$ with $LiAlH_4$ gives :

(A) $AlCl_3$

(B) BeH_2

(C) LiH

(D) LiCl

(E) $BeAlH_4$

Choose the **correct** answer from options given below :

(A) (A), (D) and (E)

(B) (A) , (B) and (D)

(C) (D) and (E)

(D) (B) , (C) and (D)

Official Ans. by NTA (B)

Sol. $2BeCl_2 + LiAlH_4 \rightarrow 2BeH_2 + LiCl + AlCl_3$

7. Borazine, also known as inorganic benzene, can be prepared by the reaction of 3-equivalents of "X" with 6-equivalents of "Y". "X" and "Y", respectively are :

(A) $B(OH)_3$ and NH_3 (B) B_2H_6 and NH_3

(C) B_2H_6 and HN_3 (D) NH_3 and B_2O_3

Official Ans. by NTA (B)

Sol. $3B_2H_6 + 6NH_3 \xrightarrow{\Delta} 2B_3N_3H_6 + 12\,H_2$

8. Which of the given reactions is not an example of disproportionation reaction ?

(A) $2H_2O_2 \rightarrow 2H_2O + O_2$

(B) $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$

(C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$

(D) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

Official Ans. by NTA (C)

Sol. $2H_2\overset{-1}{O}_2 \longrightarrow 2H_2\overset{2-}{O} + \overset{0}{O}_2$: Disproportionation

$2\overset{+4}{N}O_2 + H_2O \rightarrow H\overset{+5}{N}O_3 + H\overset{+3}{N}O_2$: Disproportionation

$MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$: reduction

$3\overset{+6}{Mn}O_4^{2-} + 4H^+ \rightarrow 2\overset{+7}{Mn}O_4^- + \overset{+4}{Mn}O_2 + 2H_2O$: Disproportionation

9. The dark purple colour of $KMnO_4$ disappears in the titration with oxalic acid in acidic medium. The overall change in the oxidation number of manganese in the reaction is :

(A) 5 (B) 1

(C) 7 (D) 2

Official Ans. by NTA (A)

Sol. In acidic medium,

$\overset{+7}{Mn}O_4^- \rightarrow Mn^{+2}$

change in ox. no. = 5

10. $\dot{C}l + CH_4 \rightarrow A + B$

A and B in the above atmospheric reaction step are

(A) C_2H_6 and Cl_2 (B) $\dot{C}HCl_2$ and H_2

(C) $\dot{C}H_3$ and HCl (D) C_2H_6 and HCl

Official Ans. by NTA (C)

Sol. $\dot{C}l + CH_4 \longrightarrow \dot{C}H_3 + HCl$

11. Which technique among the following, is most appropriate in separation of a mixture of 100 mg of p-nitrophenol and picric acid ?

(A) Steam distillation

(B) 2-5 ft long column of silica gel

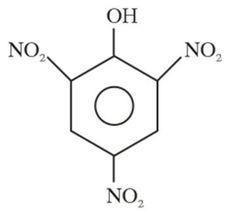
(C) Sublimation

(D) Preparative TLC (Thin Layer Chromatography)

Official Ans. by NTA (D)

Sol.

Shows inter molecular H-bonding



Shows intra molecular H-bonding

Solvent polarity has been related to R_f value of nitrocompounds.

100 mg p-nitrophenol and picric acid have different R_f value on silica gel plate

$\therefore$ Preparative TLC is best to separate 100 mg of para nitrophenol and picric acid

12. The difference in the reaction of phenol with bromine in chloroform and bromine in water medium is due to :

(A) Hyperconjugation in substrate

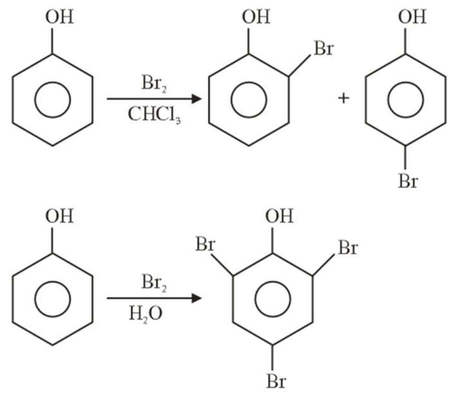
(B) Polarity of solvent

(C) Free radical formation

(D) Electromeric effect of the substrate

Official Ans. by NTA (B)

Sol.

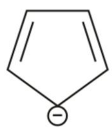


Difference in reactions is observed due to solvent polarity, which

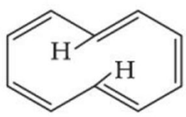
(i) Ionizes phenol to make more reactive phenoxide ion

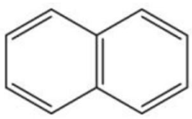
(ii) Increases electrophilicity of bromine.

