Chapter

23

Electromagnetic Induction

Magnetic Flux

(1) The total number of magnetic lines of force passing normally through an area placed in a magnetic field is equal to the magnetic flux linked with that area.

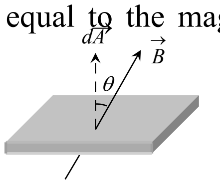


Fig. 23.1

(2) Net flux through the surface $\phi = \oint \vec{B} \cdot d\vec{A} = BA\cos\theta$

(θ is the angle between area vector and magnetic field vector)

If $\theta = 0^o$ then $\phi = BA$, If $\theta = 90^o$ then $\phi = 0$

(3) **Unit and Dimension :** Magnetic flux is a scalar quantity. It's S.I. unit is *weber* (*wb*), CGS unit is *Maxwell* or *Gauss* × *cm*^{2}; $(1\,wb = 10^8\ Maxwell)$.

(4) Other units : *Tesla* × *m*^{2} $= \frac{N \times m}{Amp} = \frac{Joule}{Amp}$ $= \frac{Volt \times \text{Coulomb}}{Amp}$

$= Volt \times sec$ = *Ohm* × *Coulomb* = *Henry* × *Amp*. It's dimensional formula $[\phi] = [ML^2T^{-2}A^{-1}]$

Faraday's Laws of Electromagnetic Induction

(1) **First law :** Whenever the number of magnetic lines of force (magnetic flux) passing through a circuit changes an emf is produced in the circuit called induced emf. The induced emf persists only as long as there is change or cutting of flux.

(2) **Second law :** The induced emf is given by rate of change of magnetic flux linked with the circuit *i.e.* $e = -\frac{d\phi}{dt}$. For *N* turns $e = -\frac{N\,d\phi}{dt}$; Negative sign indicates that induced emf (*e*) opposes the change of flux.

(3) **Other formulae :** $\phi = BA\cos\theta$; Hence ϕ will change if either, *B*, *A* or θ will change

So $e = -N\frac{d\phi}{dt} = -\frac{N(\phi_2 - \phi_1)}{\Delta t} = -\frac{NA(B_2 - B_1)\cos\theta}{\Delta t}$

$= -\frac{NBA(\cos\theta_2 - \cos\theta_1)}{\Delta t}$

Table 23.1 : Induced *i, q* and *P*

Induced current (*i*)	Induced charge (*q*)	Induced power (*P*)
$i = \frac{e}{R} = -\frac{N}{R}\cdot\frac{d\phi}{dt}$	$dq = i\,dt = -\frac{N}{R}\cdot d\phi$ Induced charge is time independent.	$P = \frac{e^2}{R} = \frac{N^2}{R}\left(\frac{d\phi}{dt}\right)^2$ It depends on time and resistance

Lenz's Law

This law gives the direction of induced emf/induced current. According to this law, the direction of induced emf or current in a circuit is such as to oppose the cause that produces it. This law is based upon law of conservation of energy.

(1) When *N*-pole of a bar magnet moves towards the coil, the flux associated with loop increases and

an emf is induced in it. Since the circuit of loop is closed, induced current also flows in it.

(2) Cause of this induced current, is approach of north pole and therefore to oppose the cause, *i.e.*, to repel the approaching north pole, the induced current in loop is in such a direction so that the front face of loop behaves as north pole. Therefore induced current as seen by observer O is in anticlockwise direction. (figure)

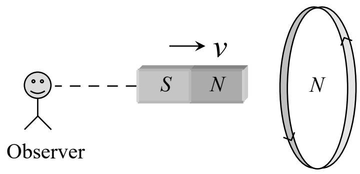
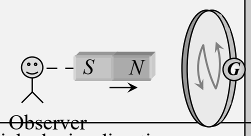
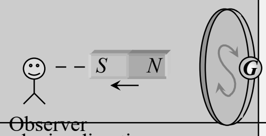
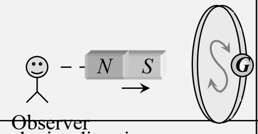
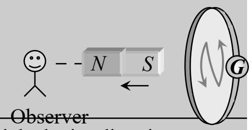


