Acceleration of Block on Horizontal Smooth Surface

(1) **When a pull is horizontal**

$R = mg$

and $F = ma$

$\therefore\ a = F/m$

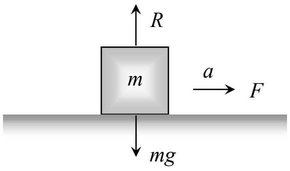


Fig : 4.22

(2) **When a pull is acting at an angle (θ) to the horizontal (upward)**

$R + F\sin\theta = mg$

$\Rightarrow R = mg - F\sin\theta$

and $F\cos\theta = ma$

$\therefore\ a = \dfrac{F\cos\theta}{m}$

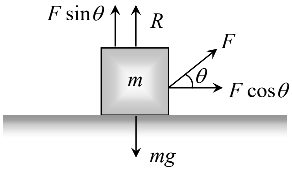


Fig : 4.23

(3) **When a push is acting at an angle (θ) to the horizontal (downward)**

$R = mg + F\sin\theta$

and $F\cos\theta = ma$

$a = \dfrac{F\cos\theta}{m}$

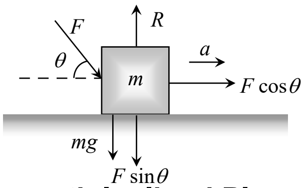


Fig : 4.24

Acceleration of Block on Smooth Inclined Plane

(1) **When inclined plane is at rest**

Normal reaction $R = mg\cos\theta$

Force along a inclined plane

$F = mg\sin\theta$; $ma = mg\ \sin\theta$

$\therefore a = g\sin\theta$

(2) **When a inclined plane given a horizontal acceleration '*b*'**

Since the body lies in an accelerating frame, an inertial force (mb) acts on it in the opposite direction.

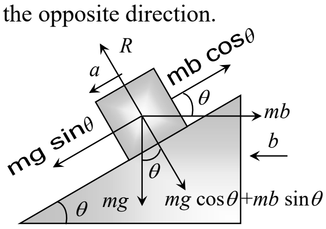


Fig : 4.25

Normal reaction $R = mg\cos\theta + mb\ sin\theta$

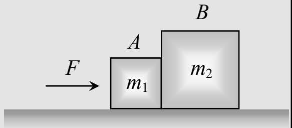
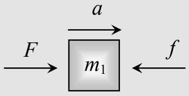
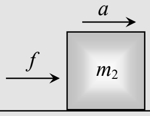
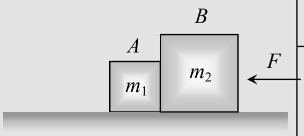
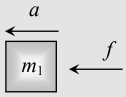
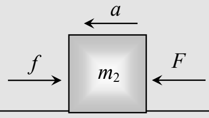
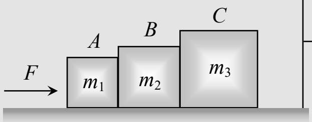
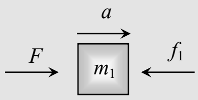
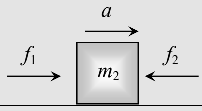
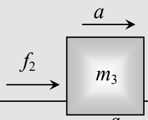
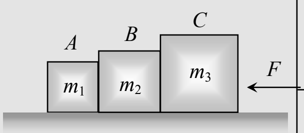
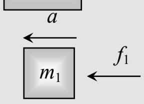
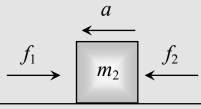
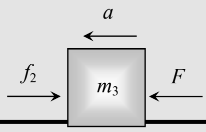
and $ma = mg\ \sin\theta - mb\cos\theta$