13. Which of the following compounds is **not** aromatic?

(A) 

(B) 

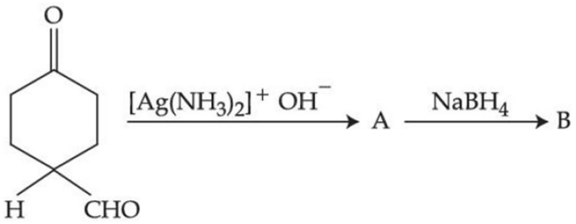
(C) 

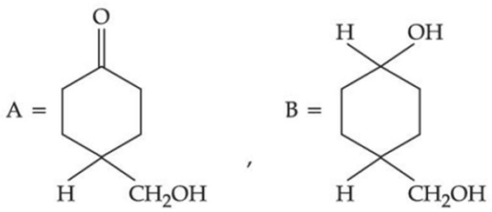
(D) 

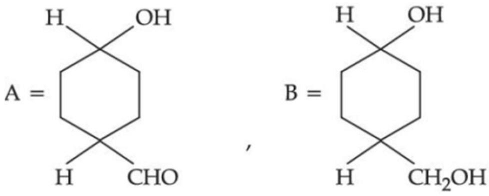
Official Ans. by NTA (C)

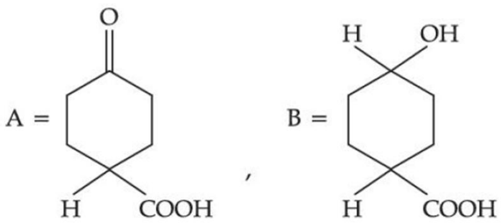
Sol. [10] Annulene, although follow $(4n + 2)\pi$ electron rule, but it is non-aromatic due to its non planar nature. It is nonplanar due to repulsion of C – H bonds present inside the ring.

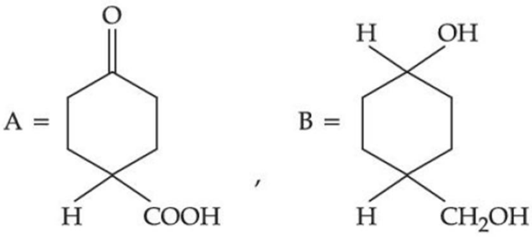
14. The products formed in the following reaction, A and B are



(A) 

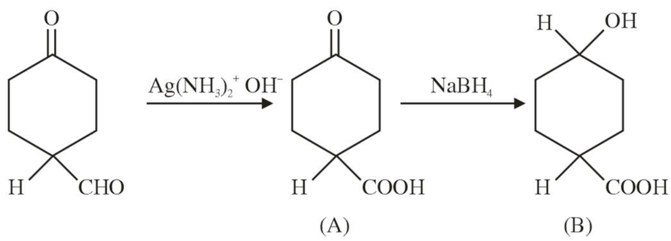
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

Sol.



$NaBH_4$ does not reduce carboxylic acid.

15. Which reactant will give the following alcohol on reaction with one mole of phenyl magnesium bromide (PhMgBr) followed by acidic hydrolysis ?

$$Ph-\underset{\displaystyle CH_3}{\overset{\displaystyle Ph}{\overset{|}{\underset{|}{C}}}}-OH$$

(A) $CH_3-C\equiv N$

(B) $Ph-C\equiv N$

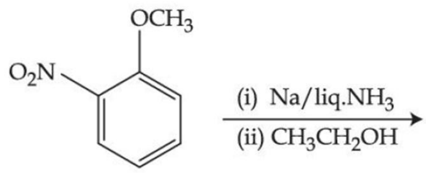
(C) $CH_3-\overset{\displaystyle O}{\overset{\|}{C}}-O-Ph$

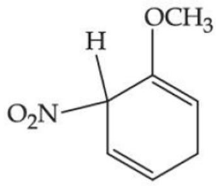
(D) $Ph-\underset{\displaystyle O}{\underset{\|}{C}}-CH_3$

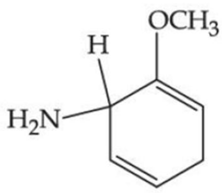
Official Ans. by NTA (D)

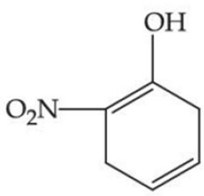
Sol. $Ph-\underset{\displaystyle O}{\underset{\|}{C}}-CH_3 \xrightarrow[\text{(ii) } H^+]{\text{(i) PhMgBr}} Ph-\underset{\displaystyle Ph}{\underset{|}{\overset{\displaystyle CH_3}{\overset{|}{C}}}}-OH$

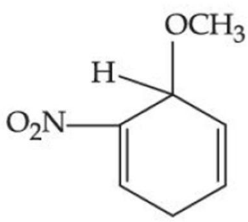
16. The major product of the following reaction is



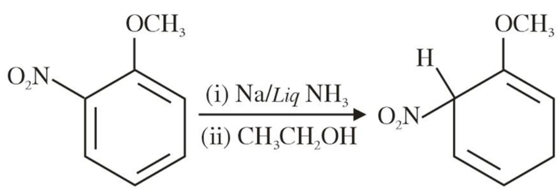
(A) 

(B) 

(C) 

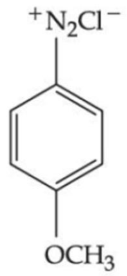
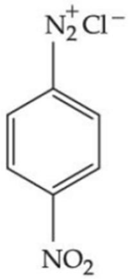
(D) 

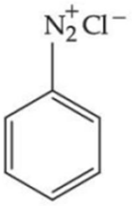
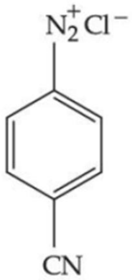
Official Ans. by NTA (A)

Sol. 

Given reaction is an example of birch reduction.