Fig. 23.2

Table 23.2 : The various positions of relative motion between the magnet and the coil

Position of magnet				
Direction of induced current	Anticlockwise direction	Clockwise direction	Clockwise direction	Anticlockwise direction
Behaviour of face of the coil	As a north pole	As a south pole	As a south pole	As a north pole
Type of magnetic force opposed	Repulsive force	Attractive force	Repulsive force	Attractive force
Magnetic field linked with the coil and it's progress as viewed from left	Cross (×), Increases	Cross (×), Decreases	Dots (·) Increases	Dots (·) Decreases

(3) If the loop is free to move the cause of induced emf in the coil can also be termed as relative motion. Therefore to oppose the cause, the relative motion between the approaching magnet and the loop should be opposed. For this, the loop will itself start moving in the direction of motion of the magnet.

(4) It is important to remember that whenever cause of induced emf is relative motion, the new motion is always in the direction of motion of the cause.

Induced Electric Field

It is non-conservative and non-electrostatic in nature. Its field lines are concentric circular closed curves.

A time varying magnetic field $\frac{dB}{dt}$ always produced induced electric field in all space surrounding it.

Induced electric field (E_{in}) is directly proportional to induced emf so $e = \oint \vec{E}_{in} \cdot d\vec{l}$ .....(i)

From Faraday's second laws $e = -\frac{d\phi}{dt}$ .....(ii)

From (i) and (ii) $e = \oint \vec{E}_{in}.d\vec{l} = -\frac{d\phi}{dt}$ This is known as integral form of Faraday's laws of EMI.

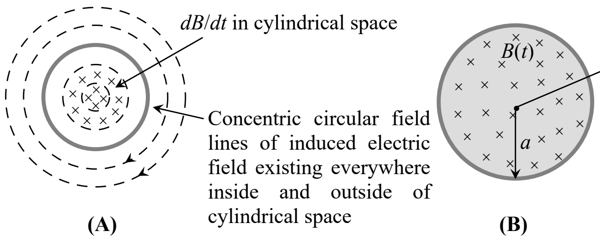


Fig. 23.3

A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius 'a' and is directed into the plane of the paper as shown, the magnitude of the induced electric field (E_{in}) at point P lies at a distance r from the centre of the circular region is calculated as follows.

So $\oint \vec{E}_{in} d\vec{l} = e = \frac{d\phi}{dt} = A\frac{dB}{dt}$ *i.e.* $E(2\pi r) = \pi a^2 \frac{dB}{dt}$

where $r \geq a$ or $E = \frac{a^2}{2r}\frac{dB}{dt}$; $E_{in} \propto \frac{1}{r}$

Dynamic (Motional) EMI Due to Translatory Motion

(1) Consider a conducting rod of length l moving with a uniform velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$, directed into the plane of the paper. Let the rod be moving to the right as shown in figure. The conducting electrons also move to the right as they are trapped within the rod.

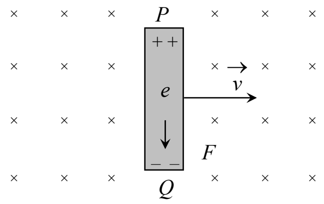


Fig. 23.4

Conducting electrons experiences a magnetic force $F_m = evB$. So they move from P to Q within the rod. The end P of the rod becomes positively charged while end Q becomes negatively charged, hence an electric field is set up within the rod which opposes the further downward movement of electrons *i.e.* an equilibrium is reached and in equilibrium $F_e = F_m$ *i.e.* $eE = evB$ or $E = vB$ $\Rightarrow$ Induced emf $e = El = Bvl$ $[E = \frac{V}{l}]$

(2) If rod is moving by making an angle θ with the direction of magnetic field or length. Induced emf $e = Bvl \sin\theta$

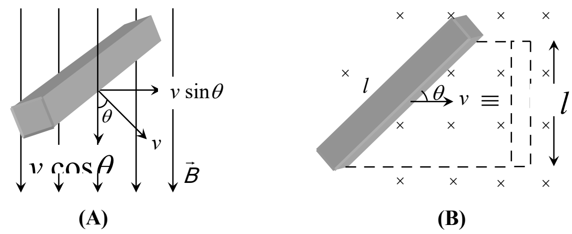


Fig. 23.5

(3) **Motion of conducting rod on an inclined plane :** When conductor start sliding from the top of an inclined plane as shown, it moves perpendicular to it's length but at an angle $(90 - \theta)$ with the direction of magnetic field.