$\therefore\quad a = g\sin\theta - b\cos\theta$

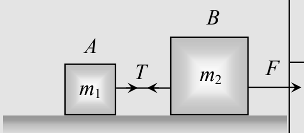
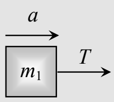
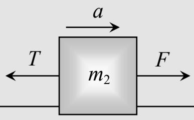
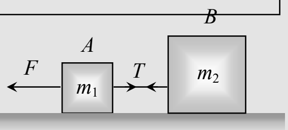
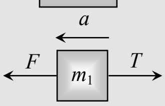
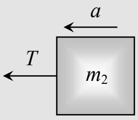
Note : ❑ The condition for the body to be at rest relative to the inclined plane : $a = g\sin\theta - b\cos\theta = 0$

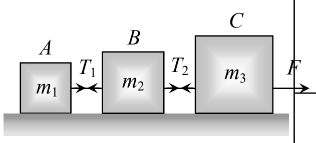
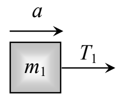
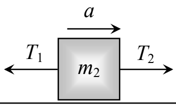
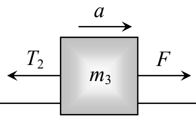
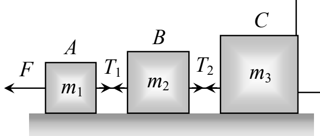
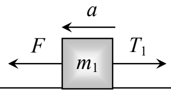
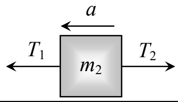
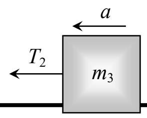
$\therefore\ b = g\tan\theta$

Motion of Blocks In Contact

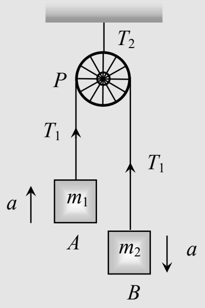
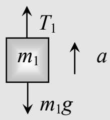
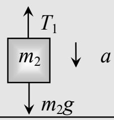
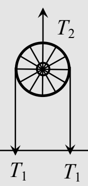
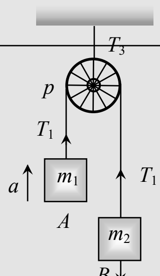
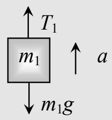
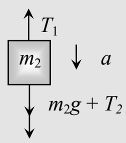
Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$
		$f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_2 - f_1 = m_2 a$	$f_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - f_2 = m_3 a$	$f_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

Motion of Blocks Connected by Mass Less String

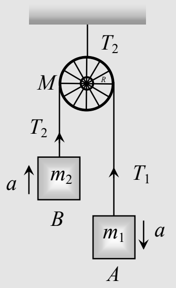
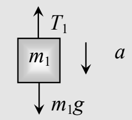
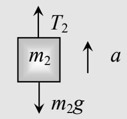
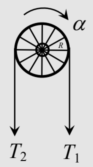
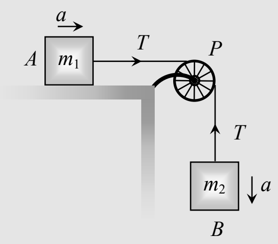
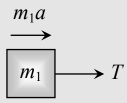
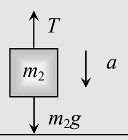
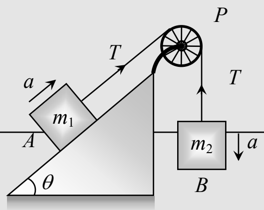
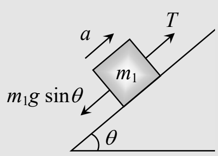
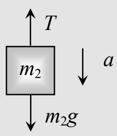
Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
			

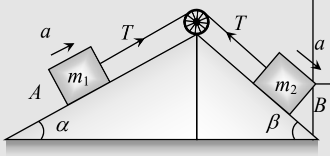
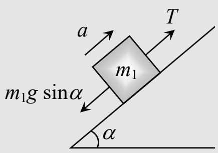
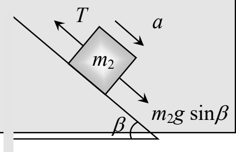
		$T = m_2 a$	$T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$
		$F - T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_1 - T_2 = m_2 a$	$T_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$T_2 = m_3 a$	$T_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