17. The correct stability order of the following diazonium salt is

(A)  (B) 

(C)  (D) 

(A) (A) > (B) > (C) > (D)

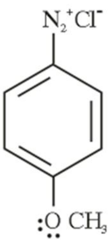
(B) (A) > (C) > (D) > (B)

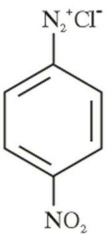
(C) (C) > (A) > (D) > (B)

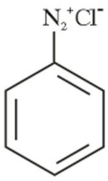
(D) (C) > (D) > (B) > (A)

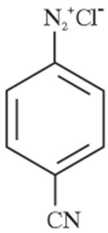
Official Ans. by NTA (B)

Sol.

(A) 
(+ M group)

(B) 
(– M & – I group)

(C) 

(D) 
(– M & – I group)

Since diazonium ion is a cation hence it is stabilized by electron donating groups and destabilized by electron withdrawing group.

Hence Stability order should be A > C > D > B.

18. Stearic acid and polyethylene glycol react to form which one of the following soap/s detergents ?

(A) Cationic detergent (B) Soap

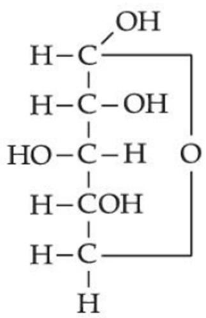
(C) Anionic detergent (D) Non-ionic detergent

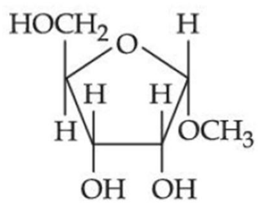
Official Ans. by NTA (D)

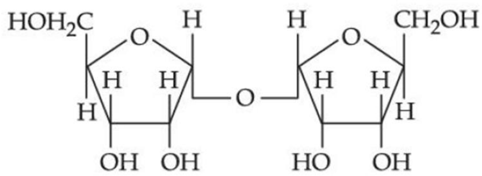
Sol.

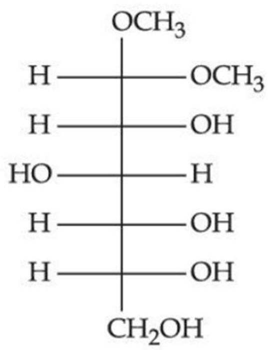
$$\underset{\text{Stearic acid}}{CH_3(CH_2)_{16}COOH} + \underset{\text{Polyethylene glycol}}{OH(CH_2CH_2O)_n CH_2CH_2OH} \xrightarrow{-H_2O} \underset{\text{Non-ionic detergent}}{CH_3(CH_2)_{16}-\overset{\overset{\displaystyle O}{\|}}{C}-O-(CH_2CH_2O)_n CH_2CH_2OH}$$

19. Which of the following is reducing sugar?

(A) 

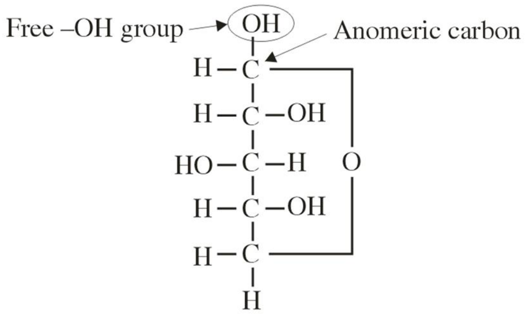
(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. If any sugar is having free –OH group at anomeric carbon then it will be a reducing sugar



20. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Experimental reaction of CH_3Cl with aniline and anhydrous $AlCl_3$ does **not** give o and p-methylaniline.

Reason (R) : The — NH_2 group of aniline becomes deactivating because of salt formation with anhydrous $AlCl_3$ and hence yields *m*-methyl aniline as the product.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

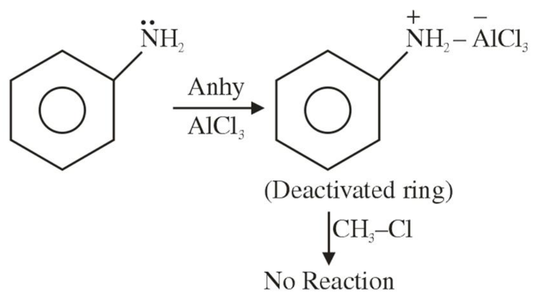
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

Official Ans. by NTA (C)

Sol.



Friedel Craft Alkylation does not occur on this deactivated ring.

SECTION-B

1. Chlorophyll extracted from the crushed green leaves was dissolved in water to make 2 L solution of Mg of concentration 48 ppm. The number of atoms of Mg in this solution is $x \times 10^{20}$ atoms. The value of x is________. (Nearest Integer)

(Given : Atomic mass of Mg is 24 g mol^{-1}, $N_A = 6.02 \times 10^{23}\ mol^{-1}$)

Official Ans. by NTA (24)

Sol. $\text{ppm} = \frac{W_{Mg}}{V_{soln}} \times 10^6 = 48$

$\Rightarrow W_{Mg} = \frac{48 \times 2 \times 1000}{10^6}$

$= 48 \times 2 \times 10^{-3}$ g

$n_{Mg} = \frac{W_{Mg}}{24} = \frac{48 \times 2 \times 10^{-3}}{24}$

$= 4 \times 10^{-3}$

Number of Mg atoms $= 4 \times 10^{-3} \times 6.02 \times 10^{23}$

$= 4 \times 6.02 \times 10^{20}$

$= 24.08 \times 10^{20}$

$\therefore\ x = 24.08$

2. A mixture of hydrogen and oxygen contains 40% hydrogen by mass when the pressure is 2.2 bar. The partial pressure of hydrogen is bar. (Nearest Integer)

