Chapter 18

Electrostatics

Electric Charge

(1) Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects.

(2) It is known that every atom is electrically neutral, containing as many electrons as the number of protons in the nucleus.

(3) Charged particles can be created by disturbing neutrality of an atom. Loss of electrons gives positive charge (as then $n_p > n_e$) and gain of electrons gives negative charge (as then $n_e > n_p$) to a particle. In charging mass of the body changes as shown below

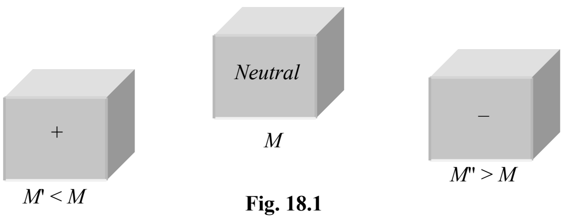


Fig. 18.1

(4) Charges with the same electrical sign repel each other, and charges with opposite electrical sign attract each other.

Fig. 18.2

(5) **Unit and dimensional formula**

S.I. unit of charge is *Ampere* × *sec* = *coulomb* (C), smaller S.I. units are mC, μC.

C.G.S. unit of charge is *Stat coulomb* or *e.s.u.* Electromagnetic unit of charge is *ab coulomb*

$$1C = 3 \times 10^9 \; statcoulomb = \frac{1}{10} \; abcoulomb \, .$$

Dimensional formula $[Q] = [AT]$

(6) **Charge is**

Transferable : It can be transferred from one body to another .
Associated with mass : Charge cannot exist without mass but reverse is not true.
Conserved : It can neither be created nor be destroyed.
Invariant : Independent of velocity of charged particle.

(7) Electric charge produces electric field ($\vec{E}$), magnetic field ($\vec{B}$) and electromagnetic radiations.

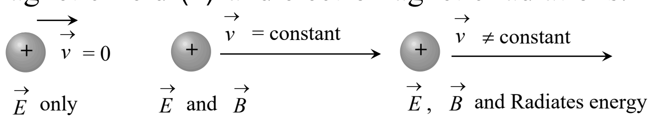
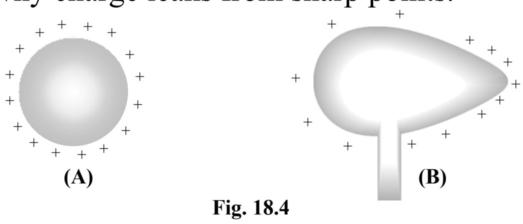


Fig. 18.3

(8) **Point charge :** A finite size body may behave like a point charge if it produces an inverse square electric field. For example an isolated charged sphere behave like a point charge at very large distance as well as very small distance close to it’s surface.

(9) **Charge on a conductor :** Charge given to a conductor always resides on it's outer surface. This is why a solid and hollow conducting sphere of same outer radius will hold maximum equal charge. If surface is uniform the charge distributes uniformly on the surface and for irregular surface the distribution of charge, *i.e.*, charge density is not uniform. It is maximum where the radius of curvature is minimum and vice versa. *i.e.*, $\sigma \propto (1/R)$. This is why charge leaks from sharp points.



(A) (B)

Fig. 18.4

(10) **Charge distribution :** It may be of two types

(i) Discrete distribution of charge : A system consisting of ultimate individual charges.

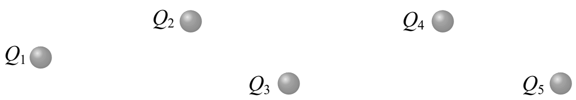


Fig. 18.5

(ii) Continuous distribution of charge : An amount of charge distribute uniformly or non-uniformly on a body. It is of following three types

(a) Line charge distribution : Charge on a line *e.g.* charged straight wire, circular charged ring *etc.*

$\lambda = \dfrac{\text{Charge}}{\text{Length}} =$ Linear charge density

→ S.I. unit is $\dfrac{C}{m}$

→ Dimension is $[L^{-1}TA]$

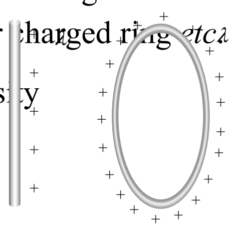


Fig. 18.6

(b) Surface charge distribution : Charge distributed on a surface *e.g.* plane sheet of charge, conducting sphere, conducting cylinder of

$\sigma = \dfrac{\text{Charge}}{\text{Area}} =$ Surface charge density

→ S.I. unit is $\dfrac{C}{m^2}$

→ Dimension is $[L^{-2}TA]$

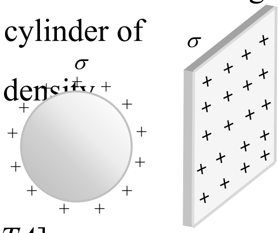


Fig. 25.7

(c) Volume charge density : Charge distributes through out the volume of the body *e.g.* charge on a dielectric sphere *etc.*

$\rho = \dfrac{\text{Charge}}{\text{Volume}} =$ Volume charge density

→ S.I. unit is $\dfrac{C}{m^3}$

→ Dimension is $[L^{-3}TA]$

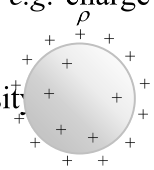


Fig. 18.8

(11) **Quantization of charge** : If the charge of an electron $(= 1.6 \times 10^{-19} C)$ is taken as elementary unit *i.e.* quanta of charge, the charge on any body will be some integral multiple of *e i.e.*,

$Q = \pm ne$ with $n = 1, 2, 3 ....$

Charge on a body can never be $\pm \frac{2}{3}e$, $\pm 17.2e$ or $\pm 10^{-5}e$ *etc.*

(12) **Comparison of charge and mass :** We are familiar with role of mass in gravitation, and we have just studied some features of electric charge. We can compare the two as shown below

Table 18.1 : Charge v/s mass

Charge	Mass
(1) Electric charge can be positive, negative or zero.	(1) Mass of a body is a positive quantity.
(2) Charge carried by a body does not depend upon velocity of the body.	(2) Mass of a body increases with its velocity as $m = \dfrac{m_0}{\sqrt{1 - v^2 / c^2}}$ where *c* is velocity of light in vacuum, *m* is the mass of the body moving with velocity *v* and m_0 is rest mass of the body.
(3) Charge is quantized.	(3) The quantization of mass is yet to be established.

(4) Electric charge is always conserved.	(4) Mass is not conserved as it can be changed into energy and vice-versa.
(5) Force between charges can be attractive or repulsive, accordingly as charges are unlike or like charges.	(5) The gravitational force between two masses is always attractive.

Methods of Charging

A body can be charged by following methods.

(1) **By friction :** By rubbing two bodies together, both positive and negative charges in equal amounts appear simultaneously due to transfer of electrons from one body to the other.

(i) When a glass rod is rubbed with silk, the rod becomes positively charged while the silk becomes negatively charged. The decrease in the mass of glass rod is equal to the total mass of electrons lost by it.

(ii) Ebonite on rubbing with wool becomes negatively charged making the wool positively charged.

(iii) Clouds also get charged by friction.

(iv) A comb moving through dry hair gets electrically charged. It starts attracting small bits of paper.

(v) During landing or take-off, the tyres of an aircraft get electrified therefore special material is used to manufacture them.

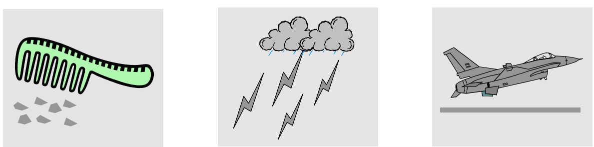


Fig. 18.9

(2) **By electrostatic induction :** If a charged body is brought near an uncharged body, one side of neutral body (closer to charged body) becomes oppositely charged while the other side becomes similarly charged.

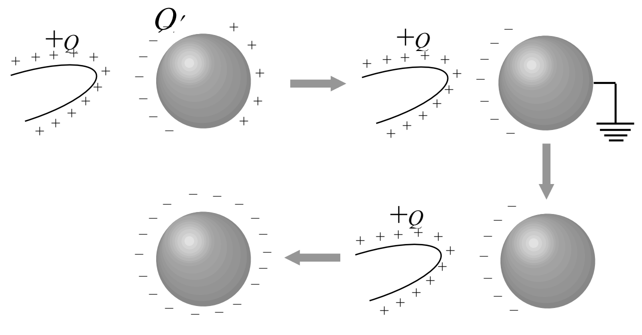


Fig. 18.10

Induced charge can be lesser or equal to inducing charge (but never greater) and its maximum value is given by $Q' = -Q\left[1 - \frac{1}{K}\right]$ where Q is the inducing charge and K is the dielectric constant of the material of the uncharged body. It is also known as specific inductive capacity **(SIC)** of the medium, or relative permittivity ε_r of the medium (relative means with respect to free space)

Table 18.2 : Different dielectric constants

Medium	*K*	Medium	*K*
Vacuum	1	Mica	6
air	1.0003	Silicon	12
Paraffin vax	2.1	Germanium	16
Rubber	3	Glycerin	50
Transformer oil	4.5	Water	80
Glass	5–10	Metal	∞

(3) **Charging by conduction :** Take two conductors, one charged and other uncharged. Bring the conductors in contact with each other. The charge (whether $-ve$ or $+ve$) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with the same sign. This is called as charging by conduction (through contact).

Electroscope

It is a simple apparatus with which the presence of electric charge on a body is detected (see figure). When metal knob is touched with a charged body, some charge is transferred to the gold leaves, which then diverges due to repulsion. The separation gives a rough idea of the amount of charge on the body. When a charged body brought near a charged electroscope, the leaves will further diverge, if the charge on body is

Fig. 18.11

similar to that on electroscope and will usually converge if opposite. If the induction effect is strong enough leaves after converging may again diverge.

Coulomb's Law

If two stationary and point charges Q_1 and Q_2 are kept at a distance *r*, then it is found that force of attraction or repulsion between them is

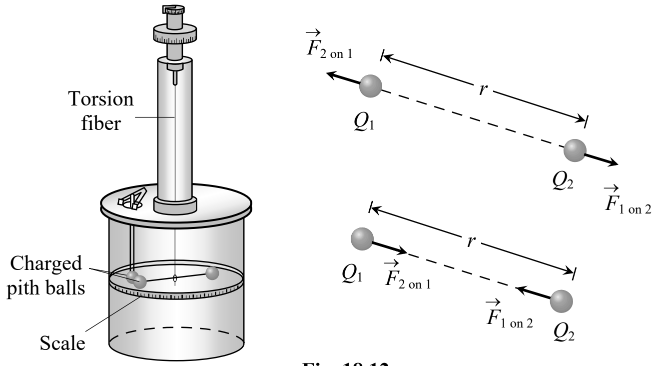


Fig. 18.12

$F \propto \dfrac{Q_1 Q_2}{r^2}$ *i.e.*, $F = \dfrac{kQ_1 Q_2}{r^2}$ (k = Proportionality constant)

In C.G.S. (for air) $k = 1$, $F = \dfrac{Q_1 Q_2}{r^2}$ *Dyne*

In S.I. (for air) $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \dfrac{N\text{-}m^2}{C^2}$

$\Rightarrow F = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{Q_1 Q_2}{r^2}$ *Newton* (1 *Newton* = 10^5 *Dyne*)

ε_0 = Absolute permittivity of air or free space

$= 8.85 \times 10^{-12} \dfrac{C^2}{N - m^2}\left(= \dfrac{Farad}{m}\right)$. It's Dimensional formula is $[M^{-1} L^{-3} T^4 A^2]$

(1) **Vector form of coulomb's law :** Vector form of Coulomb's law is $\vec{F}_{12} = K.\dfrac{Q_1 Q_2}{r^3}\vec{r}_{12} = K.\dfrac{Q_1 Q_2}{r^2}\hat{r}_{12}$, where $\hat{r}_{12}$ is the unit vector from first charge to second charge along the line joining the two charges.

(2) **Effect of medium :** When a dielectric medium is completely filled in between charges rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of *K* (**dielectric constant)**

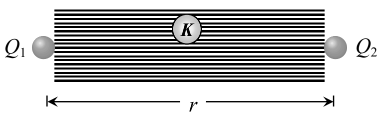


Fig. 18.13

i.e. $F_{medium} = \dfrac{F_{air}}{K} = \dfrac{1}{4\pi\varepsilon_0 K} \cdot \dfrac{Q_1 Q_2}{r^2}$

(Here $\varepsilon_0 K = \varepsilon_0 \varepsilon_r = \varepsilon$ = permittivity of medium)

If a dielectric medium (dielectric constant *K*, thickness *t*) is partially filled between the charges then effective air separation between the charges becomes $(r - t + t\sqrt{K})$

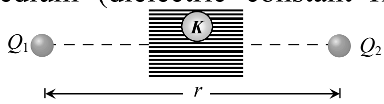


Fig. 18.14

Hence force $F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q_1 Q_2}{(r - t + t\sqrt{K})^2}$

(3) **Principle of superposition :** According to the principle of super position, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges.