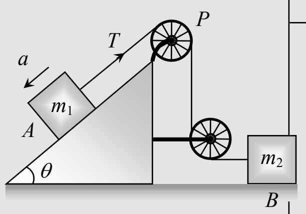
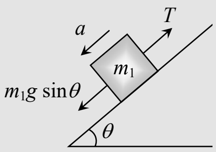
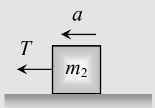
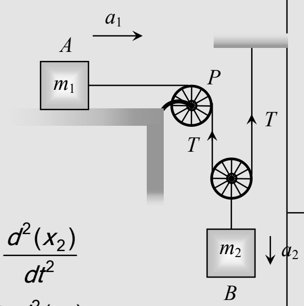
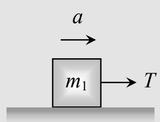
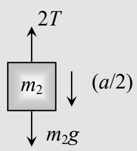
Motion of Connected Block Over A Pulley

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1 a = T_1 - m_1 g$	$T_1 = \frac{2m_1 m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T_1$	$T_2 = \frac{4m_1 m_2}{m_1 + m_2} g$
		$T_2 = 2T_1$	$a = \left[\frac{m_2 - m_1}{m_1 + m_2}\right] g$
		$m_1 a = T_1 - m_1 g$	$T_1 = \frac{2m_1[m_2 + m_3]}{m_1 + m_2 + m_3} g$
			

		$m_2a = m_2g + T_2 - T_1$	$T_2 = \dfrac{2m_1m_3}{m_1 + m_2 + m_3}g$
		$m_3a = m_3g - T_2$	$T_3 = \dfrac{4m_1[m_2 + m_3]}{m_1 + m_2 + m_3}g$
		$T_3 = 2T_1$	$a = \dfrac{[(m_2 + m_3) - m_1]g}{m_1 + m_2 + m_3}$

Condition	Free body diagram	Equation	Tension and acceleration
When pulley have a finite mass *M* and radius *R* then tension in two segments of string are different 		$m_1a = m_1g - T_1$	$a = \dfrac{m_1 - m_2}{m_1 + m_2 + \dfrac{M}{2}}$
		$m_2a = T_2 - m_2g$	$T_1 = \dfrac{m_1\left[2m_2 + \dfrac{M}{2}\right]}{m_1 + m_2 + \dfrac{M}{2}}g$
		Torque $= (T_1 - T_2)R = I\alpha$ $(T_1 - T_2)R = I\dfrac{a}{R}$ $(T_1 - T_2)R = \dfrac{1}{2}MR^2\dfrac{a}{R}$ $T_1 - T_2 = \dfrac{Ma}{2}$	$T_2 = \dfrac{m_2\left[2m_1 + \dfrac{M}{2}\right]}{m_1 + m_2 + \dfrac{M}{2}}g$
		$T = m_1a$	$a = \dfrac{m_2}{m_1 + m_2}g$
		$m_2a = m_2g - T$	$T = \dfrac{m_1m_2}{m_1 + m_2}g$
		$m_1a = T - m_1g\sin\theta$	$a = \left[\dfrac{m_2 - m_1\sin\theta}{m_1 + m_2}\right]g$
			

		$m_2a = m_2g - T$	$T = \frac{m_1m_2(1+\sin\theta)}{m_1+m_2}g$
		$T - m_1g\sin\alpha = m_1a$	$a = \frac{(m_2\sin\beta - m_1\sin\alpha)}{m_1+m_2}g$
		$m_2a = m_2g\sin\beta - T$	$T = \frac{m_1m_2(\sin\alpha+\sin\beta)}{m_1+m_2}g$