Official Ans. by NTA (2)

Sol. Let $W_{H_2} = 40\ g \Rightarrow n_{H_2} = \frac{40}{2} = 20$

$W_{O_2} = 60g \Rightarrow n_{O_2} = \frac{60}{32} = \frac{15}{8}$

$P_{H_2} = \left(\frac{20}{20 + \frac{15}{8}}\right) \times 2.2$

$= \frac{20}{20 + 1.875} \times 2.2$

$= \frac{20}{21.875} \times 2.2$

$= 2.0114$

$\simeq 2.01$ bar

3. The wavelength of an electron and a neutron will become equal when the velocity of the electron is x times the velocity of neutron. The value of x is__________. (Nearest Integer)

(Mass of electron is 9.1×10^{-31} kg and mass of neutron is 1.6×10^{-27} kg)

Official Ans. by NTA (1758)

Sol. $\nu_e = x\,\nu_N$

$\lambda_e = \lambda_N$

$$\Rightarrow \frac{h}{m_e \nu_e} = \frac{h}{m_N \nu_N}$$

$$\nu_e = \frac{m_N}{m_e}.\nu_N$$

$$= \frac{1.6 \times 10^{-27}}{9.1 \times 10^{-31}} \nu_N$$

$\nu_e = 1758.24 \times \nu_N$

$\therefore \quad x = 1758.24$

4. 2.4 g coal is burnt in a bomb calorimeter in excess of oxygen at 298 K and 1 atm pressure.

The temperature of the calorimeter rises from 298 K to 300 K. The enthalpy change during the combustion of coal is – x kJ mol^{-1}. The value of x is___________. (Nearest Integer)

(Given : Heat capacity of bomb calorimeter 20.0 kJ K^{-1} . Assume coal to be pure carbon)

Official Ans. by NTA (200)

Sol. $C\,(s) + O_2\,(g) \rightarrow CO_2\,(g)$; ΔH = -x kJ/mole

$Q = C\Delta T = 20 \text{ kJ} \times 2$

40 kJ heat is released for 2.4 g of C

For 1 mole 'C' : $Q = \frac{40}{2.4} \times 12$

$$= \frac{400}{24} \times 12 = 200 \text{ kJ/mole}$$

$Q = \Delta E = \Delta H = 200$ kJ ($\because$ $\Delta n_g = 0$)

x = 200

5. When 800 mL of 0.5 M nitric acid is heated in a beaker, its volume is reduced to half and 11.5 g of nitric acid is evaporated. The molarity of the remaining nitric acid solution is $x \times 10^{-2}$ M. (Nearest Integer)

(Molar mass of nitric acid is 63 g mol^{-1})

Official Ans. by NTA (54)

Sol. $n_{HNO_3} = 0.5 \times 0.8$

= 0.4 mole

$$(n_{HNO_3})_{remains} = 0.4 - \frac{11.5}{63}$$

$= 0.4 - 0.1825$

$= 0.2175$

$$\text{Molarity} = \frac{0.2175}{400} \times 1000$$

$$= \frac{0.2175}{0.4}$$

= 0.5437 mole/lit.

$\simeq 0.54$ mole/lit.

= 54×10^{-2} mol/lit.

6. At 298 K, the equilibrium constant is 2×10^{15} for the reaction :

$$Cu(s) + 2Ag^{+}(aq) \rightleftharpoons Cu^{2+}(aq) + 2Ag(s)$$

The equilibrium constant for the reaction

$$\frac{1}{2}Cu^{2+}(aq) + Ag(s) \rightleftharpoons \frac{1}{2}Cu(s) + Ag^{+}(aq)$$

is $x \times 10^{-8}$. The value of x is_______.

(Nearest Integer)

Official Ans. by NTA (2)

Sol. $$K'_{eq} = \frac{1}{\sqrt{K_{eq}}} = \frac{1}{\sqrt{2 \times 10^{15}}} = x \times 10^{-8}$$

$$\Rightarrow \frac{1}{\sqrt{20}} \times \frac{1}{10^7} = x \times 10^{-8}$$

$$\Rightarrow \frac{1}{\sqrt{20}} \times 10^{-7} = x \times 10^{-8}$$

$$\frac{10}{\sqrt{20}} = x$$

$$\Rightarrow x = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5} = 2.236$$

$\simeq 2.24$

7. The amount of charge in F (Faraday) required to obtain one mole of iron from Fe_3O_4 is _____. (Nearest Integer)

Official Ans. by NTA (8)

Sol. $Fe_3O_4 \xrightarrow{+8e^-} 3Fe$

Charge for 1 mole Fe = 8/3 F

= 2.67 F

8. For a reaction $A \rightarrow 2B + C$ the half lives are 100 s and 50 s when the concentration of reactant A is 0.5 and 1.0 mol L^{-1} respectively. The order of the reaction is___ _____. (Nearest Integer)

Official Ans. by NTA (2)

Sol. $t_{\frac{1}{2}} \propto \dfrac{1}{[A_0]^{n-1}}$

$[100] \propto \dfrac{1}{(0.5)^{n-1}}$

$(50) \propto \dfrac{1}{(1)^{n-1}}$

$[2]^1 = \left[\dfrac{1}{0.5}\right]^{n-1}$

$[2]^1 = [2]^{n-1}$

$n - 1 = 1$

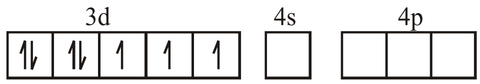
$n = 2$

order = 2

9. The difference between spin only magnetic moment values of $[Co(H_2O)_6]Cl_2$ and $[Cr(H_2O)_6]Cl_3$ is___________.

Official Ans. by NTA (0)