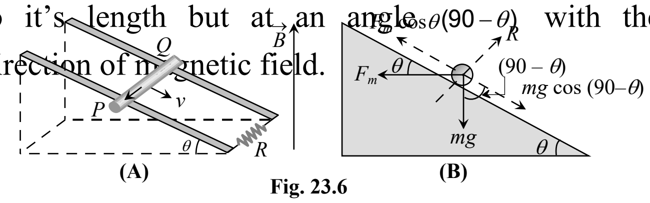


Fig. 23.6

Hence induced emf across the ends of conductor $e = Bv\sin(90-\theta)l = Bvl\cos\theta$

So induced current $i = \dfrac{Bvl\cos\theta}{R}$ (Directed from Q to P).

The forces acting on the bar are shown in following figure. The rod will move down with constant velocity only if

$$F_m\cos\theta = mg\cos(90-\theta) = mg\sin\theta \Rightarrow$$

$Bil\cos\theta = mg\sin\theta$

$$B\left(\frac{Bv_T l\cos\theta}{R}\right)l\cos\theta = mg\sin\theta \Rightarrow v_T = \frac{mgR\sin\theta}{B^2 l^2\cos^2\theta}$$

Motional Emi in Loop by Generated Area

If conducting rod moves on two parallel conducting rails as shown in following figure then phenomenon of induced emf can also be understand by the concept of generated area (The area swept of conductor in magnetic field, during it's motion)

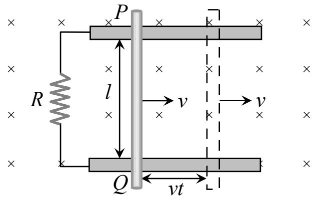


Fig. 23.7

As shown in figure in time t distance travelled by conductor $= vt$

Area generated $A = lvt$. Flux linked with this area $\phi = BA = Blvt$. Hence induced emf $|e| = \dfrac{d\phi}{dt} = Bvl$

(1) **Induced current :** $i = \dfrac{e}{R} = \dfrac{Bvl}{R}$

(2) **Magnetic force :** Conductor PQ experiences a magnetic force in opposite direction of it's motion and $F_m = Bil = B\left(\dfrac{Bvl}{R}\right)l = \dfrac{B^2vl^2}{R}$

(3) **Power dissipated in moving the conductor :** For uniform motion of rod PQ, the rate of doing mechanical work by external agent or mech. Power delivered by external source is given as $P_{mech} = P_{ext} = \dfrac{dW}{dt} = F_{ext}.v = \dfrac{B^2vl^2}{R}\times v = \dfrac{B^2v^2l^2}{R}$

(4) **Electrical power :** Also electrical power dissipated in resistance or rate of heat dissipation across resistance is given as

$$P_{thermal} = \frac{H}{t} = i^2R = \left(\frac{Bvl}{R}\right)^2.R; \quad P_{thermal} = \frac{B^2v^2l^2}{R}$$

(It is clear that $P_{mech.} = P_{thermal}$ which is consistent with the principle of conservation of energy.)

(5) **Motion of conductor rod in a vertical plane :** If conducting rod released from rest (at $t = 0$) as shown in figure then with rise in it's speed (v), induces emf (e), induced current (i), magnetic force (F_m) increases but it's weight remains constant.