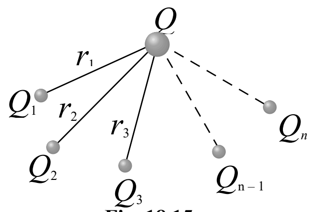


Fig. 18.15

Consider number of charge Q_1, Q_2, Q_3 …are applying force on a charge *Q*

Net force on *Q* will be

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_{n-1} + \vec{F}_n$

The magnitude of the resultant of two electric forces is given by

$F_{net} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$

and $\tan\alpha = \dfrac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$

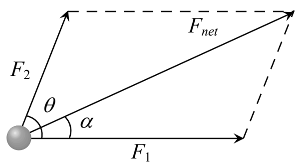


Fig. 18.16

For problem solving remember following standard results.

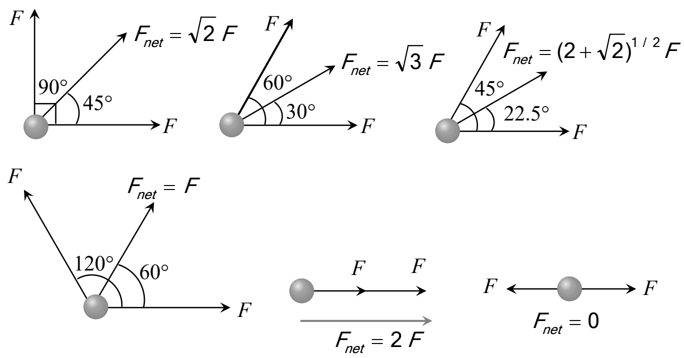


Fig. 18.17

Table 18.3 : Fundamental forces of nature

Force	Nature and formula	Range	Relative strength
Force of gravitation between two masses	Attractive $F = Gm_1m_2/r^2$, obey's Newton's third law of motion, it's a conservative force	Long range (between planets and between electron and proton)	1
Electromagnetic force (for stationary and moving charges)	Attractive as well as repulsive, obey's Newton's third law of motion, it's a conservative force	Long (upto few *kelometers*)	10^{37}
Nuclear force (between nucleons)	Exact expression is not known till date.	Short (of the order of nuclear size 10^{-15} *m*)	10^{39} (strongest)
Weak force (for processes like β decay)	Formula not known	Short (upto $10^{-15}m$)	10^{24}

Electrical Field

A positive charge or a negative charge is said to create its field around itself. Thus space around a charge in which another charged particle experiences a force is said to have electrical field in it.

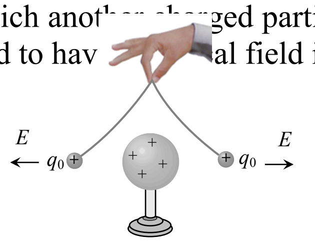


Fig. 18.18

(1) **Electric field intensity** $(\vec{E})$ **:** The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point. $\vec{E} = \frac{\vec{F}}{q_0}$

Where $q_0 \to 0$ so that presence of this charge may not affect the source charge Q and its electric field is not changed, therefore expression for electric field intensity can be better written as $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

(2) **Unit and Dimensional formula**

It's S.I. unit – $\frac{Newton}{coulomb} = \frac{volt}{meter} = \frac{Joule}{coulomb \times meter}$

and C.G.S. unit – *Dyne/stat coulomb*.

Dimension : $[E] = [MLT^{-3}A^{-1}]$

(3) **Direction of electric field :** Electric field (intensity) $\vec{E}$ is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge.

(4) **Relation between electric force and electric field :** In an electric field $\vec{E}$ a charge (Q) experiences a force $\vec{F} = Q\vec{E}$. If charge is positive then force is directed in the direction of field while if charge is negative force acts on it in the opposite direction of field

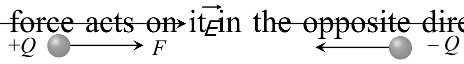


Fig. 18.19

(5) **Super position of electric field** (electric field at a point due to various charges) : The resultant electric field at any point is equal to the vector sum of electric fields at that point due to various charges *i.e.* $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$

(6) **Electric field due to continuous distribution of charge :** A system of closely spaced electric charges forms a continuous charge distribution. To find the field of a continuous charge distribution, we divide the charge into infinitesimal charge elements. Each infinitesimal charge element is then considered, as a point charge and electric field $\vec{dE}$ is determined due to this charge at given point. The Net field at the given point is the summation of fields of all the elements. *i.e.*, $\vec{E} = \int \vec{dE}$.

Electric Potential

(1) **Definition :** Potential at a point in a field is defined as the amount of work done in bringing a unit positive test charge, from infinity to that point along any arbitrary path (infinity is point of zero potential). Electric potential is a scalar quantity, it is denoted by *V*; $V = \frac{W}{q_0}$

(2) **Unit and dimensional formula**

S. I. unit : $\frac{Joule}{Coulomb} = volt$

C.G.S. unit : *Stat volt* (e.s.u.); **1 *volt*** $= \frac{1}{300}$ ***Stat volt***

Dimension : $[V] = [ML^2T^{-3}A^{-1}]$

(3) **Types of electric potential :** According to the nature of charge potential is of two types

(i) Positive potential : Due to positive charge.

(ii) Negative potential : Due to negative charge.

(4) **Potential of a system of point charges :** Consider P is a point at which net electric potential is to be determined due to several charges. So net potential at P

$$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2} + k\frac{Q_3}{r_3} + k\frac{(-Q_4)}{r_4} + \ldots$$

In general $V = \sum_{i=1}^{X} \frac{kQ_i}{r_i}$

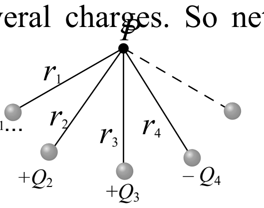


Fig. 18.20

(5) **Electric potential due to a continuous charge distribution :** The potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided *i.e.*,

$$V = \int dV, \quad = \int \frac{dQ}{4\pi\varepsilon_0 r}$$

(6) **Graphical representation of potential :** As we move on the line joining two charges then variation of potential with distance is shown below

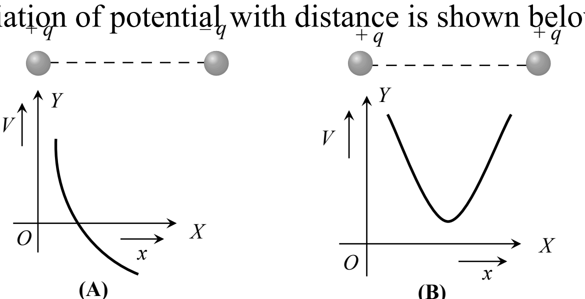


Fig. 18.21

(7) **Potential difference :** In an electric field potential difference between two points A and B is defined as equal to the amount of work done (by external agent) in moving a unit positive charge from point A to point B *i.e.*, $V_B - V_A = \frac{W}{q_0}$

Electric Field and Potential Due to Various Charge Distribution

(1) **Point charge :** Electric field and potential at point P due to a point charge Q is

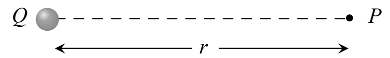


Fig. 18.22

$$E = k\frac{Q}{r^2} \text{ or } \vec{E} = k\frac{Q}{r^2}\hat{r} \quad \left(k = \frac{1}{4\pi\varepsilon_0}\right), \quad V = k\frac{Q}{r}$$

Graph

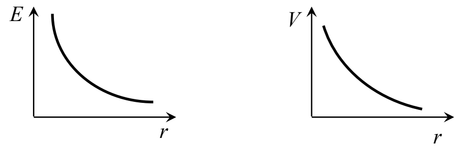


Fig. 18.23

(2) **Line charge:** Electric field and potential due to a charged straight conducting wire of length l and charge density λ

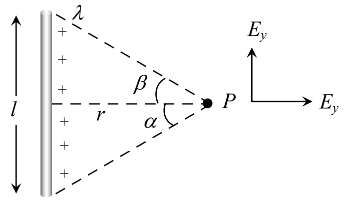


Fig. 18.24

$$E_x = \frac{k\lambda}{r}(\sin\alpha + \sin\beta) \text{ and } E_y = \frac{k\lambda}{r}(\cos\beta - \cos\alpha)$$

$$V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left[\frac{\sqrt{r^2+l^2} - l}{\sqrt{r^2+l^2} + l}\right]$$

(i) If point P lies at perpendicular bisector of wire *i.e.* $\alpha = \beta$; $E_x = \frac{2k\lambda}{r}\sin\alpha$ and $E_y = 0$

(ii) If wire is infinitely long *i.e.* $l \to \infty$ so $\alpha = \beta = \frac{\pi}{2}$; $E_x = \frac{2k\lambda}{r}$ and $E_y = 0 \Rightarrow E_{net} = \frac{\lambda}{2\pi\varepsilon_0 r}$ and $V = \frac{-\lambda}{2\pi\varepsilon_0}\log_e r + c$

(iii) If point P lies near one end of infinitely long wire *i.e.* $\alpha = 0$, and $\beta = \frac{\pi}{2}$

$$|E_x| = |E_y| = \frac{k\lambda}{r}$$

$$\Rightarrow E_{net} = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\,k\lambda}{r}$$

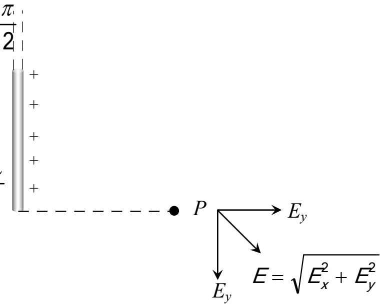


Fig. 18.25

(3) **Charged circular ring :** Suppose we have a charged circular ring of radius R and charge Q. On it's axis electric field and potential is to be determined, at a point 'x' distance away from the centre of the ring.

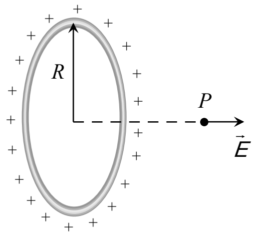


Fig. 18.26

At point P

$$E = \frac{kQx}{(x^2+R^2)^{3/2}},\quad V = \frac{kQ}{\sqrt{x^2+R^2}}$$

At centre $x = 0$ so $E_{centre} = 0$ and $V_{centre} = \frac{kQ}{R}$

At a point on the axis such that $x >> R$ $E = \frac{kQ}{x^2}$, $V = \frac{kQ}{x}$

If $x = \pm\frac{R}{\sqrt{2}}$, $E_{\max} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 a^2}$ and $V_{\max} = \frac{Q}{2\sqrt{6}\pi\varepsilon_0}$

Graph

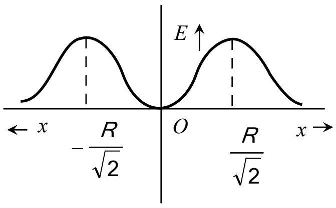
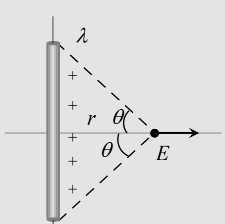
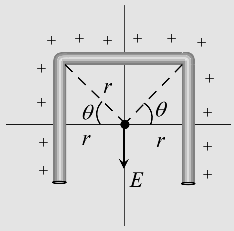
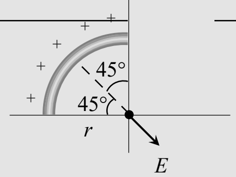
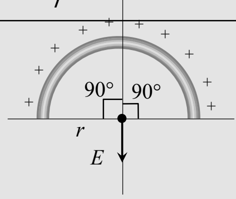
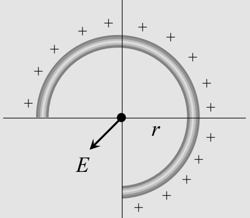
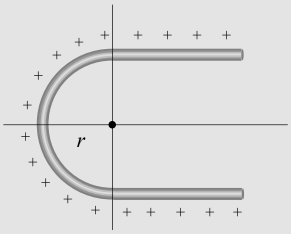


Fig. 18.27

(4) **Some more results of line charge :** If a thin plastic rod having charge density λ is bent in the following shapes then electric field at P in different situations shown in the following table

Table 18.4 : Bending of charged rod

	
$E = \frac{2k\lambda}{r}\sin\theta$	$E = \frac{2k\lambda}{r}\cos\theta$
	
$E = \frac{\sqrt{2}\,k\lambda}{r}$	$E = \frac{2k\lambda}{r}$
	
$E = \frac{\sqrt{2}\,k\lambda}{r}$	$E = 0$

(5) **Charged cylinder**

(i) Non-conducting uniformly charged cylinder

(ii) Conducting charged cylinder

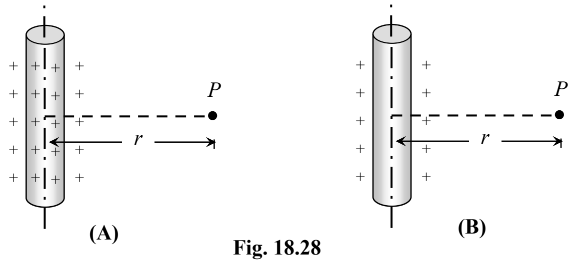


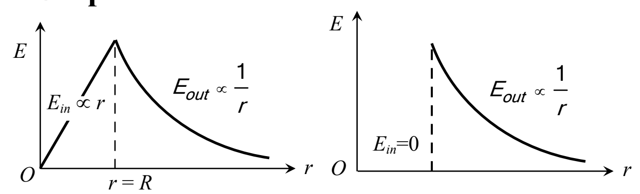
Fig. 18.28

If point of observation (P) lies outside the cylinder then for both type of cylindrical charge distribution $E_{out} = \frac{\lambda}{2\pi\varepsilon_0 r}$, and $V_{out} = \frac{-\lambda}{2\pi\varepsilon_0}\log_e r + c$