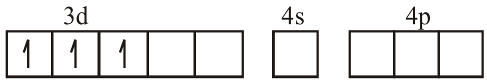
Sol. $[Co(H_2O)_6]^{2+}$

Co^{+2} : 

number of unpaired e^- = 3

$\mu = \sqrt{15}BM$

$[Cr(H_2O)_6]^{3+}$

Cr^{+3} : 

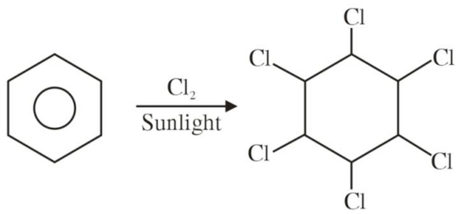
number of unpaired e^- = 3

$\mu = \sqrt{15}BM$

Difference in spin only magnetic moment = 0

10. In the presence of sunlight, benzene reacts with Cl_2 to give product, X. The number of hydrogens in X is_________.

Official Ans. by NTA (6)

Sol. 

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let f: R → R be a continuous function such that f (3x) – f (x) = x. If f (8) = 7, then f (14) is equal to :

(A) 4 (B) 10

(C) 11 (D) 16

Official Ans. by NTA (B)

Sol. f(x) – f(x/3) = x/3

$f(x/3) - f(x/3^2) = x/3^2$

.... on adding

$$f(x)-\lim_{n\to\infty} f\left(\frac{x}{3^n}\right)=x\left(\frac{1}{3}+\frac{1}{3^2}....\infty\right)$$

$$f(x)-f(0)=\frac{x}{2}$$

f(8) = 7 ; f(0) = 3

f(x) = x/2 + 3

f(14) = 10

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true ?

(A) $\arg z_2 = \pi - \tan^{-1} 3$

(B) $\arg (z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$

(C) $|z_2| = \sqrt{10}$

(D) $|2z_1 - z_2| = 5$

Official Ans. by NTA (D)

Sol. $AB = AO.\ z^{-i\pi/2} = -2 + i$

So OB = (–2 + i) + (1 + 2i)

$z_2 = -1 + 3i$

$\therefore |2z_1 - z_2| = \sqrt{10}$

3. If the system of linear equations.

8x + y + 4z = –2

x + y + z = 0

λx – 3y = μ

has infinitely many solutions, then the distance of the point $\left(\lambda,\mu,-\frac{1}{2}\right)$ from the plane 8x + y + 4z + 2 = 0 is :

(A) $3\sqrt{5}$ (B) 4

(C) $\frac{26}{9}$ (D) $\frac{10}{3}$

Official Ans. by NTA (D)

Sol. $$D=\begin{vmatrix}8 & 1 & 4\\ 1 & 1 & 1\\ \lambda & -3 & 0\end{vmatrix}=0\Rightarrow\lambda=4$$

Also $D_1 = D_2 = D_3 = 0$

So μ = –2

Point $\left(4,-2,-\frac{1}{2}\right)$

Distance from plane = $\frac{10}{3}$

4. Let A be a 2 × 2 matrix with det (A) = –1 and det ((A + I) (Adj (A) + I)) = 4. Then the sum of the diagonal elements of A can be :

(A) –1 (B) 2

(C) 1 (D) $-\sqrt{2}$

Official Ans. by NTA (B)

Sol. Let $A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$; ad – bc = –1

$|A+I||\text{adj } A+I| = 4$

⇒ ad – bc + a + d + 1 = 2 or –2

a + d = 2 or –2

5. The odd natural number a, such that the area of the region bounded by y= 1, y = 3, x = 0, x = y^a is $\frac{364}{3}$, equal to :

(A) 3 (B) 5

(C) 7 (D) 9

Official Ans. by NTA (B)

Sol. $A=\int_1^3 y^a.dy=\left.\frac{y^{a+1}}{a+1}\right|_1^3=\frac{364}{3}$

$\Rightarrow a=5$

6. Consider two G.Ps. 2, 2^2, 2^3, ... and 4, 4^2, 4^3,.... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :

(A) 560 (B) 1540

(C) 1330 (D) 2600

Official Ans. by NTA (C)

Sol. $\left(\left(2^1 2^2.....2^{60}\right)\left(4^1.4^2......4^n\right)\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$\left(2^{30\times61}4^{\frac{n(n+1)}{2}}\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$2^{1830+n^2+n}=2^{\frac{(225)(60+n)}{8}}$

$=8n^2-217n+1140=0$

$n=20,\ \frac{57}{8}$

$\sum_{k=1}^{n} nk-k^2=\frac{n^2(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}$

= 1330

7. If the function

$$f(x)=\begin{cases}\dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)-\{0\}\\ k & ,x=0\end{cases}$$

is continuous at x = 0, then k is equal to :

(A) 1 (B) –1

(C) e (D) 0

Official Ans. by NTA (A)

Sol. $\lim_{x\to0}\frac{\left(\ln\left(1+x^2+x^4\right)\right)\cos x}{1-\cos^2 x}$

$$\lim_{x\to0}\frac{\left(\frac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\right)x^2\left(1+x^2\right)\cos x}{\left(\frac{\sin^2 x}{x^2}\right)x^2}=1$$