Rod will achieve a constant maximum (terminal) velocity v_T if $F_m = mg$

So $\dfrac{B^2v_T^2l^2}{R} = mg$

$\Rightarrow v_T = \dfrac{mgR}{B^2l^2}$

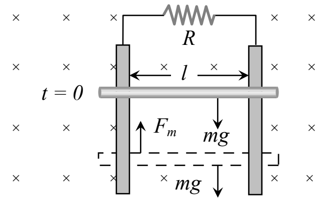


Fig. 23.8

Special cases

Motion of train and aeroplane in earth's magnetic field

(A) **Fig. 23.9** **(B)**

Induced emf across the axle of the wheels of the train and it is across the tips of the wing of the aeroplane is given by $e = B_v lv$ where l = length of the axle or distance between the tips of the wings of plane, B_v = vertical component of earth's magnetic field and v = speed of train or plane.

Motional EMI Due to Rotational Motion

(1) **Conducting rod :** A conducting rod of length l whose one end is fixed, is rotated about the axis passing through it's fixed end and perpendicular to it's length with constant angular velocity ω. Magnetic field (B) is perpendicular to the plane of the paper.

emf induces across the ends of the rod

where ν = frequency (revolution per sec) and T = Time period.

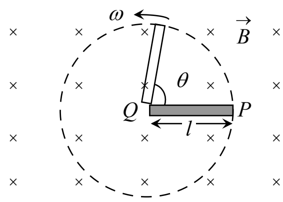


Fig. 23.10

(2) **Cycle wheel** : A conducting wheel each spoke of length l is rotating with angular velocity ω in a given magnetic field as shown below in fig.

Due to flux cutting each metal spoke becomes identical cell of emf e (say), all such identical cells connected in parallel fashion $e_{net} = e$ (emf of single cell). Let N be the number of spokes hence $e_{net} = \frac{1}{2}Bwl^2; \omega = 2\pi\nu$

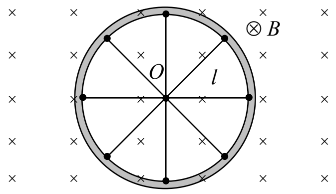


Fig. 23.11

Here $e_{net} \propto N^o$ *i.e.* total emf does not depends on number of spokes '*N*'.

(3) **Faraday copper disc generator :** A metal disc can be assumed to made of uncountable radial conductors when metal disc rotates in transverse magnetic field these radial conductors cuts away magnetic field lines and because of this flux cutting all becomes identical cells each of emf 'e' where $e = \frac{1}{2}B\omega r^2$,

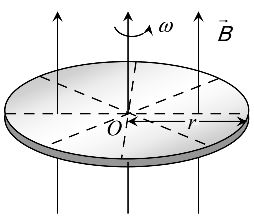


Fig. 23.12

(4) **Semicircular conducting loop** : If a semi-circular conducting loop (*ACD*) of radius '*r*' with centre at *O*, the plane of loop being in the plane of paper. The loop is now made to rotate with a constant angular velocity ω, about an axis passing through *O* and perpendicular to the plane of paper. The effective resistance of the loop is *R*.

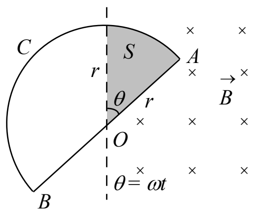


Fig. 23.13

In time t the area swept by the loop in the field *i.e.* region II $A = \frac{1}{2}r(r\theta) = \frac{1}{2}r^2\omega t$; $\frac{dA}{dt} = \frac{r^2\omega}{2}$

Flux link with the rotating loop at time t $\phi = BA$

Hence induced emf in the loop in magnitude $|e| = \frac{d\phi}{dt} = B\frac{dA}{dt} = \frac{B\omega r^2}{2}$ and induced current $i = \frac{|e|}{R} = \frac{B\omega r^2}{2R}$

Periodic EMI

Suppose a rectangular coil having N turns placed initially in a magnetic field such that magnetic field is perpendicular to it's plane as shown.