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1g\sin\theta - T = m_1a$	$a = \frac{m_1g\sin\theta}{m_1+m_2}$
		$T = m_2a$	$T = \frac{2m_1m_2}{4m_1+m_2}g$
 As $\frac{d^2(x_2)}{dt^2} = \frac{1}{2}\frac{d^2(x_1)}{dt^2}$ $\therefore a_2 = \frac{a_1}{2}$ a_1 = acceleration of block *A* a_2 = acceleration of block *B*		$T = m_1a$	$a_1 = a = \frac{2m_2g}{4m_1+m_2}$ $a_2 = \frac{m_2g}{4m_1+m_2}$ $T = \frac{2m_1m_2g}{4m_1+m_2}$
		$m_2\frac{a}{2} = m_2g - 2T$	

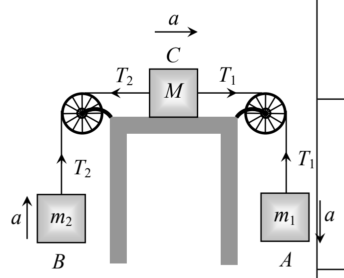
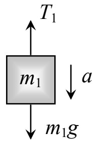
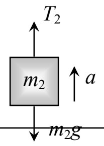
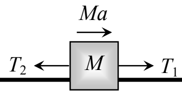
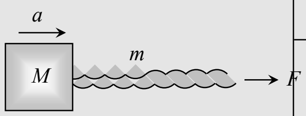
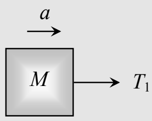
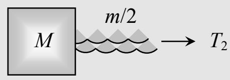
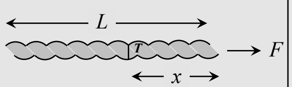
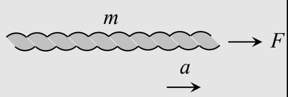
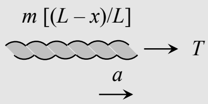
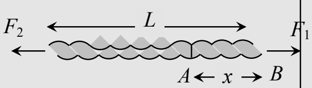
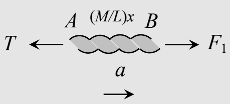
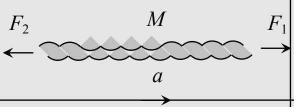
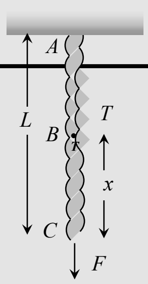
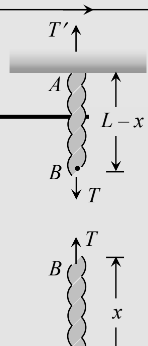
		$m_1 a = m_1 g - T_1$	$a = \frac{(m_1 - m_2)}{[m_1 + m_2 + M]} g$
		$m_2 a = T_2 - m_2 g$	$T_1 = \frac{m_1(2m_2 + M)}{[m_1 + m_2 + M]} g$
		$T_1 - T_2 = Ma$	$T_2 = \frac{m_2(2m_1 + M)}{[m_1 + m_2 + M]} g$

Table 4.3 : Motion of massive string

Condition	Free body diagram	Equation	Tension and acceleration
	 T_1 = force applied by the string on the block	$F = (M + m)a$ $T_1 = Ma$	$a = \frac{F}{M + m}$ $T_1 = M\frac{F}{(M + m)}$
	 T_2 = Tension at mid point of the rope	$T_2 = \left(M + \frac{m}{2}\right)a$	$T_2 = \frac{(2M + m)}{2(M + m)}F$
 m = Mass of string T = Tension in string at a distance x from the end where the force is applied		$F = ma$	$a = F/m$
		$T = m\left(\frac{L - x}{L}\right)a$	$T = \left(\frac{L - x}{L}\right)F$
 M = Mass of uniform string L = Length of string		$F_1 - T = \frac{Mxa}{L}$	$a = \frac{F_1 - F_2}{M}$
		$F_1 - F_2 = Ma$	$T = F_1\left(1 - \frac{x}{L}\right) + F_2\left(\frac{x}{L}\right)$
		$T' = \frac{M}{L}(L - x)g + T$	$T' = F + Mg$