$\therefore k=1$

8. If f(x) = $\begin{cases}x+a & , x\le0\\ |x-4|, & x>0\end{cases}$ and

g(x) = $\begin{cases}x+1 & , x<0\\ (x-4)^2+b, & x\ge0\end{cases}$

are continuous on R, then (gof) (2) + (fog) (–2) is equal to :

(A) –10 (B) 10

(C) 8 (D) –8

Official Ans. by NTA (D)

Sol. $f(x)=\begin{cases}x+a & ; x\le0\\ |x-4|; & x>0\end{cases}$; $g(x)=\begin{cases}x+1 & ; x<0\\ (x-4)^2+b; & x\ge0\end{cases}$

For continuity a = 4 and b = –15

$g\left(f(2)\right)+f\left(g(-2)\right)$

$=g(2)+f(-1)=-8$

9. Let f(x) = $\begin{cases} x^3 - x^2 + 10x - 7, & x \le 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$

Then the set of all values of b, for which f(x) has maximum value at x = 1, is :

(A) (–6, –2)

(B) (2, 6)

(C) [–6, –2) $\cup$ (2, 6]

(D) [$-\sqrt{6}$, –2) $\cup$ (2, $\sqrt{6}$]

Official Ans. by NTA (C)

Sol. $f(1) = 3$

For $x < 1$, $f'(x) = 3x^2 - 2x + 10 > 0$

$\Rightarrow f(x)$ is increasing

For $x > 1$, $f'(x) < 0$

$\Rightarrow$ function is decreasing.

$\lim_{x \to 1^+} f(x) = -2 + \log_2(b^2 - 4)$

For maximum value at x = 1

$3 \ge -2 + \log_2(b^2 - 4)$

$32 \ge b^2 - 4 > 0$

$b \in [-6, -2) \cup (2, 6]$

10. If a = $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{2n}{n^2 + k^2}$ and f(x) = $\sqrt{\frac{1 - \cos x}{1 + \cos x}}$, $x \in (0, 1)$, then :

(A) $2\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(B) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right) = \sqrt{2}$

(C) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(D) $f\left(\frac{a}{2}\right) = \sqrt{2} f'\left(\frac{a}{2}\right)$

Official Ans. by NTA (C)

Sol. $a = \frac{1}{n} \sum_{k=1}^{n} \frac{2}{1 + \left(\frac{k}{n}\right)^2} = \int_0^1 \frac{2}{1 + x^2} dx = \frac{\pi}{2}$

$f(x) = \tan\left(\frac{x}{2}\right); \ x \in (0, 1)$

$f\left(\frac{\pi}{4}\right) = \sqrt{2} - 1$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{2} \sec^2\left(\frac{\pi}{8}\right) = \frac{\sqrt{2}}{\sqrt{2} + 1}$

$f'\left(\frac{\pi}{4}\right) = \sqrt{2} f\left(\frac{\pi}{4}\right)$

11. If $\frac{dy}{dx}$ + 2y tan x = sin x, 0 < x < $\frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right)$ = 0, then the maximum value of y(x) is

(A) $\frac{1}{8}$

(B) $\frac{3}{4}$

(C) $\frac{1}{4}$

(D) $\frac{3}{8}$

Official Ans. by NTA (A)

Sol. $\frac{dy}{dx} + 2y \tan x = \sin x$

I.F = $e^{\int 2\tan x \, dx} = e^{\ln(\sec x)^2} = \sec^2 x$

$y(\sec^2 x) = \int \sin x \sec^2 x \, dx + C$

$y.\sec^2 x = \sec x + C$

Put $x = \frac{\pi}{3}, \ y = 0$

$y = \cos x - 2\cos^2 x$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$\therefore y_{max} = \frac{1}{8}$

12. A point P moves so that the sum of squares of its distances from the points (1, 2) and (–2, 1) is 14. Let f(x, y) = 0 be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the point C, D . Then the area of the quadrilateral ACBD is equal to

(A) $\frac{9}{2}$ (B) $\frac{3\sqrt{17}}{2}$

(C) $\frac{3\sqrt{17}}{4}$ (D) 9

Official Ans. by NTA (B)

Sol. $(x-1)^2+(y-2)^2+(x+2)^2+(y-1)^2=14$

$\Rightarrow x^2+y^2+x-3y-2=0$

Put x = 0

$\Rightarrow y^2-3y-2=0$

$\Rightarrow y=\frac{3\pm\sqrt{17}}{2}$

Put y = 0

$\Rightarrow x^2+x-2=0$

$(x+2)(x-1)=0$

$\therefore A(-2,0), B(1,0), C\left(0,\frac{3+\sqrt{17}}{2}\right), D\left(0,\frac{3-\sqrt{17}}{2}\right)$

Area = $\frac{1}{2}.3.\sqrt{17}=\frac{3\sqrt{17}}{2}$

13. Let the tangent drawn to the parabola $y^2=24x$ at the point (α,β) is perpendicular to the line 2x +2y = 5. Then the normal to the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ at the point $(\alpha+4,\beta+4)$ does NOT pass through the point :

(A) (25, 10) (B) (20, 12)
(C) (30, 8) (D) (15, 13)

Official Ans. by NTA (D)