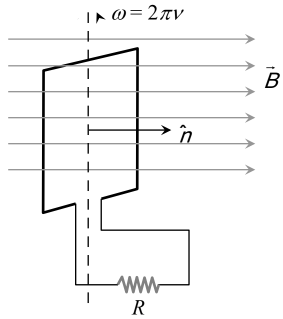


Fig. 23.14

ω – Angular speed

ν – Frequency of rotation of coil

R – Resistance of coil

For uniform rotational motion with ω, the flux linked with coil at any time t

$$\phi = NBA\cos\theta = NBA\cos\omega t$$

$\phi = \phi_0 \cos\omega t$ where $\phi_0 = NBA$ = maximum flux

(1) **Induced emf in coil :** Induced emf also changes in periodic manner that's why this phenomenon called periodic EMI

$e = -\frac{d\phi}{dt} = NBA\omega\sin\omega t \Rightarrow e = e_0 \sin\omega t$ where e_0 = emf amplitude or max. emf $= NBA\omega = \phi_0\omega$

(2) **Induced current :** At any time t, $i = \frac{e}{R} = \frac{e_0}{R}\sin\omega t = i_0 \sin\omega t$ where i_0 = current amplitude or max. current $i_0 = \frac{e_0}{R} = \frac{NBA\omega}{R} = \frac{\phi_0\omega}{R}$

Inductance

(1) Inductance is that property of electrical circuits which opposes any change in the current in the circuit.

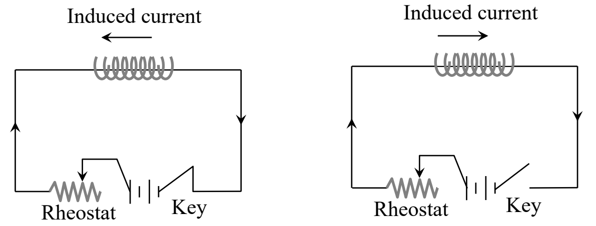
(2) Inductance is inherent property of electrical circuits. It will always be found in an electrical circuit whether we want it or not.

(3) A straight wire carrying current with no iron part in the circuit will have lesser value of inductance.

(4) Inductance is analogous to inertia in mechanics, because inductance of an electrical circuit opposes any change of current in the circuit.

Self Induction

Whenever the electric current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, in accordance with Faraday's laws of electromagnetic induction, an emf is induced in the coil or the circuit which opposes the change that causes it. This phenomenon is called 'self induction' and the emf induced is called back emf, current so produced in the coil is called induced current.



(A) Main current increasing **(B)** Main current decreasing

Fig. 23.15

(1) **Coefficient of self-induction :** Number of flux linkages with the coil is proportional to the current *i*. *i.e.* $N\phi \propto i$ or $N\phi = Li$ (N is the number of turns in coil and $N\phi$ – total flux linkage). Hence $L = \frac{N\phi}{i}$ = coefficient of self-induction.

(2) If i = 1*amp*, N = 1 then, $L = \phi$ *i.e.* the coefficient of self induction of a coil is equal to the flux linked with the coil when the current in it is 1 *amp*.

(3) By Faraday's second law induced emf $e = -N\frac{d\phi}{dt}$. Which gives $e = -L\frac{di}{dt}$; If $\frac{di}{dt} = 1\, amp/sec$ then $|e| = L$.

Hence coefficient of self induction is equal to the emf induced in the coil when the rate of change of current in the coil is unity.

(4) **Units and dimensional formula of '*L*' :** It's S.I. unit

$$\frac{weber}{Amp} = \frac{Tesla \times m^2}{Amp} = \frac{N \times m}{Amp^2} = \frac{Joule}{Amp^2} = \frac{Coulomb \times volt}{Amp^2}$$

$= \frac{volt \times \sec}{amp} = ohm \times \sec$. But practical unit is henry (***H***). It's dimensional formula $[L] = [ML^2T^{-2}A^{-2}]$