If point of observation lies at surface *i.e.* $r = R$ so for both cylinder $E_{suface} = \frac{\lambda}{2\pi\varepsilon_0 R}$ and $V_{surface} = \frac{-\lambda}{2\pi\varepsilon_0}\log_e R + c$

If point of observation lies inside the cylinder then for conducting cylinder $E_{in} = 0$ and for non-conducting $E_{in} = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$

Graph

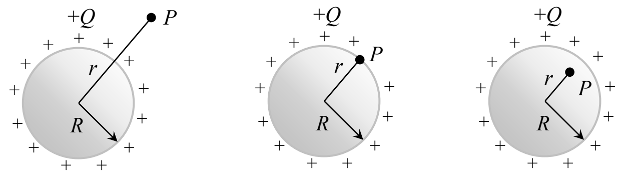


(A) For non-conducting cylinder **(B)** For conducting cylinder

Fig. 18.29

(6) **Charged Conducting sphere (or shell of charge) :** If charge on a conducting sphere of radius R is Q (and σ = surface charge density) as shown in

figure then electric field and potential in different situation are



(A) Outside **(B)** At the surface **(C)** Inside

Fig. 18.30

(i) **Out side the sphere :** If point P lies outside the sphere

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \text{ and } V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} = \frac{\sigma R^2}{\varepsilon_0 r}$$

$(Q = \sigma \times A = \sigma \times 4\,\pi R^2)$

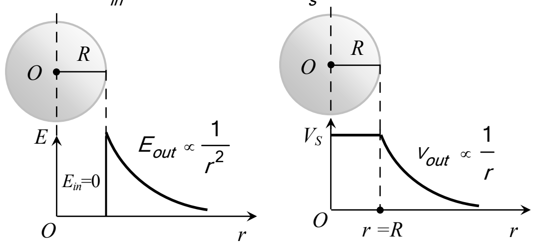
(ii) **At the surface of sphere :** At surface $r = R$

So, $E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0}$ and $V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$

(iii) **Inside the sphere :** Inside the conducting charge sphere electric field is zero and potential remains constant every where and equals to the potential at the surface.

$E_{in} = 0$ and V_{in} = constant $= V_s$

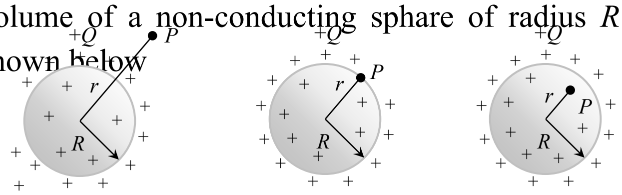
Graph



(A) **(B)**

Fig. 18.31

(7) **Uniformly charged non-conducting sphere :** Suppose charge Q is uniformly distributed in the volume of a non-conducting sphare of radius R as shown below



(A) Outside **(B)** At the surface **(C)** Inside

Fig. 18.32

(i) **Outside the sphere :** If point P lies outside the sphere

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} \text{ and } V_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r}$$

If the sphere has uniform volume charge density $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$

then $E_{out} = \frac{\rho R^3}{3\varepsilon_0 r^2}$ and $V_{out} = \frac{\rho R^3}{3\varepsilon_0 r}$

(ii) **At the surface of sphere :** At surface $r = R$

$$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\rho R}{3\varepsilon_0} \text{ and } V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\rho R^2}{3\varepsilon_0}$$

(iii) **Inside the sphere :** At a distance r from the centre

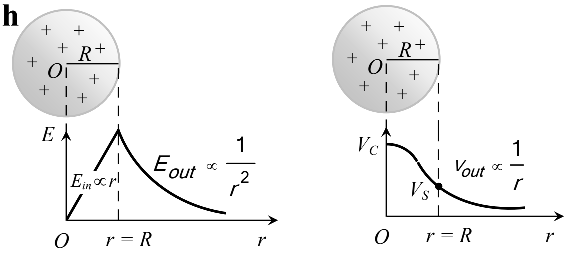
$$E_{in} = \frac{1}{4\pi\varepsilon_0}.\frac{Qr}{R^3} = \frac{\rho r}{3\varepsilon_0} \quad \{E_{in} \propto r\}$$

and $V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q[3R^2 - r^2]}{2R^3} = \frac{\rho(3R^2 - r^2)}{6\varepsilon_0}$

At centre $r = 0$ so, $V_{\text{centre}} = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{3}{2}V_s$

i.e., $V_{centre} > V_{surface} > V_{out}$

Graph



(A) **(B)**

Fig. 18.33

(8) **Infinite thin plane sheet of charge :** Consider a thin infinite non-conducting plane sheet having uniform surface charge density is σ. Electric field and potential near the sheet is as follows

$E = \frac{\sigma}{2\varepsilon_0}$ $(E \propto r^o)$

and $V = -\frac{\sigma r}{2\varepsilon_0} + C$

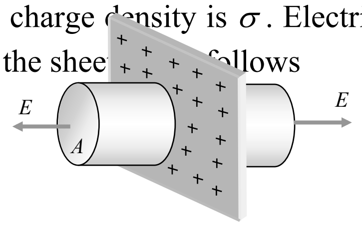


Fig. 18.34

(9) **Electric field due to two thin infinite plane parallel sheet of charge :** Consider two large, uniformly charged parallel. Plates A and B, having surface charge densities are σ_A and σ_B respectively. Suppose net electric field at point P, Q and R is to be calculated.

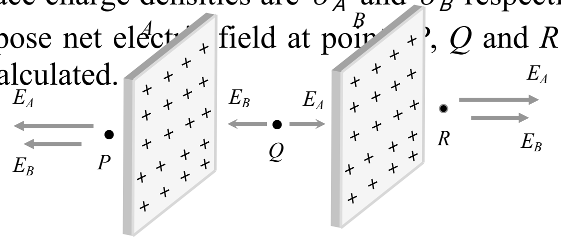


Fig. 18.35

At P, $E_P = -(E_A + E_B) = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

At Q, $E_Q = (E_A - E_B) = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$

At R, $E_R = (E_A + E_B) = \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

Special case

(i) If $\sigma_A = \sigma_B = \sigma$ then $E_P = E_R = \sigma/\varepsilon_0$ and $E_q = 0$

(ii) If $\sigma_A = \sigma$ and $\sigma_B = -\sigma$ then $E_P = E_R = 0$ and $E_Q = \sigma/\varepsilon_0$

(10) **Hemispherical charged body**

At centre O, $E = \frac{\sigma}{4\varepsilon_0}$

$V = \frac{\sigma R}{2\varepsilon_0}$

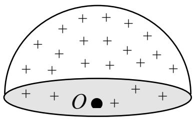


Fig. 18.36

(11) **Uniformly charged disc :** At a distance x from centre O on it's axis

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + R^2} - x\right]$$

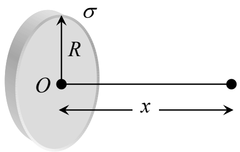


Fig. 18.37

If $x \to 0$, $E \simeq \frac{\sigma}{2\varepsilon_0}$ *i.e.* for points situated near the disc, it behaves as an infinite sheet of charge.

Potential Due to Concentric Spheres

(1) If two concentric conducting shells of radii r_1 and $r_2 (r_2 > r_1)$ carrying uniformly distributed charges Q_1 and Q_2 respectively. Potential at the surface of each shell

$$V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{r_2}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{r_2}$$

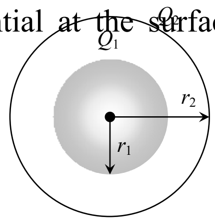


Fig. 18.38

(2) The figure shows three conducting concentric shell of radii a, b and c ($a < b < c$) having charges Q_a, Q_b and Q_c respectively

Potential at A;

$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{a} + \frac{Q_b}{b} + \frac{Q_c}{c}\right]$$

Potential at B;

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{b} + \frac{Q_b}{b} + \frac{Q_c}{c}\right]$$

Potential at C;

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{c} + \frac{Q_b}{c} + \frac{Q_c}{c}\right]$$

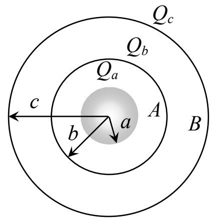


Fig. 18.39

(3) The figure shows two concentric spheres having radii r_1 and r_2 respectively ($r_2 > r_1$). If charge on inner sphere is $+Q$ and outer sphere is earthed then

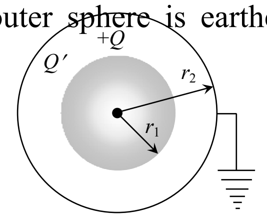


Fig. 18.40

(i) Potential at the surface of outer sphere

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q'}{r_2} = 0$$

$\Rightarrow Q' = -Q$

(ii) Potential of the inner sphere

$$V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{r_2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

(4) In the above case if outer sphere is given a charge $+Q$ and inner sphere is earthed then

(i) In this case potential at the surface of inner sphere is zero, so if Q' is the charge induced on inner sphere

then $V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q'}{r_1} + \frac{Q}{r_2}\right] = 0$

i.e., $Q' = -\frac{r_1}{r_2}Q$

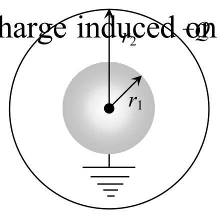


Fig. 18.41

(Charge on inner sphere is less than that of the outer sphere.)

(ii) Potential at the surface of outer sphere

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q'}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_2}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0 r_2}\left[-Q\frac{r_1}{r_2}+Q\right] = \frac{Q}{4\pi\varepsilon_0 r_2}\left[1-\frac{r_1}{r_2}\right]$$

Relation Between Electric Field and Potential

(1) In an electric field rate of change of potential with distance is known as **potential gradient.**

(2) Potential gradient is a vector quantity and it's direction is opposite to that of electric field.

(3) Potential gradient relates with electric field according to the following relation $E = -\frac{dV}{dr}$; This relation gives another unit of electric field is $\frac{volt}{meter}$.

(4) In the above relation negative sign indicates that in the direction of electric field potential decreases.

(5) Negative of the slope of the *V*-*r* graph denotes intensity of electric field *i.e.* $\tan\theta = \frac{V}{r} = -E$

(6) In space around a charge distribution we can also write $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

where $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$ and $E_z = -\frac{\partial V}{\partial z}$

(7) With the help of formula $E = -\frac{dV}{dr}$, potential difference between any two points in an electric field can be determined by knowing the boundary conditions

$$dV = -\int_{r_1}^{r_2}\vec{E}.\vec{dr} = -\int_{r_1}^{r_2} E.\,dr\cos\theta$$

Electric Lines of Force

(1) **Definition :** The electric field in a region is represented by continuous lines (also called lines of force). Field line is an imaginary line along which a positive test charge will move if left free.

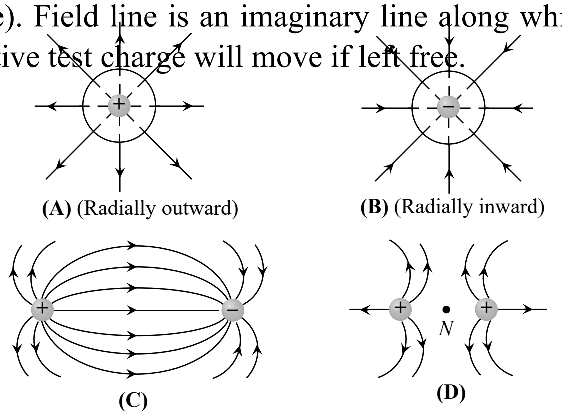


Fig. 18.42

(2) **Properties of electric lines of force**

(i) Electric field lines come out of positive charge and go into the negative charge.

(ii) Tangent to the field line at any point gives the direction of the field at that point.

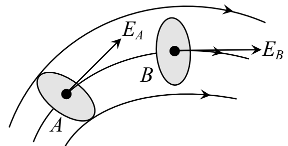


Fig. 18.43

(iii) Field lines never intersect each other.

(iv) Field lines are always normal to conducting surface.

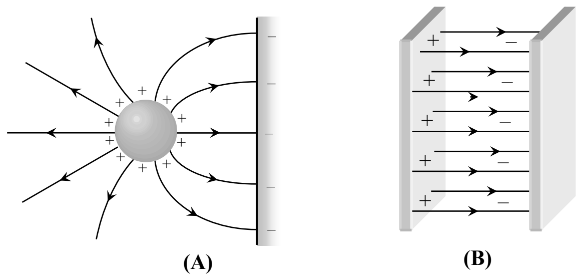


Fig. 18.44

(v) Field lines do not exist inside a conductor.

(vi) The electric field lines never form closed loops. (While magnetic lines of forces form closed loop)

(vii) The number of lines originating or terminating on a charge is proportional to the magnitude of charge *i.e.* $|Q| \propto$ number of lines. In the following figure $|Q_A| > |Q_B|$

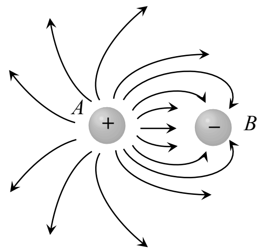
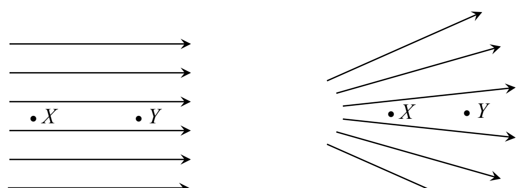


Fig. 18.45

(ix) If the lines of forces are equidistant and parallel straight lines the field is uniform and if either lines of force are not equidistant or straight



line or both the field will be non uniform, also the density of field lines is proportional to the strength of the electric field.