Sol. Tangent at (α,β) has slope 1

$\beta^2=24\alpha$

Equation of tangent $y\beta=12(x+\alpha)$, $\frac{12}{\beta}=1$

$\Rightarrow \alpha=6, \beta=12$

$\therefore(\alpha+4,\beta+4)=(10,16)$

Normal at (10, 16) to $\frac{x^2}{36}-\frac{y^2}{144}=1$ is

2x + 5y = 100

14. The length of the perpendicular from the point (1, –2, 5) on the line passing through (1, 2, 4) and parallel to the line x + y – z = 0 = x – 2y + 3z – 5 is :

(A) $\sqrt{\frac{21}{2}}$ (B) $\sqrt{\frac{9}{2}}$

(C) $\sqrt{\frac{73}{2}}$ (D) 1

Official Ans. by NTA (A)

Sol. d.r's of the line = $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -1\\ 1 & -2 & 3\end{vmatrix}=\hat{i}-4\hat{j}-3\hat{k}$

$\therefore$ equation of line is

$\vec{r}=\hat{i}+2\hat{j}+4\hat{k}+\lambda(\hat{i}-4\hat{j}-3\hat{k})$

Let A(1, 2, 4) and P be $(1+\lambda, 2-4\lambda, 4-3\lambda)$

$\therefore \overline{PA}.(\hat{i}-4\hat{j}-3\hat{k})=0$

$\lambda=\frac{1}{2}$

$\Rightarrow P\left(\frac{1}{2},2,\frac{-5}{2}\right)$

$|AP|=\sqrt{\frac{21}{2}}$

15. Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}, \alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to

(A) $\frac{15}{2}$ (B) 8

(C) $\frac{13}{2}$ (D) 7

Official Ans. by NTA (D)

Sol. $\vec{a} \times \vec{b} = (1-\alpha)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}$

Projection of $\vec{a} \times \vec{b}$ on $-\hat{i} + 2\hat{j} - 2\hat{k}$

$$= \frac{(\vec{a} \times \vec{b}).(-\hat{i} + 2\hat{j} - 2\hat{k})}{3} = 30$$

$\Rightarrow 2\alpha^2 - \alpha - 91 = 0$

$\Rightarrow \alpha = 7, -\frac{13}{2}$

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X=1) = \frac{4}{243}$, then $P(X = 4 \text{ or } 5)$ is equal to :

(A) $\frac{5}{9}$ (B) $\frac{64}{81}$

(C) $\frac{16}{27}$ (D) $\frac{145}{243}$

Official Ans. by NTA (C)

Sol. $np = \alpha$ .....(1)

$npq = \alpha/3$ .....(2)

From (1) & (2)

$q = 1/3$ & $p = 2/3$

${}^{n}C_1\, q^{n-1}p^1 = \frac{4}{243}$

$\frac{n}{3^n} = \frac{2}{243}$

$n = 6$

$P(4 \text{ or } 5) = {}^{6}C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^{6}C_5\left(\frac{2}{3}\right)^5.\left(\frac{1}{3}\right)^0$

$= \frac{16}{27}$

17. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to :

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$

(C) $\frac{5}{4}$ (D) 1

Official Ans. by NTA (D)

Sol. $0 \le P(E_i) \le 1$ for i = 1, 2, 3

$\Rightarrow -2/3 \le p \le 1$

E_1 & E_2 & E_3 are mutually exclusive

$P(E_1) + P(E_2) + P(E_3) \le 1$

$\Rightarrow 2/3 \le p \le 1$

$p_1 = 1$, $p_2 = 2/3$

$p_1 + p_2 = 5/3$

18. Let

$S = \{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\}$. Then

$n(S) + \sum_{\theta \in S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right)\right)$ is equal to :

(A) 0 (B) –2

(C) –4 (D) 12

Official Ans. by NTA (C)

Sol. $8^{2\sin^2\theta} + 8^{2-2\sin^2\theta} = 16$

$y + \frac{64}{y} = 16$

$\Rightarrow y = 8$

$\Rightarrow \sin^2\theta = ½$

$$n(S) + \sum_{\theta \in S}\frac{1}{\cos(\pi/4 + 2\theta)\sin(\pi/4 + 2\theta)}$$

$= 4 + (-2) \times 4 = -4$

19. $\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$ is equal to:

(A) 1 (B) 2

(C) $\frac{1}{4}$ (D) $\frac{5}{4}$

Official Ans. by NTA (B)

Sol. $\tan\left(2(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8})+\tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left[2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

$= 2$

20. The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is :

(A) a tautology

(B) a contradiction

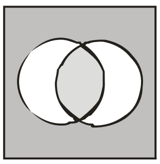
(C) equivalent to $(p \Rightarrow q) \wedge q$

(D) equivalent to $(p \Rightarrow q) \wedge p$

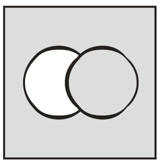
Official Ans. by NTA (D)

Sol. $(\sim(p \Leftrightarrow \sim q)) \wedge q \equiv (p \Leftrightarrow q) \wedge q$

$(p \Leftrightarrow q) \wedge q \equiv p \wedge q$



p⇔q



p→q

SECTION-B

1. If for some p, q, r $\in$ R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to-

Official Ans. by NTA (272)

Sol. $(px - q)^2 + (qx - r)^2 = 0$

$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = -4$

$\Rightarrow \frac{q^2+r^2}{p^2} = 272$

2. The number of 5-digit natural numbers, such that the product of their digits is 36, is

Official Ans. by NTA (180)

Sol. $3\times\frac{5!}{2!2!}+\frac{5!}{3!\times 2!}+\frac{5!}{2!}+\frac{5!}{3!}=180$

3. The series of positive multiples of 3 is divided into sets : {3}, {6, 9,12}, {15, 18, 21, 24, 27},... Then the sum of the elements in the 11^{th} set is equal to__________,

Official Ans. by NTA (6993)

Sol. $S_{11} = 3[101+102+......+121]$

$=\frac{3}{2}(222)\times 21 = 6993$

4. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is

Official Ans. by NTA (3)

Sol. $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$

$\Rightarrow (x-1)^2 (x + 1)(x^5 + 3x - 1) = 0$

Let $f(x) = x^5 + 3x - 1$

$f'(x) > 0 \ \forall\ x \in R$

Hence 3 real distinct roots.