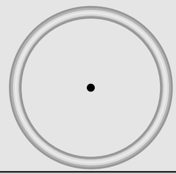
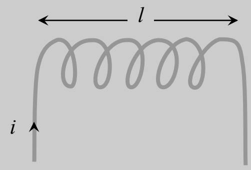
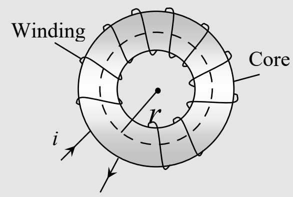
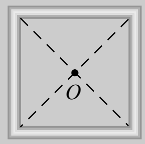
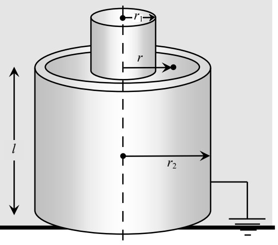
(5) **Dependence of self inductance (*L*) :** '*L*' does not depend upon current flowing or change in current flowing but it depends upon number of turns (N), Area of cross section (A) and permeability of medium (μ).

'*L*' does not play any role till there is a constant current flowing in the circuit. '*L*' comes in to the picture only when there is a change in current.

(6) **Magnetic potential energy of inductor :** In building a steady current in the circuit, the source emf has to do work against of self inductance of coil and whatever energy consumed for this work stored in magnetic field of coil this energy called as magnetic potential energy (U) of coil

$$U = \int_0^i Li\,di = \frac{1}{2}Li^2 \text{ ; Also } U = \frac{1}{2}(Li)i = \frac{N\phi i}{2}$$

(7) **The various formulae for *L***

Condition	Figure
Circular coil $L = \frac{\mu_0 \pi N^2 r}{2}$	
Solenoid $L = \frac{\mu_0 \mu_r N^2 A}{l} = \frac{\mu N^2 A}{l} (\mu = \mu_0 \mu_r)$	
Toroid $L = \frac{\mu_0 N^2 r}{2}$	
Square coil $L = \frac{2\sqrt{2}\mu_0 N^2 a}{\pi}$	
Coaxial cylinders $L = \frac{\mu_0}{2\pi r}\log_e \frac{r_2}{r_1}$ $\frac{2.303}{2\pi r}\mu_0 \log_{10}\frac{r_2}{r_1}$	

Mutual Induction

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is called 'mutual induction'.

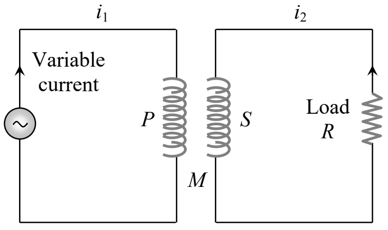


Fig. 23.16

(1) **Coefficient of mutual induction :** Total flux linked with the secondary due to current in the primary is $N_2\phi_2$ and $N_2\phi_2 \propto i_1 \Rightarrow N_2\phi_2 = Mi_1$ where N_1 - Number of turns in primary; N_2 - Number of turns in secondary; ϕ_2 - Flux linked with each turn of secondary; i_1 - Current flowing through primary; M-Coefficient of mutual induction or mutual inductance.

(2) According to Faraday's second law emf induces in secondary $e_2 = -N_2\frac{d\phi_2}{dt}$; $e_2 = -M\frac{di_1}{dt}$

(3) If $\frac{di_1}{dt} = \frac{1 Amp}{sec}$ then $|e_2| = M$. Hence coefficient of mutual induction is equal to the emf induced in the secondary coil when rate of change of current in primary coil is unity.

(4) **Units and dimensional formula of M :** Similar to self-inductance (L)

(5) **Dependence of mutual inductance**

(i) Number of turns (N_1, N_2) of both coils

(ii) Coefficient of self inductances (L_1, L_2) of both the coils

(iii) Area of cross-section of coils

(iv) Magnetic permeability of medium between the coils (μ_r) or nature of material on which two coils are wound

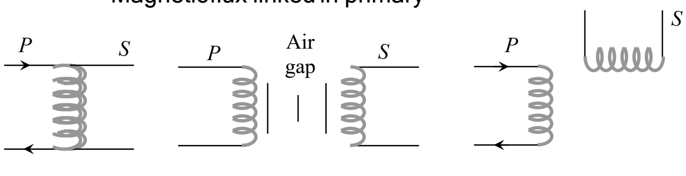
(v) Distance between two coils (As d increases so M decreases)