Equipotential Surface

For a given charge distribution, locus of all points having same potential is called "equipotential surface" regarding equipotential surface following points should keep in mind :

(1) The density of the equipotential lines gives an idea about the magnitude of electric field. Higher the density larger the field strength.

(2) The direction of electric field is perpendicular to the equipotential surfaces or lines.

(3) The equipotential surfaces produced by a point charge or a spherically charge distribution are a family of concentric spheres.

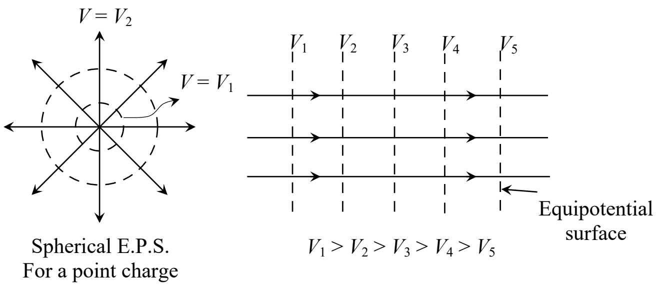


Fig. 18.47

(4) For a uniform electric field, the equipotential surfaces are a family of plane perpendicular to the field lines.

(5) A metallic surface of any shape is an equipotential surface.

(6) Equipotential surfaces can never cross each other

(7) The work done in moving a charge along an equipotential surface is always zero.

Motion of Charge Particle in Electric Field

(1) **When charged particle initially at rest is placed in the uniform field**

Suppose a charge particle having charge Q and mass m is initially at rest in an electric field of strength E. The particle will experience an electric force which causes it's motion.

(i) **Force and acceleration :** The force experienced by the charged particle is $F = QE$.

Acceleration produced by this force is $a = \frac{F}{m} = \frac{QE}{m}$

(ii) **Velocity :** Suppose at point A particle is at rest and in time t, it reaches the point B where it's velocity becomes v. Also if ΔV = Potential difference between A and B, S = Separation between A and B

$$\Rightarrow \quad v = \frac{QEt}{m} = \sqrt{\frac{2Q\Delta V}{m}}$$

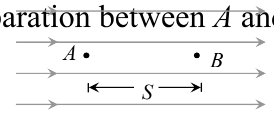


Fig. 18.48

(iii) **Momentum :** Momentum $p = mv$, $p = m \times \frac{QEt}{m} = QEt$

or $p = m \times \sqrt{\frac{2Q\Delta V}{m}} = \sqrt{2mQ\Delta V}$

(iv) **Kinetic energy :** Kinetic energy gained by the particle in time t is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{QEt}{m}\right)^2 = \frac{Q^2E^2t^2}{2m}$

or $K = \frac{1}{2}m \times \frac{2QV}{m} = Q\Delta V$

(v) **Work done :** According to work energy theorem we can say that gain in kinetic energy = work done in displacement of charge *i.e.* $W = Q\Delta V$

where ΔV = Potential difference between the two position of charge Q. ($\Delta V = \vec{E}.\Delta\vec{r} = E\Delta r\cos\theta$ where θ is the angle between direction of electric field and direction of motion of charge).

If charge Q is given a displacement $\vec{r} = (r_1\hat{i} + r_2\hat{j} + r_3\hat{k})$ in an electric field $\vec{E} = (E_1\hat{i} + E_2\hat{j} + E_3\hat{k})$. The work done is $W = Q(\vec{E}.\vec{r}) = Q(E_1r_1 + E_2r_2 + E_3r_3)$.

Work done in displacing a charge in an electric field is path independent.

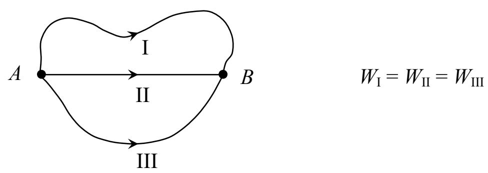


Fig. 18.49

(2) **When a charged particle enters with an initial velocity at right angle to the uniform field**

When charged particle enters perpendicularly in an electric field, it describe a parabolic path as shown

(i) **Equation of trajectory :** Throughout the motion particle has uniform velocity along x-axis and horizontal displacement (x) is given by the equation $x = ut$

Since the motion of the particle is accelerated along y–axis

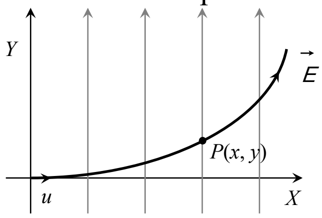


Fig. 18.50

So $y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{u}\right)^2$; this is the equation of parabola which shows $y \propto x^2$

(ii) **Velocity at any instant :** At any instant t, $v_x = u$ and $v_y = \frac{QEt}{m}$ so $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{Q^2E^2t^2}{m^2}}$

If β is the angle made by v with x-axis than

$$\tan\beta = \frac{v_y}{v_x} = \frac{QEt}{mu}.$$

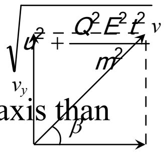


Fig. 18.51

Equilibrium of Charges

(1) **Definition :** A charge is said to be in equilibrium, if net force acting on it is zero. A system of charges is said to be in equilibrium if each charge is separately in equilibrium.

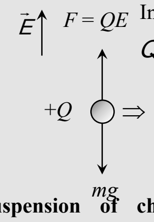
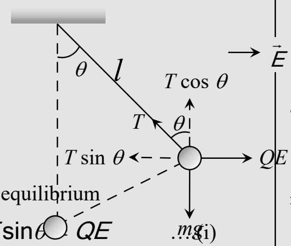
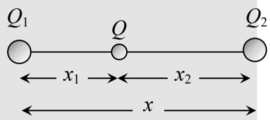
(2) **Type of equilibrium :** Equilibrium can be divided in following type:

(i) **Stable equilibrium :** After displacing a charged particle from it's equilibrium position, if it returns back then it is said to be in stable equilibrium. If U is the potential energy then in case of stable equilibrium $\frac{d^2U}{dx^2}$ is positive *i.e., U* is minimum.

(ii) **Unstable equilibrium :** After displacing a charged particle from it's equilibrium position, if it never returns back then it is said to be in unstable equilibrium and in unstable equilibrium $\frac{d^2U}{dx^2}$ is negative *i.e., U* is maximum.

(iii) **Neutral equilibrium :** After displacing a charged particle from it's equilibrium position if it neither comes back, nor moves away but remains in the position in which it was kept it is said to be in neutral equilibrium and in neutral equilibrium $\frac{d^2U}{dx^2}$ is zero *i.e., U* is constant

Table 18.5 : Different cases of equilibrium of charge

Suspended charge	System of three collinear charge
Freely suspended charge  In equilibrium $QE = mg$ $\Rightarrow E = \frac{mg}{Q}$ **Suspension of charge from string**  In equilibrium $T\sin\theta = QE$ ....(i) $T\cos\theta = mg$ ....(ii) From equations (i) and (ii) $T = \sqrt{(QE)^2 + (mg)^2}$ and $\tan\theta = \frac{QE}{mg}$	In the following figure three charges Q_1, Q and Q_2 are kept along a straight line, charge Q will be in equilibrium if and only if |Force applied by charge Q_1| = |Force applied by charge Q_2|  *i.e.* $\frac{Q_1Q}{x_1^2} = \frac{Q_2Q}{x_2^2}$ $\Rightarrow \frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$ This is the necessary condition for Q to be in equilibrium. If all the three charges (Q_1, Q and Q_2) are similar, Q will be in stable equilibrium. If extreme charges are similar while charge Q is of different nature so Q will be in unstable equilibrium.

Time Period of Oscillation of a Charged Body

(1) **Simple pendulum based :** If a simple pendulum having length l and mass of bob m oscillates about it's mean position than it's time period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$

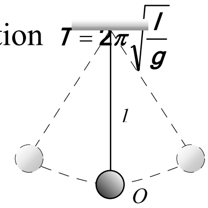


Fig. 18.52

Case-1 : If some charge say $+Q$ is given to bob and an electric field E is applied in the direction as shown in figure then equilibrium position of charged bob (point charge) changes from O to O'.

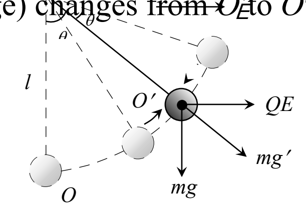


Fig. 18.53

On displacing the bob from it's equilibrium position $0'$. It will oscillate under the effective acceleration g', where

$mg' = \sqrt{(mg)^2 + (QE)^2}$ $\Rightarrow g' = \sqrt{g^2 + (QE/m)^2}$. Hence the new time period is $T_1 = 2\pi\sqrt{\dfrac{l}{g'}}$

$$= 2\pi\sqrt{\frac{l}{\left(g^2 + (QE/m)^2\right)^{\frac{1}{2}}}}$$

Since $g' > g$, so $T_1 < T$ *i.e.* time period of pendulum will decrease.

Case-2 : If electric field is applied in the downward direction then.

Effective acceleration

$g' = g + QE/m$

So new time period

$$T_2 = 2\pi\sqrt{\frac{l}{g + (QE/m)}}$$

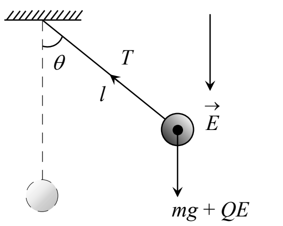


Fig. 18.54

$T_2 < T$

Case-3 : In case 2 if electric field is applied in upward direction then, effective acceleration.

$g' = g - QE/m$

So new time period

$$T_3 = 2\pi\sqrt{\frac{l}{g - (QE/m)}}$$

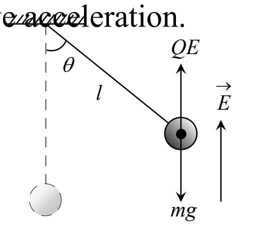


Fig. 18.55

$T_3 > T$

(2) **Charged circular ring :** A thin stationary ring of radius R has a positive charge $+Q$ unit. If a negative charge $-q$ (mass m) is placed at a small distance x from the centre. Then motion of the particle will be simple harmonic motion.

Having time period $T = 2\pi\sqrt{\dfrac{4\pi\varepsilon_0 mR^3}{Qq}}$

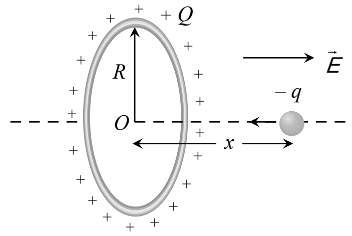


Fig. 18.56

(3) **Spring mass system :** A block of mass m containing a negative charge $-Q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant k as shown. If electric field E applied as shown in figure the block experiences an electric force, hence spring compress and block comes in new position. This is called the equilibrium position of block under the influence of electric field. If block compressed further or stretched, it execute oscillation having time period $T = 2\pi\sqrt{\dfrac{m}{k}}$.

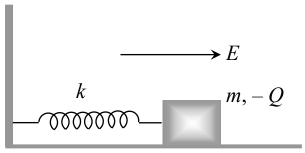


Fig. 18.57

Maximum compression in the spring due to electric field $= \dfrac{QE}{k}$

Neutral Point and Zero Potential

A neutral point is a point where resultant electrical field is zero.

(1) **Neutral point Due to a system of two like point charge :** For this case neutral point is obtained at an internal point along the line joining two like charges.

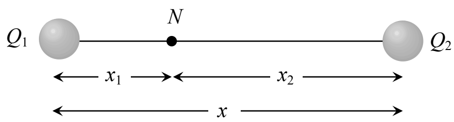


Fig. 18.58

If N is the neutral point at a distance x_1 from Q_1 and at a distance $x_2 (= x - x_1)$ from Q_2 then

At N **|E.F. due to Q_1| = |E.F. due to Q_2|**

i.e., $\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q_1}{x_1^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q_2}{x_2^2} \Rightarrow \dfrac{Q_1}{Q_2} = \left(\dfrac{x_1}{x_2}\right)^2$

Short Trick : $x_1 = \dfrac{x}{\sqrt{Q_2/Q_1}+1}$ and $x_2 = \dfrac{x}{\sqrt{Q_1/Q_2}+1}$

(2) **Neutral point due to a system of two unlike point charge :** For this condition neutral point lies at an external point along the line joining two unlike charges. Suppose two unlike charge Q_1 and Q_2 separated by a distance *x* from each other.

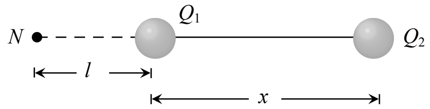


Fig. 18.59

Here neutral point lies outside the line joining two unlike charges and also it lies nearer to charge which is smaller in magnitude.

If $|Q_1| < |Q_2|$ then neutral point will be obtained on the side of Q_1, suppose it is at a distance *l* from Q_1 so $l = \dfrac{x}{\left(\sqrt{Q_2/Q_1}-1\right)}$

(3) **Zero potential due to a system of two point charge**

(i) If both charges are like then resultant potential is not zero at any finite point.