5. If the coefficients of x and x^2 in the expansion of $(1 + x)^p(1 - x)^q$, p, q $\le$ 15, are –3 and–5 respectively, then the coefficient of x^3 is equal to_______________.

Official Ans. by NTA (23)

Sol. Since coefficient of x is –3

$\Rightarrow {}^pC_1 - {}^qC_1 = -3$

$\Rightarrow p - q = -3$ ....(1)

Comparing coefficients of x^2

$-{}^pC_1{}^qC_1 + {}^pC_2 + {}^qC_2 = -5$

$-pq+\frac{p(p-1)}{2}+\frac{q(q-1)}{2} = -5$ ....(2)

Solving (1) and (2)

$p = 8, q = 11$

Coefficient of x^3 is

$-{}^qC_3 + {}^pC_3 + {}^pC_1{}^qC_2 - {}^pC_2{}^qC_1$

$= -{}^{11}C_3 + {}^8C_3 + {}^8C_1{}^{11}C_2 - {}^8C_2{}^{11}C_1$

$= 23$

6. If

$n(2n+1)\int_0^1 (1-x^n)^{2n} dx = 1177 \int_0^1 (1-x^n)^{2n+1} dx$, then $n \in N$ is equal to _______

Official Ans. by NTA (24)

Sol. Let $I_1 = \int_0^1 (1-x^n)^{2n} dx$, $I_2 = \int_0^1 (1-x^n)^{2n+1} dx$

$I_2 = \int_0^1 (1-x^n)^{2n+1} . 1\, dx$

$= (1-x^n)^{2n+1}.x\Big|_0^1 - \int_0^1 (2n+1)(1-x^n)^{2n}(-nx^{n-1})x\, dx$

$I_2 = -n(2n+1)\{I_2 - I_1\}$

$(2n^2+n+1)I_2 = n(2n+1)I_1$

$\frac{I_1}{I_2} = \frac{2n^2+n+1}{n(2n+1)} = \frac{1177}{n(2n+1)}$

$\Rightarrow 2n^2 + n - 1176 = 0 \Rightarrow n = 24$

7. Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and x(>3) be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to _______

Official Ans. by NTA (6)

Sol. $x^4 = 3yx.y' - 3y^2$

$\Rightarrow 3xy\frac{dy}{dx} = 3y^2 + x^4$

Put $y^2 = t, y\frac{dy}{dx} = \frac{1}{2}\frac{dt}{dx}$

$\frac{dt}{dx} - \frac{2}{x}t = \frac{2}{3}x^3$

$\therefore \frac{t}{x^2} = \frac{x^2}{3} + C$

$\Rightarrow \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

Put (3, 3) , C = –2

$\therefore \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

$3y^2 = x^4 - 6x^2$

$x^4 - 6x^2 = 1080$

$\therefore x = 6$

8. The equations of the sides AB, BC and CA of a triangle ABC are 2x + y = 0, x + py = 15a and x – y = 3 respectively. If its orthocentre is (2, a), $-\frac{1}{2} < a < 2$, then p is equal to

Official Ans. by NTA (3)

Sol. Coordinates of A(1, –2), $B\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$ and

orthocentre H(2, a)

Slope of AH = p

a + 2 = p .....(1)

Slope of BH = –1

31a – 2ab = 15a + 4p–2 .....(2)

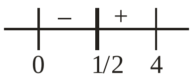
From (1) and (2)

a = 1 & p = 3

9. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, 8a – 1) but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to-

Official Ans. by NTA (45)

Sol. $f'(x) = 4x - \frac{1}{x}$



$a = \frac{1}{2}$

Let $P(x_1, y_1)$ be any point on $y^2 = 4ax$

$\frac{1}{y_1} = \frac{3 - y_1}{4 - x_1} \Rightarrow y_1^2 - 6y_1 + 8 = 0$

$y_1 = 2, 4$

$\Rightarrow$ P(8, 4) as P(2, 2) rejected

Equation of normal at P.

$y - 4 = -4(x - 8)$

$\frac{x}{9} + \frac{y}{36} = 1$

$\alpha = 9, \beta = 36$

$\alpha + \beta = 45$

10. Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). Then the square of the area of the triangle PQR is_____________.

Official Ans. by NTA (153)

Sol. Let $(2\lambda - 1, 3\lambda - 2, 2\lambda + 1)$ be any point on the line

$(2\lambda - 5)^2 + (3\lambda - 4)^2 + (2\lambda - 6)^2 = 26$

$\lambda = 1, 3$

Q (1, 1, 3) ; R (5, 7, 7) ; P (4, 2, 7)

Area of triangle PQR = ½ $\left|\vec{PQ} \times \vec{PR}\right|$

$= \sqrt{153}$