(vi) Orientation between primary and secondary coil (for 90° orientation no flux relation $M = 0$)

(vii) Coupling factor 'K' between primary and secondary coil

(6) **Relation between M, L_1 and L_2 :** For two magnetically coupled coils $M = k\sqrt{L_1 L_2}$; where k – coefficient of coupling or coupling factor which is defined as

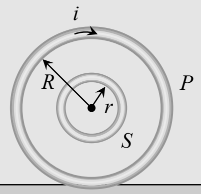
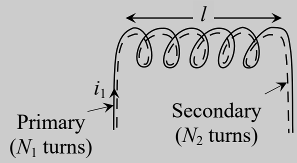
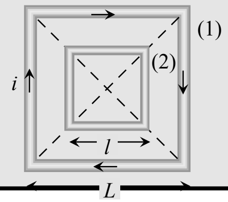
$$k = \frac{\text{Magnetic flux linked in secondary}}{\text{Magnetic flux linked in primary}}; \quad 0 \le k \le 1$$



(A) $k = 1$ (B) $0 < k < 1$ (C) $k = 0$

Fig. 23.17

(7) **The various formulae for M :**

Condition	Figure
Two concentric coplaner circular coils $M = \frac{\pi\mu_0 N_1 N_2 r^2}{2R}$	
Two Solenoids $M = \frac{\mu_0 N_1 N_2 A}{l}$	
Two concentric coplaner square coils $M = \frac{\mu_0 2\sqrt{2} N_1 N_2 l^2}{\pi L}$	

Combination of Inductance

(1) **Series :** If two coils of self-inductances L_1 and L_2 having mutual inductance are in series and are far from each other, so that the mutual induction between them is negligible, then net self inductance $L_S = L_1 + L_2$

When they are situated close to each other, then net inductance $L_S = L_1 + L_2 \pm 2M$

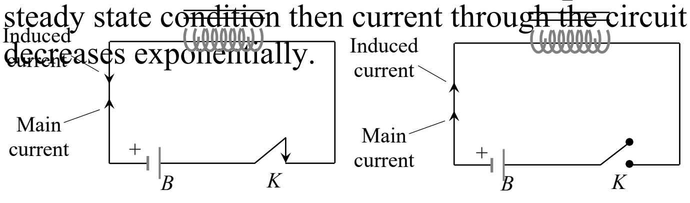
(2) **Parallel :** If two coils of self-inductances L_1 and L_2 having mutual inductance are connected in parallel and are far from each other, then net inductance L is $\frac{1}{L_P}=\frac{1}{L_1}+\frac{1}{L_2} \Rightarrow L_P=\frac{L_1L_2}{L_1+L_2}$

When they are situated close to each other, then

$$L_P=\frac{L_1L_2-M^2}{L_1+L_2\pm 2M}$$

Growth and Decay of Current In *LR*- Circuit

If a circuit containing a pure inductor L and a resistor R in series with a battery and a key then on closing the circuit current through the circuit rises exponentially and reaches up to a certain maximum value (steady state). If circuit is opened from it's steady state condition then current through the circuit decreases exponentially.



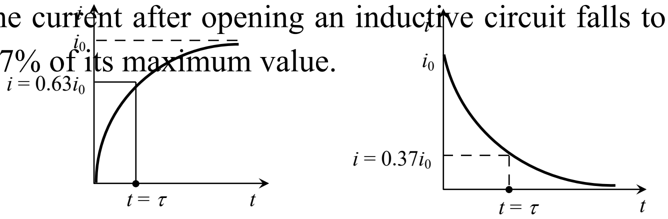
(A) Growth of current **(B)** Decay of current

Fig. 23.18

(1) The value of current at any instant of time t after closing the circuit (*i.e.* during the rising of current) is given by $i=i_0\left[1-e^{-\frac{R}{L}t}\right]$; where $i_0=i_{\max}=\frac{E}{R}=$ steady state current.