(ii) If the charges are unequal and unlike then all such points where resultant potential is zero lies on a closed curve.

(iii) Along the line joining the two charge, two such points exist, one lies inside and one lies outside the charges on the line joining the charges. Both the above points lie nearer the smaller charge.

For internal point

(It is assumed that $|Q_1| < |Q_2|$).

At *P*, $\dfrac{Q_1}{x_1} = \dfrac{Q_2}{(x - x_1)}$

$\Rightarrow \quad x_1 = \dfrac{x}{(Q_2/Q_1 + 1)}$

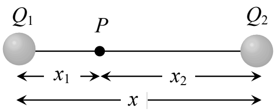


Fig. 18.60

For External point

At *P*, $\dfrac{Q_1}{x_1} = \dfrac{Q_2}{(x + x_1)}$

$\Rightarrow \quad x_1 = \dfrac{x}{(Q_2/Q_1 - 1)}$

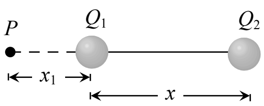


Fig. 18.61

Electrostatic Potential Energy

(1) Work done in bringing the given charge from infinity to a point in the electric field is known as potential energy of the charge. Potential can also be written as potential energy per unit charge. *i.e.* $V = \dfrac{W}{Q} = \dfrac{U}{Q}$.

(2) **Potential energy of a system of two charge**

Potential energy of Q_1 = Potential energy of Q_2 = potential energy of system $U = k\dfrac{Q_1Q_2}{r}$

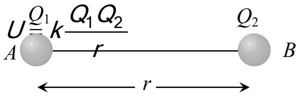


Fig. 18.62

In C.G.S. $U = \dfrac{Q_1Q_2}{r}$

(3) **Potential energy of a system of *n* charge**

It is given by $U = \dfrac{k}{2}\sum_{\substack{i,j \\ i \neq j}}^{n} \dfrac{Q_iQ_j}{r_{ij}}$ $\quad \left(k = \dfrac{1}{4\pi\varepsilon_0}\right)$

The factor of $\dfrac{1}{2}$ is applied only with the summation sign because on expanding the summation each pair is counted twice.

For a system of 3 charges $U = k\left(\dfrac{Q_1Q_2}{r_{12}} + \dfrac{Q_2Q_3}{r_{23}} + \dfrac{Q_1Q_3}{r_{13}}\right)$

(4) **Work energy relation :** If a charge moves from one position to another position in an electric field so it's potential energy change and work done by external force for this change is $W = U_f - U_i$

(5) **Electron volt (*eV*) :** It is the smallest practical unit of energy used in atomic and nuclear physics. As electron volt is defined as "the energy acquired by a particle having one quantum of charge (1*e*), when accelerated by 1*volt*" *i.e.* $1eV = 1.6 \times 10^{-19}\,C \times \dfrac{1J}{C} = 1.6 \times 10^{-19}\,J = 1.6 \times 10^{-12}\,erg$

(6) **Electric potential energy of a uniformly charged sphere :** Consider a uniformly charged sphere of radius *R* having a total charge *Q*. The electric potential energy of this sphere is equal to the work done in bringing the charges from infinity to assemble the sphere. $U = \dfrac{3Q^2}{20\pi\varepsilon_0 R}$

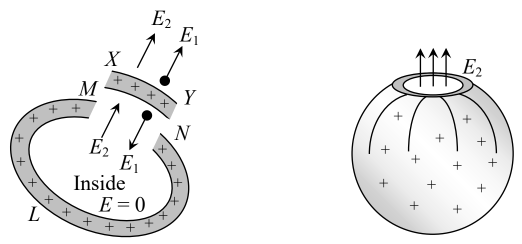
(7) **Electric potential energy of a uniformly**

charged thin spherical shell : It is given by the following formula $U = \frac{Q^2}{8\pi\varepsilon_0 R}$

(8) **Energy density :** The energy stored per unit volume around a point in an electric field is given by $U_e = \frac{U}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$. If in place of vacuum some medium is present then $U_e = \frac{1}{2}\varepsilon_0\varepsilon_r E^2$

Force on a Charged Conductor

To find force on a charged conductor (due to repulsion of like charges) imagine a small part *XY* to be cut and just separated from the rest of the conductor *MLN*. The field in the cavity due to the rest of the conductor is E_2, while field due to small part is E_1. Then



(A) (B)

Fig. 18.63

Inside the conductor $E = E_1 - E_2 = 0$ or $E_1 = E_2$

Outside the conductor $E = E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$

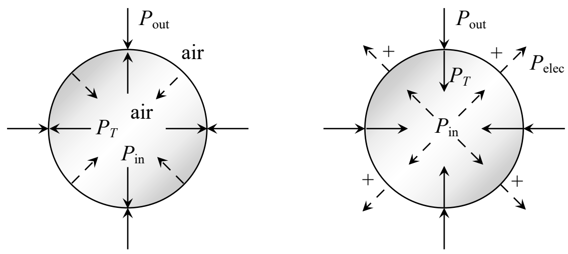
Thus $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$

(1) To find force, imagine charged part *XY* (having charge $\sigma\, dA$ placed in the cavity *MN* having field E_2). Thus force $dF = (\sigma\, dA)E_2$ or $dF = \frac{\sigma^2}{2\varepsilon_0}dA$. The force per unit area or electrostatic pressure p = $\frac{dF}{dA} = \frac{\sigma^2}{2\varepsilon_0}$

(2) The force is always outwards as $(\pm\sigma)^2$ is positive *i.e.,* whether charged positively or negatively, this force will try to expand the charged body. [A soap bubble or rubber balloon expands on charging to it (charge of any kind + or –)].

Equilibrium of Charged Soap Bubble

(1) For a charged soap bubble of radius *R* and surface tension *T* and charge density σ. The pressure due to surface tension $4\frac{T}{R}$ and atmospheric pressure P_{out} act radially inwards and the electrical pressure (P_{el}) acts radially outward.



(A) Uncharged bubble **(B)** Charged bubble

Fig. 18.64

(2) The total pressure inside the soap bubble

$$P_{\text{in}} = P_{\text{out}} + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$$

(3) Excess pressure inside the charged soap bubble

$$P_{\text{in}} - P_{\text{out}} = P_{\text{excess}} = \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}.$$

(4) If air pressure inside and outside are assumed equal then $P_{\text{in}} = P_{\text{out}}$ *i.e.,* $P_{\text{excess}} = 0$. So, $\frac{4T}{R} = \frac{\sigma^2}{2\varepsilon_0}$

(i) Charge density : Since $\frac{4T}{R} = \frac{\sigma^2}{2\varepsilon_0} \Rightarrow$ $\sigma = \sqrt{\frac{8\varepsilon_0 T}{R}} = \sqrt{\frac{2T}{\pi kR}}$

(ii) Radius of bubble $R = \frac{8\varepsilon_0 T}{\sigma^2}$

(iii) Surface tension $T = \frac{\sigma^2 R}{8\varepsilon_0}$

(iv) Total charge on the bubble $Q = 8\pi R\sqrt{2\varepsilon_0 TR}$

(v) Electric field intensity at the surface of the bubble

$$E = \sqrt{\frac{8T}{\varepsilon_0 R}} = \sqrt{\frac{32\pi kT}{R}}$$

(vi) Electric potential at the surface $V = \sqrt{32\pi RTk} = \sqrt{\frac{8RT}{\varepsilon_0}}$

Electric Dipole

System of two equal and opposite charges separated by a small fixed distance is called a dipole.

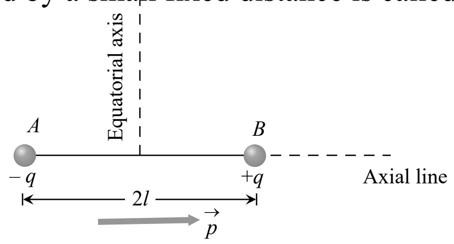


Fig. 18.65

(1) **Dipole moment :** It is a vector quantity and is directed from negative charge to positive charge along the axis. It is denoted as $\vec{p}$ and is defined as the product of the magnitude of either of the charge and the dipole length *i.e.* $\vec{p} = q(2\vec{l})$

Its S.I. unit is ***coulomb-metre*** or ***Debye*** (1 *Debye* = 3.3 × 10^{-30} *C* × *m*) and its dimensions are $M^0L^1T^1A^1$.

(2) When a dielectric is placed in an electric field, its atoms or molecules are considered as tiny dipoles.

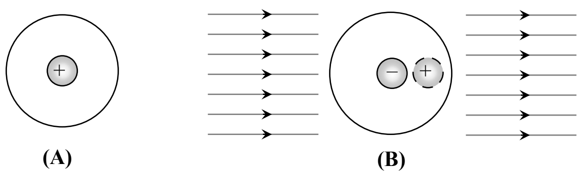


Fig. 18.66

Water (H_2O), Chloroform ($CHCl_3$), Ammonia (NH_3), HCl, CO molecules are some example of permanent electric dipole.

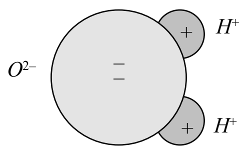


Fig. 18.67

(3) **Electric field and potential due to an electric dipole :** If *a*, *e* and *g* are three points on axial, equatorial and general position at a distance *r* from the centre of dipole

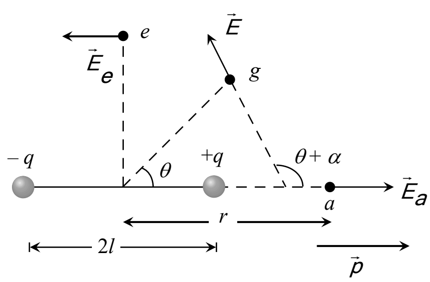


Fig. 18.68

(i) **At axial point :** Electric field and potential are given as

$$E_a = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \text{ (directed from } -q \text{ to } +q)$$

$V_a = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$. Angle between $\vec{E}_a$ and $\vec{p}$ is 0°.

(ii) **At equatorial point** : $E_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$ (directed from $+q$ to $-q$) and $V_e = 0$. Angle between $\vec{E}_e$ and $\vec{p}$ is 180°.

(iii) **At general point :** $E_g = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}\sqrt{(3\cos^2\theta + 1)}$ and $V_g = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$. Angle between $\vec{E}$ and $\vec{p}$ is $(\theta + \alpha)$ (where $\tan\alpha = \frac{1}{2}\tan\theta$)

(4) **Dipole in an external electric field :** When a dipole is kept in an uniform electric field. The net force experienced by the dipole is zero as shown in fig.

The net torque experienced by the dipole is

$\tau = pE\sin\theta$

$\vec{\tau} = \vec{p} \times \vec{E}$

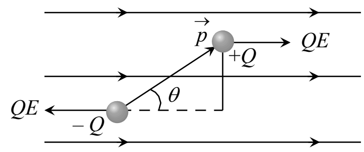


Fig. 18.69

Hence due to torque so produced, dipole align itself in the direction of electric field. This is the position of stable equilibrium of dipole.

(i) **Work done in rotation :** Suppose initially, dipole is kept in a uniform electric field at an angle θ_1. Now to turn it through an angle θ_2 (with the field)

Work done $W = pE(\cos\theta_1 - \cos\theta_2)$

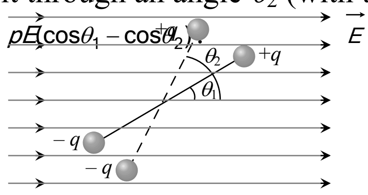
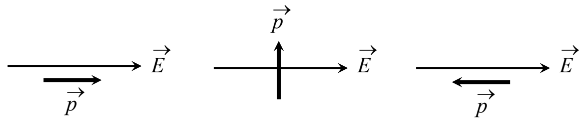


Fig. 18.70

If $\theta_1 = 0^o$ and $\theta_2 = \theta$ *i.e.* initially dipole is kept along the field then it turn through θ so work done $W = pE(1-\cos\theta)$

(ii) **Potential energy of dipole :** It is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction, *i.e.* from above formula of work.

If $\theta_1 = 90^o$ and $\theta_2 = \theta \Rightarrow W = U = -pE\cos\theta$



$\theta = 0^o$	$\theta = 90^O$	$\theta = 180^o$
Stable equilibrium	Not in equilibrium	Unstable equilibrium
$\tau = 0$	$\tau_{max} = pE$	$\tau = 0$
$W = 0$	$W = pE$	$W_{max} = 2pE$
$U_{min} = -pE$	$U = 0$	$U_{max} = pE$

(iii) **Equilibrium of dipole :** When $\theta = 0^o$ *i.e.* dipole is placed along the electric field it is said to be in stable equilibrium, because after turning it through a small angle, dipole tries to align itself again in the direction of electric field.

When $\theta = 180^o$ *i.e.* dipole is placed opposite to electric field, it is said to be in unstable equilibrium.

(iv) **Oscillation of dipole :** In a uniform electric field if a dipole is slightly displaced from it's stable equilibrium position it executes angular SHM having period of oscillation.

$T = 2\pi\sqrt{\frac{I}{pE}}$ where I = moment of inertia of dipole about the axis passing through it's centre and perpendicular to it's length.