Mass of segment $BC = \left(\frac{M}{L}\right)x$			

Spring Balance and Physical Balance

(1) **Spring balance :** When its upper end is fixed with rigid support and body of mass m hung from its lower end. Spring is stretched and the weight of the body can be measured by the reading of spring balance $R = W = mg$

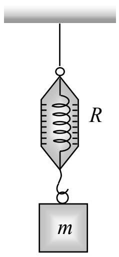


Fig : 4.26

The mechanism of weighing machine is same as that of spring balance.

Effect of frame of reference : In inertial frame of reference the reading of spring balance shows the actual weight of the body but in non-inertial frame of reference reading of spring balance increases or decreases in accordance with the direction of acceleration

(2) **Physical balance :** In physical balance actually we compare the mass of body in both the pans. Here we does not calculate the absolute weight of the body.

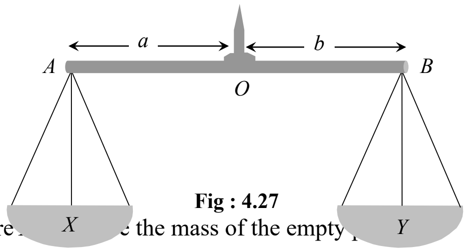


Fig : 4.27

Here … the mass of the empty …

(i) Perfect physical balance :

Weight of the pan should be equal *i.e.* $X = Y$

and the needle must in middle of the beam *i.e.* $a = b$.

Effect of frame of reference : If the physical balance is perfect then there will be no effect of frame of reference (either inertial or non-inertial) on the measurement. It is always errorless.

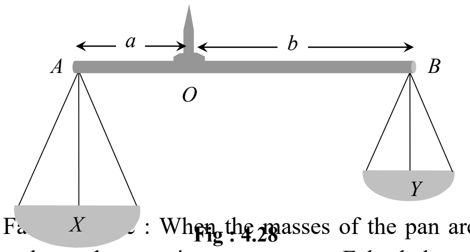


Fig : 4.28

(ii) Fa… : When the masses of the pan are not equal then balance shows the error in measurement. False balance may be of two types

(a) If the beam of physical balance is horizontal (when the pans are empty) but the arms are not equal

$X > Y$ and $a < b$

For rotational equilibrium about point 'O'

$Xa = Yb$ …(i)

In this physical balance if a body of weight W is placed in pan X then to balance it we have to put a weight W_1 in pan Y.

For rotational equilibrium about point 'O'

$(X + W)a = (Y + W_1)b$ …(ii)

Now if the pans are changed then to balance the body we have to put a weight W_2 in pan X.

For rotational equilibrium about point 'O'

$(X + W_2)a = (Y + W)b$ …(iii)

From (i), (ii) and (iii)

True weight $W = \sqrt{W_1 W_2}$

(b) If the beam of physical balance is not horizontal (when the pans are empty) and the arms are equal

i.e. $X > Y$ and $a = b$

In this physical balance if a body of weight W is placed in X Pan then to balance it.

We have to put a weight W_1 in Y Pan

For equilibrium $X + W = Y + W_1$ …(i)

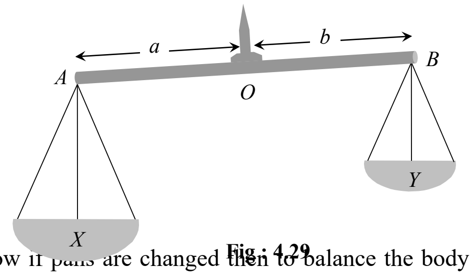


Fig : 4.29

Now if pans are changed then to balance the body we have to put a weight W_2 in X Pan.