(5) **Electric dipole in non-uniform electric field :** In non-uniform electric field $F_{net} \neq 0, \tau_{net} \neq 0$

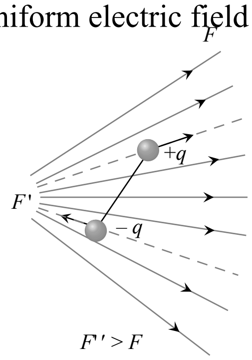
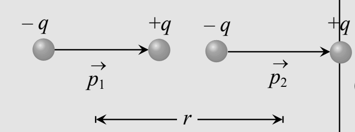
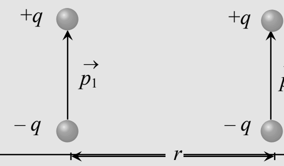
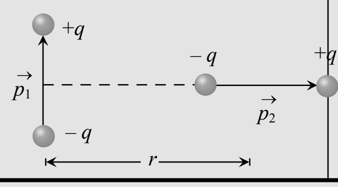


Fig. 18.71

Motion of the dipole is combination of translatory and rotatory motion

Table 18.6 : Dipole-dipole interaction

Relative position of dipole	Force	Potential energy
	$\frac{1}{4\pi\varepsilon_0}\cdot\frac{6p_1p_2}{r^4}$ (attractive)	$\frac{1}{4\pi\varepsilon_0}\cdot\frac{2p_1p_2}{r^3}$
	$\frac{1}{4\pi\varepsilon_0}\cdot\frac{3p_1p_2}{r^4}$ (repulsive)	$\frac{1}{4\pi\varepsilon_0}\cdot\frac{p_1p_2}{r^3}$
	$\frac{1}{4\pi\varepsilon_0}\cdot\frac{3p_1p_2}{r^4}$ (perpendicular to r)	0

Electric Flux

Electric flux is a measure of 'flow' of electric field through a surface. It is equal to the product of an area element and the perpendicular component of $\vec{E}$, integrated over a surface.

(1) Flux of electric field $\vec{E}$ through any area $\vec{A}$ is defined as.

$\phi = E.A\cos\theta$ or

$\phi = \vec{E}.\vec{A}$

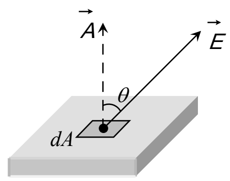
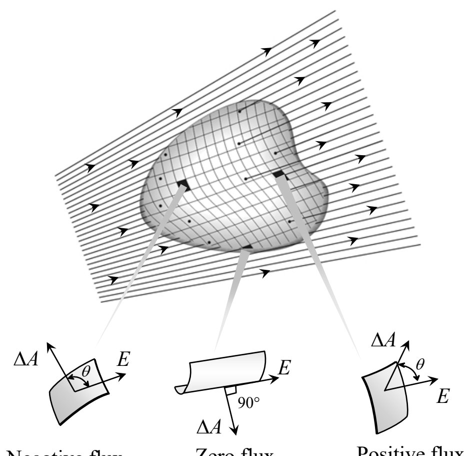


Fig. 18.72

(2) In case of variable electric field or curved area. $\phi = \int \vec{E}.\vec{dA}$

(3) It's S.I. Unit is ($Volt \times m$) or $\frac{N\text{-}m^2}{C}$

(4) For a closed body outward flux is taken to be positive while inward flux is taken to be negative.



Gauss's Law and it's Application

(1) According to this law, the total flux linked with a closed surface called Gaussian surface. (The surface need not be a real physical surface, it can also be an hypothetical one) is $(1/\varepsilon_0)$ times the charge enclosed by the closed surface *i.e.*, $\phi = \oint_s \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_o}(Q_{enc})$

(2) Electric field in $\oint \vec{E}.\vec{dA}$ is complete electric field. It may be partly due to charge with in the surface and partly due to charge outside the surface. However if there is no charge enclosed in the Gaussian surface, then $\oint \vec{E}.\vec{dA} = 0$.

(3) The electric field $\vec{E}$ is resulting from all charge, both those inside and those outside the Gaussian surface.

(Keep in mind, the electric field due to a charge outside the Gaussian surface contributes zero net flux through the surface, Because as many lines due to that charge enter the surface as leave it).

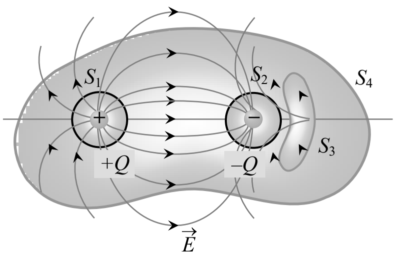


Fig. 18.74

Flux from surface $S_1 = +\frac{Q}{\varepsilon_0}$, Flux from surface $S_2 = -\frac{Q}{\varepsilon_0}$, and flux from S_3= flux from surface $S_4 = 0$

Application of Gauss's law : See flux emergence in the following cases

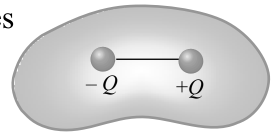


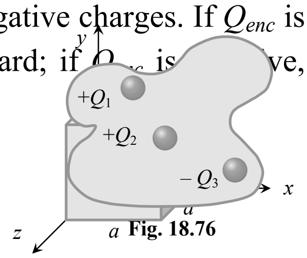
Fig. 18.75

(1) If a dipole is enclosed by a surface

$\because Q_{enc} = 0$

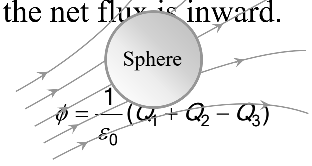
$\Rightarrow \phi = 0$

(2) The net charge Q_{enc} is the algebraic sum of all the enclosed positive, and negative charges. If Q_{enc} is positive the net flux is outward; if Q_{enc} is negative, the net flux is inward.



$\phi = \frac{1}{\varepsilon_0}(Q_1 + Q_2 - Q_3)$

Fig. 18.76



(A) $\phi_T = 0$ (B) $\phi_{in} = \phi_{out} = Ea^2 \Rightarrow \phi_T = 0$

Fig. 18.77

(3) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non-uniform) total flux linked with it will be zero

(4) If a hemispherical body is placed in uniform electric field then flux linked with the curved surface calculated as follows

$\phi_{Curved} + \phi_{Circular} = 0$

$\phi_{Curved} = -\phi_{Circular}$

$= -(E \times \pi R^2 \cos 180°)$

$= +\pi R^2 E$

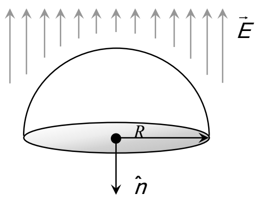


Fig. 18.78

(5) If a hemispherical body is placed in non-uniform electric field as shown below, then flux linked with the circular surface calculated as follows

$\phi_{Circular} = -\phi_{Curved}$

$\phi_{Circular} = -(E \times 2\pi R^2 \cos 0°)$

$= -2\pi R^2 E$

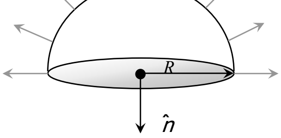


Fig. 18.79

(6) If charge is kept at the centre of cube

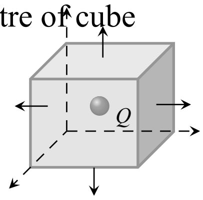
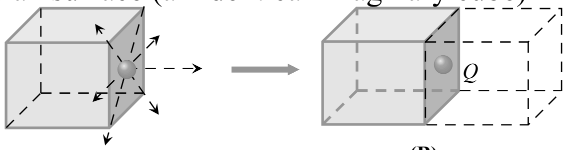


Fig. 18.80

$\phi_{total} = \frac{1}{\varepsilon_0}.(Q)$ $\phi_{face} = \frac{Q}{6\varepsilon_0}$

$\phi_{corner} = \frac{Q}{8\varepsilon_0}$ $\phi_{edge} = \frac{Q}{12\varepsilon_0}$

(7) If charge is kept at the centre of a face : First we should enclosed the charge by assuming a Gaussian surface (an identical imaginary cube)



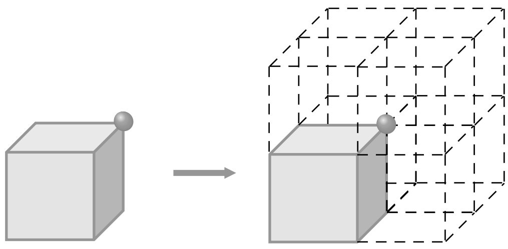
(A) (B)

Fig. 18.81

Total flux emerges from the system (Two cubes) $\phi_{total} = \frac{Q}{\varepsilon_0}$

Flux from given cube (*i.e.* from 5 face only) $\phi_{cube} = \frac{Q}{2\varepsilon_0}$

(8) If a charge is kept at the corner of a cube



(A) (B)

Fig. 18.82

For enclosing the charge seven more cubes are required so total flux from the 8 cube system is $\phi_T = \frac{Q}{\varepsilon_0}$. Flux from given cube $\phi_{cube} = \frac{Q}{8\varepsilon_0}$. Flux from one face opposite to change, of the given cube $\phi_{face} = \frac{Q/8\varepsilon_0}{3} = \frac{Q}{24\varepsilon_0}$ (Because only three faces are seen).

(9) A long straight wire of charge density λ penetrates a hollow body as shown. The flux emerges from the body is

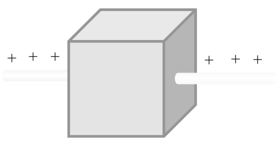
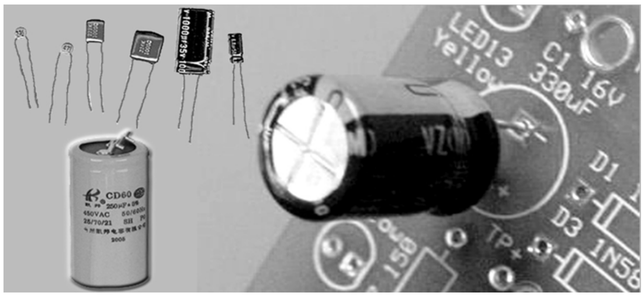


Fig. 18.83

$\phi = \lambda \times$ (Length of the wire inside the body)

Capacitance



(1) **Capacitance of a conductor :** Charge given to a conductor increases it's potential *i.e.*, $Q \propto V \Rightarrow Q = CV$

Where C is a proportionality constant, called capacity or capacitance of conductor. Hence capacitance is the ability of conductor to hold the charge.

(2) It's S.I. unit is $\frac{Coulomb}{Volt} = Farad\ (F)$

Smaller S.I. units are *mF*, *μF*, *nF* and *pF* ($1mF = 10^{-3}F$, $1\mu F = 10^{-6}F$, $1nF = 10^{-9}F$, $1pF = 1\mu\mu F = 10^{-12}F$)

(3) It's C.G.S. unit is *Stat Farad* $1F = 9 \times 10^{11}$ *Stat Farad*.

(4) It's dimension : $[C] = [M^{-1}L^{-2}T^4A^2]$.

(5) Capacity of a body is independent of charge given to the body or it's potential raised and depends on shape and size only.

(6) **Capacity of an isolated spherical conductor :** When charge Q is given to a spherical conductor of radius R, then potential at the surface of sphere is

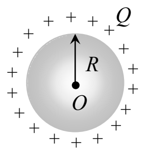


Fig. 18.84

$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow \frac{Q}{V} = 4\pi\varepsilon_0 R$

$C = 4\pi\varepsilon_0 R = \frac{1}{9\times 10^9}.R$

If earth is assumed to be a conducting sphere having radius $R = 6400\ km$. It's theoretical capacitance $C = 711\ \mu F$. But for all practical purpose capacitance of earth is taken infinity and its potential $V = 0$.

(7) **Energy of a charged conductor :** Electrostatic potential energy of a conductor carrying

charge Q, capacitance C and potential V is given by

$$U=\frac{1}{2}QV=\frac{1}{2}CV^2=\frac{Q^2}{2C}$$

Combination of Charged Drops

Suppose we have n identical drops each having Radius – r, Capacitance – c, Charge – q, Potential – v and Energy – u.

If these drops are combined to form a big drop of Radius – R, Capacitance – C, Charge – Q, Potential – V and Energy – U then

(1) **Charge on big drop :** $Q=nq$

(2) **Radius of big drop :** Volume of big drop = n × volume of a single drop *i.e.*, $\frac{4}{3}\pi R^3=n\times\frac{4}{3}\pi r^3$, $R=n^{1/3}r$

(3) **Capacitance of big drop :** $C=n^{1/3}c$

(4) **Potential of big drop :** $V=\frac{Q}{C}=\frac{nq}{n^{1/3}c}$

$V=n^{2/3}v$

(5) **Energy of big drop :** $U=\frac{1}{2}CV^2=\frac{1}{2}(n^{1/3}c)(n^{2/3}v)^2$

$$U=n^{5/3}u$$

(6) **Energy difference :** Total energy of big drop is greater than the total energy all smaller drop. Hence energy difference

$$\Delta U=U-nu=U-n\times\frac{U}{n^{5/3}}=U\left(1-\frac{1}{n^{2/3}}\right)$$

Redistribution of Charges and Loss of Energy

When two charged conductors joined together through a conducting wire, charge begins to flow from one conductor to another from higher potential to lower potential.

This flow of charge stops when they attain the same potential.

Due to flow of charge, loss of energy also takes place in the form of heat through the connecting wire.

Suppose there are two spherical conductors of radii r_1 and r_2, having charge Q_1 and Q_2, potential V_1 and V_2, energies U_1 and U_2 and capacitance C_1 and C_2 respectively.

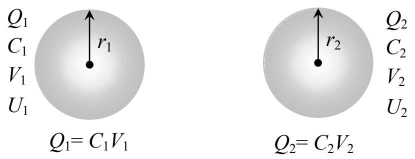


Fig. 18.85

If these two spheres are connected through a conducting wire, then alteration of charge, potential and energy takes place.

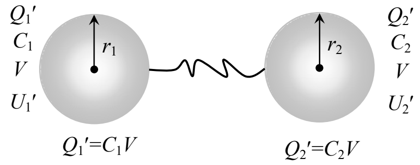


Fig. 18.86

(1) **New charge :** According to the conservation of charge

$$Q_1+Q_2=Q_1'+Q_2'=Q \text{ (say), also } \frac{Q_1'}{Q_2'}=\frac{C_1}{C_2}=\frac{r_1}{r_2}$$

$\Rightarrow \quad Q_2'=Q\left[\frac{r_2}{r_1+r_2}\right]$ and similarly $Q_1'=Q\left[\frac{r_1}{r_1+r_2}\right]$

(2) **Common potential :** Common potential

$$(V)=\frac{\text{Total charge}}{\text{Total capacity}}=\frac{Q_1+Q_2}{C_1+C_2}=\frac{Q_1'+Q_2'}{C_1+C_2}$$

$$=\frac{C_1V_1+C_2V_2}{C_1+C_2}$$

(3) **Energy loss :** The loss of energy due to redistribution of charge is given by

$$\Delta U=U_i-U_f=\frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$$

Capacitor or Condenser

(1) A capacitor is a device that stores electric energy. or A capacitor is a pair of two conductors of any shape, which are close to each other and have equal and opposite charge.

(2) The capacitance of a capacitor is defined as the magnitude of the charge Q on the positive plate divided by the magnitude of the potential difference V between the plates *i.e.*, $C=\frac{Q}{V}$

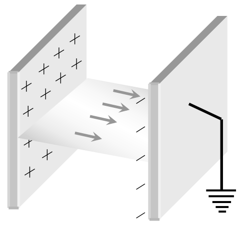


Fig. 18.87

(3) A capacitor get's charged when a battery is connected across the plates. Once capacitor get's fully charged, flow of charge carriers stops in the

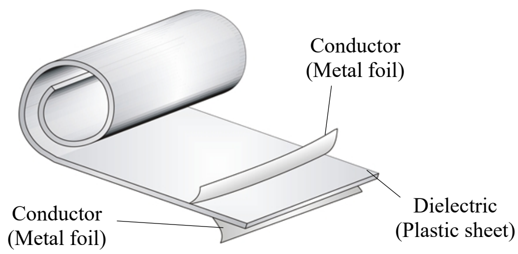
circuit and in this condition potential difference across the plates of capacitor is same as the potential difference across the terminals of battery.

(4) Net charge on a capacitor is always zero, but when we speak of the charge Q on a capacitor, we are referring to the magnitude of the charge on each plate.

(5) **Energy stored :** When a capacitor is charged by a voltage source (say battery) it stores the electric energy. If C = Capacitance of capacitor; Q = Charge on capacitor and V = Potential difference across capacitor then energy stored in capacitor $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$

In charging capacitor by battery half the energy supplied is stored in the capacitor and remaining half energy (1/2 QV) is lost in the form of heat.

Dielectric



Dielectrics are insulating (non-conducting) materials which transmits electric effect without conducting.

Dielectrics are of two types

(1) **Polar dielectrics :** A polar molecule has permanent electric dipole moment ($\vec{p}$) in the absence of electric field also. But a polar dielectric has net dipole moment zero in the absence of electric field because polar mly oriented as shown in figure

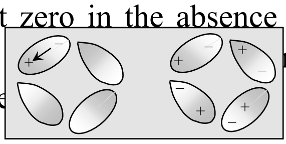


Fig. 18.88

In the presence of electric field polar molecules tends to line up in the direction of electric field, and the substance has finite dipole moment *e.g.* water, Alcohol, CO_2, NH_3, *HCl etc*. are made of polar atoms/molecules.

(2) **Non polar dielectric :** In non-polar molecules, Each molecule has zero dipole moment in its normal state.

When electric field is applied, molecules becomes induced electric dipole *e.g.* N_2, O_2, Benzene, Methane *etc*. are made of non-polar atoms/molecules

In general, any non-conducting, material can be called as a dielectric but broadly non conducting material having non polar molecules referred to as dielectric.

(3) **Polarization of a dielectric slab :** It is the process of inducing equal and opposite charges on the two faces of the dielectric on the application of electric field.

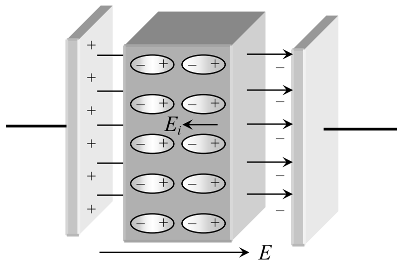


Fig. 18.89

(i) Electric field between the plates in the presence of dielectric medium is $E = E - E_i$ where E = Main field, E' = Induced field.

(ii) Dielectric constant of dielectric medium is defined as :

$$\frac{E}{E'} = \frac{\text{Electric field between the plates with air}}{\text{Electric field between the plates with medium}} = K$$

(iii) K is also known as relative permittivity (ε_r) of the material or **SIC** (Specific Inductive Capacitance)

(4) **Dielectric breakdown and dielectric strength :** If a very high electric field is created in a dielectric,. The dielectric then behaves like a conductor. This phenomenon is known as **dielectric breakdown.**

The maximum value of electric field (or potential gradient) that a dielectric material can tolerate without it's electric breakdown is called it's **dielectric strength.**

S.I. unit of dielectric strength of a material is $\frac{V}{m}$ but practical unit is $\frac{kV}{mm}$.

Capacity of Various Capacitor

(1) **Parallel plate capacitor :** It consists of two parallel metallic plates (may be circular, rectangular, square) separated by a small distance. If A = Effective overlapping area of each plate.

(i) Electric field between the plates : $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

(ii) Potential difference between the plates : $V = E \times d = \frac{\sigma d}{\varepsilon_0}$

(iii) Capacitance : $C = \frac{\varepsilon_0 A}{d}$. In C.G.S. : $C = \frac{A}{4\pi d}$

(iv) If a dielectric medium of dielectric constant K is filled completely between the plates then capacitance increases by K times *i.e.* $C = \frac{K\varepsilon_0 A}{d} \Rightarrow C = KC$

(v) The capacitance of parallel plate capacitor depends on A ($C \propto A$) and d $\left(C \propto \frac{1}{d}\right)$. It does not depend on the charge on the plates or the potential difference between the plates.

(vi) If a dielectric slab is partially filled between the plates

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

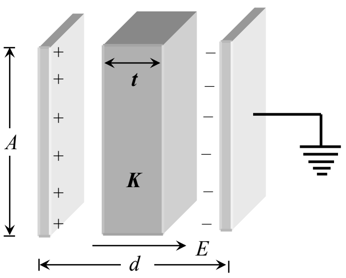


Fig. 18.90

(vii) If a number of dielectric slabs are inserted between the plate as shown

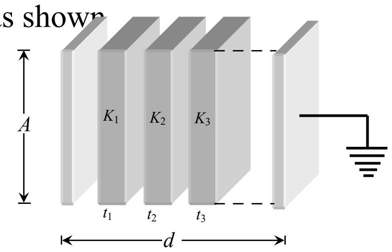


Fig. 18.91

$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + ...) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + ...\right)}$$

(viii) When a metallic slab is inserted between the plates

$$C = \frac{\varepsilon_0 A}{(d - t)}$$

If metallic slab fills the complete space between the plates (*i.e.* $t = d$) or both plates are joined through a metallic wire then capacitance becomes infinite.

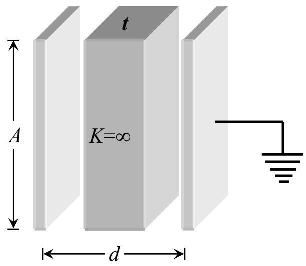


Fig. 18.92

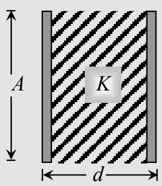
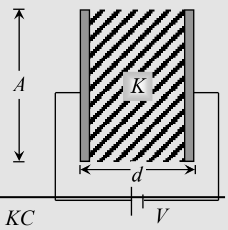
(ix) Force between the plates of a parallel plate capacitor.

$$|F| = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{CV^2}{2d}$$

(x) Energy density between the plates of a parallel plate capacitor.

Energy density $= \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$.

Table 18.7 : Variation of different variable (*Q*, *C*, *V*, *E* and *U*) of parallel plate capacitor

Quantity	Battery is Removed	Battery Remains connected
		
Capacity	$C' = KC$	$C' = KC$
Charge	$Q' = Q$	$Q' = KQ$
Potential	$V' = V/K$	$V' = V$
Intensity	$E' = E/K$	$E' = E$
Energy	$U' = U/K$	$U'' = KU$

(2) **Spherical capacitor :** It consists of two concentric conducting spheres of radii a and b ($a < b$). Inner sphere is given charge $+Q$, while outer sphere is earthed

(i) Potential difference : Between the spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

(ii) Capacitance : $C = 4\pi\varepsilon_0 \cdot \frac{ab}{b-a}$

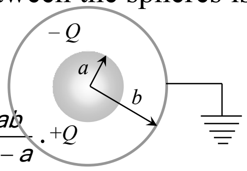


Fig. 18.93

In C.G.S. $C = \frac{ab}{b-a}$. In the presence of dielectric medium (dielectric constant K) between the spheres $C = 4\pi\varepsilon_0 K \frac{ab}{b-a}$

(iii) If outer sphere is given a charge $+Q$ while inner sphere is earthed

Induced charge on the inner sphere $Q = -\frac{a}{b}.Q$ and capacitance of the system $C = 4\pi\varepsilon_0 \cdot \frac{b^2}{b-a}$

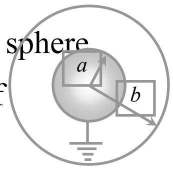


Fig. 18.94

This arrangement is not a capacitor. But it's capacitance is equivalent to the sum of capacitance of spherical capacitor and spherical conductor *i.e.*

$$4\pi\varepsilon_0 \cdot \frac{b^2}{b-a} = 4\pi\varepsilon_0 \frac{ab}{b-a} + 4\pi\varepsilon_0 b$$

(3) **Cylindrical capacitor :** It consists of two concentric cylinders of radii a and b ($a < b$), inner cylinder is given charge $+Q$ while outer cylinder is earthed. Common length of the cylinders is l then

$$C = \frac{2\pi\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)}$$

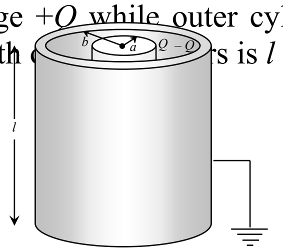


Fig. 18.95

Grouping of Capacitor

(1) **Series grouping**

(i) Charge on each capacitor remains same and equals to the main charge supplied by the battery but potential difference distributes *i.e.* $V = V_1 + V_2 + V_3$

(ii) Equivalent capacitance

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \text{ or } C_{eq} = (C_1^{-1} + C_2^{-1} + C_3^{-1})^{-1}$$

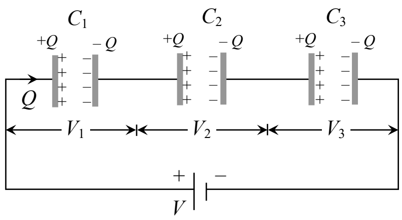


Fig. 18.96

(iii) In series combination potential difference and energy distributes in the reverse ratio of capacitance *i.e.*,

$$V \propto \frac{1}{C} \text{ and } U \propto \frac{1}{C}.$$

(iv) If two capacitors having capacitances C_1 and C_2 are connected in series then $C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\text{Multiplication}}{\text{Addition}}$

$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right).V \text{ and } V_2 = \left(\frac{C_1}{C_1 + C_2}\right).V$$

(v) If n identical capacitors each having capacitances C are connected in series with supply voltage V then Equivalent capacitance $C_{eq} = \frac{C}{n}$ and Potential difference across each capacitor $V' = \frac{V}{n}$.

(vi) If n identical plates are arranged as shown below, they constitute $(n-1)$ capacitors in series. If each capacitors having capacitance $\frac{\varepsilon_0 A}{d}$ then $C_{eq} = \frac{\varepsilon_0 A}{(n-1)d}$

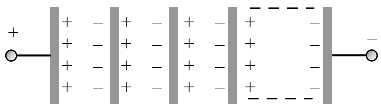


Fig. 18.97

In this situation except two extreme plates each plate is common to adjacent capacitors.