For equilibrium $X + W_2 = Y + W$ …(ii)

From (i) and (ii)

True weight $W = \dfrac{W_1 + W_2}{2}$

Modification of Newton's Laws of motion

According to Newton, time and space are absolute. The velocity of observer has no effect on it. But, according to special theory of relativity Newton's laws are true, as long as we are dealing with velocities which are small compare to velocity of light. Hence the time and space measured by two observers in relative motion are not same. Some conclusions drawn by the special theory of relativity about mass, time and distance which are as follows :

(1) Let the length of a rod at rest with respect to an observer is L_0. If the rod moves with velocity v w.r.t. observer and its length is L,

then $L = L_0\sqrt{1 - v^2 / c^2}$

where, c is the velocity of light.

Now, as v increases L decreases, hence the length will appear shrinking.

(2) Let a clock reads T_0 for an observer at rest. If the clock moves with velocity v and clock reads T with respect to observer, then

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, the clock in motion will appear slow.

(3) Let the mass of a body is m_0 at rest with respect to an

observer. Now, the body moves with velocity v with respect to observer and its mass is m, then $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

m_0 is called the rest mass.

Hence, the mass increases with the increases of velocity.

Note : ❑ If $v << c$, *i.e.*, velocity of the body is very small *w.r.t.* velocity of light, then $m = m_0$. *i.e.*, in the practice there will be no change in the mass.

❑ If v is comparable to c, then $m > m_0$ *i.e.*, mass will increase.

❑ If $v = c$, then $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{v^2}}}$ or $m = \frac{m_0}{0} = \infty$. Hence, the mass becomes infinite, which is not possible, thus the speed cannot be equal to the velocity of light.

❑ The velocity of particles can be accelerated up to a certain limit. Even in cyclotron the speed of charged particles cannot be increased beyond a certain limit.

Tips & Tricks

- Inertia is proportional to mass of the body.
- Force cause acceleration.
- In the absence of the force, a body moves along a straight line path.
- A system or a body is said to be in equilibrium, when the net force acting on it is zero.
- If a number of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, ...$ act on the body, then it is in equilibrium when $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... = \vec{0}$
- A body in equilibrium cannot change the direction of motion.
- Four types of forces exist in nature. They are – gravitation (F_g), electromagnetic (F_{em}), weak force (F_w) and nuclear force (F_n).

  $(F_g) : (F_w) : (F_{em}) : (F_n) :: 1 : 10^{25} : 10^{36} : 10^{38}$
- If a body moves along a curved path, then it is certainly acted upon by a force.
- A single isolated force cannot exist.
- Forces in nature always occur in pairs.
- Newton's first law of the motion defines the force.
- Absolute units of force remains the same throughout the universe while gravitational units of force varies from place to place as they depend upon the value of 'g'.
- Newton's second law of motion gives the measure of force i.e. $F = ma$.
- Force is a vector quantity.
- Absolute units of force are dyne in CGS system and newton (N) in *SI*.
- $1\ N = 10^5$ *dyne*.
- Gravitational units of force are *gf* (or *gwt*) in *CGS* system and *kgf* (or *kgwt*) in *SI*.
- 1 *gf* = 980 *dyne* and 1 *kgf* = 9.8 *N*
- The beam balance compares masses.
- Acceleration of a horse-cart system is

  $a = \frac{H - F}{M + m}$

  where H = Horizontal component of reaction; F = force of friction; M = mass of horse; m = mass of cart.
- The weight of the body measured by the spring balance in a lift is equal to the apparent weight.
- Apparent weight of a freely falling body = ZERO, (state of weightlessness).
- If the person climbs up along the rope with acceleration a, then tension in the rope will be $m(g+a)$
- If the person climbs down along the rope with acceleration, then tension in the rope will be $m(g - a)$
- When the person climbs up or down with uniform speed, tension in the string will be mg.
- A body starting from rest moves along a smooth inclined plane of length l, height h and having angle of inclination θ.

  (i) Its acceleration down the plane is $g \sin\theta$.