(2) **Parallel grouping**

(i) Potential difference across each capacitor remains same and equal to the applied potential difference but charge distributes *i.e.* $Q = Q_1 + Q_2 + Q_3$

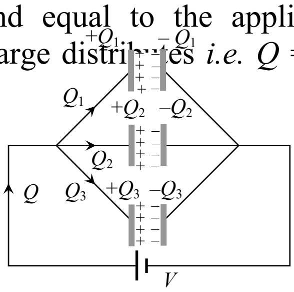


Fig. 18.98

(ii) $C_{eq} = C_1 + C_2 + C_3$

(iii) In parallel combination charge and energy distributes in the ratio of capacitance *i.e.* $Q \propto C$ and $U \propto C$

(iv) If two capacitors having capacitance C_1 and C_2 respectively are connected in parallel then $C_{eq} = C_1 + C_2$

$$Q_1 = \left(\frac{C_1}{C_1 + C_2}\right).Q \text{ and } Q_2 = \left(\frac{C_2}{C_1 + C_2}\right).Q$$

(v) If n identical capacitors are connected in parallel

Equivalent capacitance $C_{eq} = nC$ and Charge on each capacitor $Q = \frac{Q}{n}$

If n identical plates are arranged such that even numbered of plates are connected together and odd numbered plates are connected together, then $(n - 1)$ capacitors will be formed and they will be in parallel grouping.

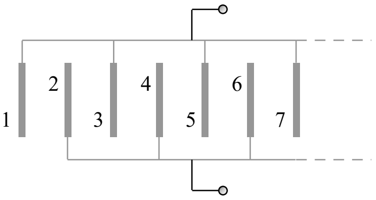


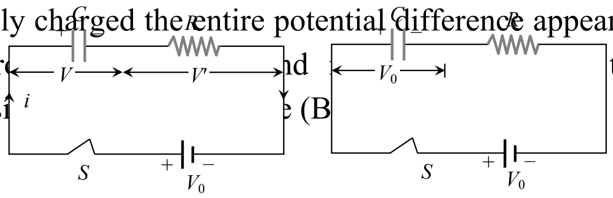
Fig. 18.99

Equivalent capacitance $C = (n-1)\,C$

where C = capacitance of a capacitor $= \frac{\varepsilon_0 A}{d}$

Charging and Discharging of Capacitor in Series *RC* Circuit

As shown in the following figure (A) when switch S is closed, capacitor start charging. In this transient state potential difference appears across capacitor as well as resistor. When capacitor gets fully charged the entire potential difference appeared acr... nd ... the resi... (B



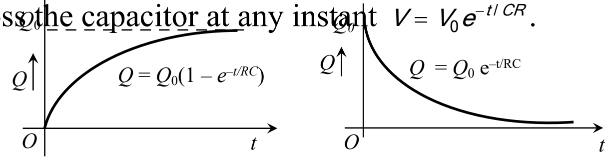
(A) Transient state **(B)** Steady state

Fig. 18.100

(i) **Charging :** In transient state of charging charge on the capacitor at any instant $Q = Q_0\left(1 - e^{\frac{-t}{RC}}\right)$ and potential difference across the capacitor at any instant $V = V_0\left(1 - e^{\frac{-t}{RC}}\right)$

(Here Q and V are the instantaneous values of charge and potential difference while maximum charge on capacitor is $Q_0 = CV_0$)

(ii) **Discharging :** After the completion of charging, if battery is removed capacitor starts discharging. In transient state charge on the capacitor at any instant $Q = Q_0 e^{-t/RC}$ and potential difference across the capacitor at any instant $V = V_0 e^{-t/CR}$.



Charge on the capacitor increases with time during charging

Charge on the capacitor decreases with time during discharging

Fig. 18.101

(iii) **Time constant (τ) :** The quantity RC is called the time constant as it has the dimension of time during charging if $t = \tau = RC$, $Q = Q_0(1 - e^{-1}) = 0.63\,Q_0 =$ 63% of Q_0 $\left(\frac{1}{e} = 0.37\right)$ **or** during discharging it is defined as the time during which charge on a capacitor falls to 0.37 times (37%) of the initial charge on the capacitor.

Kirchhoff's Law for Capacitor Circuits

According to Kirchhoff's junction law $\sum q = 0$ and Kirchhoff's second law (Loop law) states that in a close loop of an electric circuit $\sum V = 0$

Use following sign convention while solving the problems.

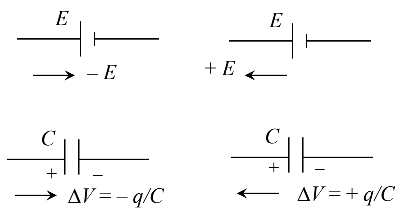
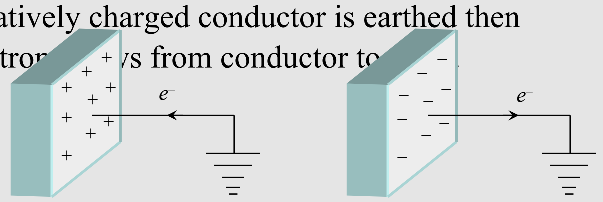


Fig. 18.102

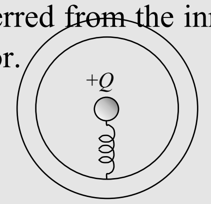
When an arrangement of capacitors cannot be simplified by the method of successive reduction, then we need to apply the Kirchhoff's laws to solve the circuit.

Tips & Tricks

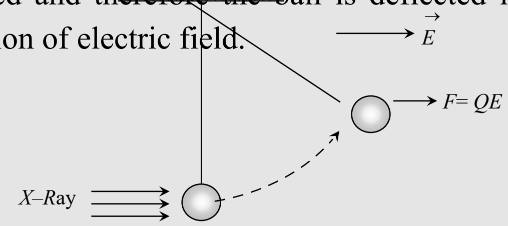
- After earthing a positively charged conductor electrons flow from earth to conductor and if a negatively charged conductor is earthed then electrons flows from conductor to earth.



- When a charged spherical conductor placed inside a hollow insulated conductor and connected through a fine conducting wire the charge will be completely transferred from the inner conductor to the outer conductor.



- Lightening-rod arrestors are made up of conductors with one of their ends earthed while the other sharp, and protects a building from lightening either by neutralising or conducting the charge of the cloud to the ground.
- With rise in temperature dielectric constant of liquid decreases.
- If X-rays are incident on a charged electroscope, due to ionisation of air by X-rays the electroscope will get discharged and hence its leaves will collapse. However, if the electroscope is evacuated. X-rays will cause photoelectric effect with gold and so the leaves will further diverge if it is positively charged (or uncharged) and will converge if it is negatively charged.
- Two point charges separated by a distance r in vacuum and a force F acting between them. After filling a dielectric medium having dielectric constant K completely between the charges, force between them decreases. To maintain the force as before separation between them has to be changed to $r\sqrt{K}$. This distance known as effective air separation.
- No point charge produces electric field at it's own position.
- The electric field on the surface of a conductor is directly proportional to the surface charge density at that point *i.e,* $E \propto \sigma$
- Two charged spheres having radii r_1 and r_2, charge densities σ_1 and σ_2 respectively, then the ratio of electric field on their surfaces will be $\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{r_2^2}{r_1^2}$ $\left\{\sigma = \frac{Q}{4\pi r^2}\right\}$
- In air, if intensity of electric field exceeds the value $3 \times 10^6\ N/C$, air ionizes.
- A small ball is suspended in a uniform electric field with the help of an insulated thread. If a high energy X–ray beam falls on the ball, *X*-rays knock out electrons from the ball so the ball is positively charged and therefore the ball is deflected in the direction of electric field.



- Electric field is always directed from higher potential to lower potential.
- A positive charge if left free in electric field always moves from higher potential to lower potential while a negative charge moves from lower potential to higher potential.

✍ An electric potential can exist at a point in a region where the electric field is zero and it's vice versa.

✍ It is a common misconception that the path traced by a positive test charge is a field line but actually the path traced by a unit positive test charge represents a field line only when it moves along a straight line.

✍ An electric field is completely characterized by two physical quantities Potential and Intensity. Force characteristic of the field is intensity and work characteristic of the field is potential.

✍ For a short dipole, electric field intensity at a point on the axial line is double the electric field intensity at a point on the equatorial line of electric dipole *i.e.* $E_{axial} = 2E_{equatorial}$

✍ It is interesting to note that dipole field $E \propto \frac{1}{r^3}$ decreases much rapidly as compared to the field of a point charge $\left(E \propto \frac{1}{r^2}\right)$.

✍ Franklin (*i.e.*, *e.s.u.* of charge) is the smallest unit of charge while faraday is largest (1 *Faraday* = 96500 *C*).

✍ The *e.s.u.* of charge is also called stat coulomb or Franklin (*Fr*) and is related to *e.m.u.* of charge through the relation $\frac{\text{emu of charge}}{\text{esu of charge}} = 3 \times 10^{10}$

✍ Recently it has been discovered that elementary particles such as proton or neutron are composed of quarks having charge $(\pm 1/3)e$ and $(\pm 2/3)e$. However, as quarks do not exist in free state, the quanta of charge is still e.

✍ Inducting body neither gains nor loses charge.

✍ Dielectric constant of an insulator can not be ∞

✍ For metals in electrostatics $K = \infty$ and so $Q' = -Q$, *i.e.* in metals induced charge is equal and opposite to inducing charge.

✍ A truck carrying explosives has a metal chain touching the ground, to conduct away the charge produced by friction.

✍ Coulombs law is valid at a distance greater than $10^{-15} m$.

✍ Ratio of gravitational force and electrostatic force between (i) Two electrons is **10^{-43}/1.** (ii) Two protons is **10^{-36}/1**

(iii) One proton and one electron **10^{-39}/1**.

✍ Decreasing order to fundamental forces $F_{Nuclear} > F_{Electromagnetic} > F_{Weak} > F_{Gravitational}$

✍ At the centre of the line joining two equal and opposite charge $V = 0$ but $E \neq 0$.

✍ At the centre of the line joining two equal and similar charge $V \neq 0, E = 0$.

✍ Electric field intensity and electric potential due to a point charge q, at a distance $t_1 + t_2$ where t_1 is thickness of medium of dielectric constant K_1 and t_2 is thickness of medium of dielectric constant K_2 are :

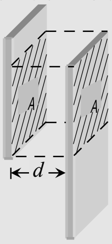
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(t_1\sqrt{K_1} + t_2\sqrt{K_2})^2}\ ;\ V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(t_1\sqrt{K_1} + t_2\sqrt{K_2})}$$

✍ If an electron (charge e and mass m) is moving on a circular path of radius r about a positively charge infinitely long linear charge, (charge density λ) then the velocity of electron in dynamic equilibrium will be $v = \sqrt{\frac{e\lambda}{2\pi\varepsilon_0 m}}$.

✍ A metal plate is charged uniformly with a surface charge density σ. An electron of energy W is fired towards the charged metal plate from a distance d, then for no collision of electron with plate $d = \frac{W\varepsilon_0}{e\sigma}$

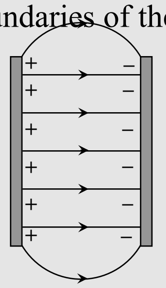
✍ It is a very common misconception that a capacitor stores charge but actually a capacitor stores electric energy in the electrostatic field between the plates.

✍ Two plates of unequal area can also form a capacitor, but effective overlapping area is considered.



✍ Capacitance of a parallel plate capacitor doesn't depends upon the charge given, potential raised or nature of metals and thickness of plates.

✍ The distance between the plates is kept small to avoid fringing or edge effect (non-uniformity of the field) at the boundaries of the plates.



✍ Spherical conductor is equivalent to a spherical capacitor with it's outer sphere of infinite radius.

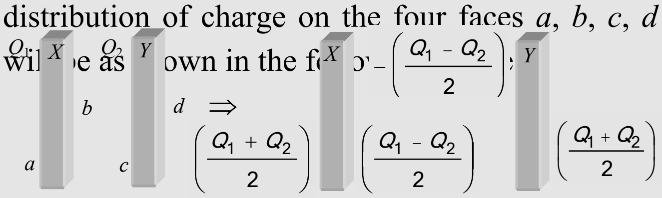
✍ A spherical capacitor behaves as a parallel plate capacitor if it's spherical surfaces have large radii and are close to each other.

✍ The intensity of electric field between the plates of a parallel plate capacitor ($E = \sigma/\varepsilon_0$) *does not depend upon the distance between them.*

✍ The plates of a parallel plate capacitor are being moved away with some velocity. If the plate separation at any instant of time is 'd' then the rate of change of capacitance with time is proportional to $\frac{1}{d^2}$.

✍ Radial and non-uniform electric field exists between the spherical surfaces of spherical capacitor.

✍ Two large conducting plates X and Y kept close to each other. The plate X is given a charge Q_1 while plate Y is given a charge $Q_2 (Q_1 > Q_2)$, the distribution of charge on the four faces a, b, c, d will be as shown in the following figure.



✍ When dielectric is partially filled between the plates of a parallel plate capacitor then it's capacitance increases but potential difference decreases. To maintain the capacitance and potential difference of capacitor as before separation between the plates has to be increased say by d. In such case

$$K = \frac{t}{t - d'}$$

✍ In series combination equivalent capacitance is always lesser than that of either of the individual capacitors. In parallel combination, equivalent capacitance is always greater than the maximum capacitance of either capacitor in network.

✍ If n identical capacitors are connected in parallel which are charged to a potential V. If these are separated and connected in series then potential difference of combination will be nV.

✍ Two capacitors of capacitances C_1 and C_2 are charged to potential of V_1 and V_2 respectively. After disconnecting from batteries they are again connected to each other with reverse polarity *i.e.*, positive plate of a capacitor connected to negative plate of other. Then common potential is given by

$$V = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}.$$