  (ii) Its velocity at the bottom of the inclined plane will be $\sqrt{2gh} = \sqrt{2gl \sin\theta}$.

  (iii) Time taken to reach the bottom will be

  $$t = \sqrt{\frac{2l}{g \sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$$

  (iv) If the angle of inclination is changed keeping the height constant then

  $\frac{t_1}{t_2} = \frac{\sin\theta_2}{\sin\theta_1}$
- For an isolated system (on which no external force acts), the total momentum remains conserved (Law of conservation of momentum).
- The change in momentum of a body depends on the magnitude and direction of the applied force and the period of time over which it is applied *i.e.* it depends on its impulse.
- Guns recoil when fired, because of the law of conservation of momentum. The positive momentum gained by the bullet is equal to negative recoil momentum of the gun

and so the total momentum before and after the firing of the gun is zero.

✍ Recoil velocity of the gun is $\vec{V} = \frac{-m}{M}\vec{v}$

✍ where m = mass of bullet, M = mass of gun and $\vec{v}$ = muzzle velocity of bullet.

✍ The rocket pushes itself forwards by pushing the jet of exhaust gases backwards.

✍ Upthrust on the rocket = $u \times \frac{dm}{dt}$.

where u = velocity of escaping gases relative to rocket and $\frac{dm}{dt}$ = rate of consumption of fuel.

✍ Initial thrust on rocket = $m(g + a)$, where a is the acceleration of the rocket.

✍ Upward acceleration of rocket = $\frac{u}{m} \times \frac{dm}{dt}$.

✍ Impulse, $\vec{I} = \vec{F} \times \Delta t =$ change in momentum

✍ Unit of impulse is *N-s.*

✍ Action and reaction forces never act on the same body. They act on different bodies. If they act on the same body, the resultant force on the body will be zero i.e., the body will be in equilibrium.

✍ Action and reaction forces are equal in magnitude but opposite in direction.

✍ Action and reaction forces act along the line joining the centres of two bodies.

✍ Newton's third law is applicable whether the bodies are at rest or in motion.

✍ The non-inertial character of the earth is evident from the fact that a falling object does not fall straight down but slightly deflects to the east.

OT Ordinary Thinking

Objective Questions

First law of motion

1. A rider on horse back falls when horse starts running all of a sudden because **[MP PMT 1982]**

(a) Rider is taken back

(b) Rider is suddenly afraid of falling

(c) Inertia of rest keeps the upper part of body at rest whereas lower part of the body moves forward with the horse

(d) None of the above

2. When a train stops suddenly, passengers in the running train feel an instant jerk in the forward direction because **[MP PMT 1982]**

(a) The back of seat suddenly pushes the passengers forward

(b) Inertia of rest stops the train and takes the body forward

(c) Upper part of the body continues to be in the state of motion whereas the lower part of the body in contact with seat remains at rest

(d) Nothing can be said due to insufficient data

3. Inertia is that property of a body by virtue of which the body is **[MGIMS Wardha 1982]**

(a) Unable to change by itself the state of rest

(b) Unable to change by itself the state of uniform motion

(c) Unable to change by itself the direction of motion

(d) Unable to change by itself the state of rest and of uniform linear motion

4. A man getting down a running bus falls forward because **[CPMT 1981]**

(a) Due to inertia of rest, road is left behind and man reaches forward

(b) Due to inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as they touch the road

(c) He leans forward as a matter of habit

(d) Of the combined effect of all the three factors stated in (*a*), (*b*) and (*c*)

5. A boy sitting on the topmost berth in the compartment of a train which is just going to stop on a railway station, drops an apple aiming at the open hand of his brother sitting vertically below his hands at a distance of about 2 *meter*. The apple will fall **[CPMT 1986]**

(a) Precisely on the hand of his brother

(b) Slightly away from the hand of his brother in the direction of motion of the train

(c) Slightly away from the hand of his brother in the direction opposite to the direction of motion of the train

(d) None of the above