(2) The value of current at any instant of time t after opening from the steady state condition (*i.e.* during the decaying of current) is given by $i=i_0 e^{-\frac{R}{L}t}$

(3) **Time constant (τ) :** It is given as $\tau=\frac{L}{R}$; It's unit is *second*. In other words the time interval, during which the current in an inductive circuit rises to 63% of its maximum value at make, is defined as time constant or it is the time interval, during which the current after opening an inductive circuit falls to 37% of its maximum value.



(A) **Fig. 23.19** **(B)**

LC- Oscillation

When a charged capacitor C having an initial charge q_0 is discharged through an inductance L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. We also assume an idealized situation in which energy is not radiated away from the circuit. The total energy associated with the circuit is constant.

Frequency of oscillation is given by

$$\omega=\frac{1}{\sqrt{LC}}\frac{rad}{sec}$$

or $\nu=\frac{1}{2\pi\sqrt{LC}}Hz$

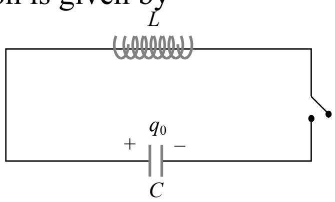


Fig. 23.20

Eddy Current

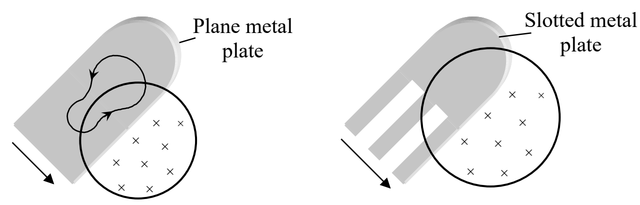
When a changing magnetic flux is applied to a bulk piece of conducting material then circulating currents called eddy currents are induced in the material. Because the resistance of the bulk conductor is usually low, eddy currents often have large magnitudes and heat up the conductor.

(1) These are circulating currents like eddies in water.

(2) Experimental concept given by Focault hence also named as "Focault current".

(3) The production of eddy currents in a metallic block leads to the loss of electric energy in the form of heat.

(4) By Lamination, slotting processes the resistance path for circulation of eddy current increases, resulting in to weakening them and also reducing losses causes by them



(A) Strong eddies produced Cause excessive electro magnetic damping

(B) Feeble eddies Gradual damping

(5) **Application of eddy currents :** Though most of the times eddy currents are undesirable but they find some useful applications as enumerated below

(i) **Dead-beat galvanometer :** A dead beat galvanometer means one whose pointer comes to rest in the final equilibrium position immediately without any oscillation about the equilibrium position when a current is passed in its coil.

This is achieved by winding the coil on a metallic frame the large eddy currents induced in the frame provide electromagnetic damping.

(ii) **Electric-brakes :** When the train is running its wheel is moving in air and when the train is to be stopped by electric breaks the wheel is made to move in a field created by electromagnet. Eddy currents induced in the wheels due to the changing flux oppose the cause and stop the train.

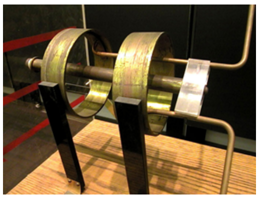
(iii) **Induction furnace :** Joule's heat causes the melting of a metal piece placed in a rapidly changing magnetic field.

(iv) **Speedometer :** In the speedometer of an automobile, a magnet is geared to the main shaft of the vehicle and it rotates according to the speed of the vehicle. The magnet is mounted in an aluminium cylinder with the help of hair springs. When the magnet rotates, it produces eddy currents in the drum and drags it through an angle, which indicates the speed of the vehicle on a calibrated scale.

(v) **Energy meter :** In energy meters, the armature coil carries a metallic aluminium disc which rotates between the poles of a pair of permanent horse shoe magnets. As the armature rotates, the current induced in the disc tends to oppose the motion of the armature coil. Due to this braking effect, deflection is proportional to the energy consumed.

dc Motor

It is an electrical machine which converts electrical energy into mechanical energy.



(1) **Principle :** It is based on the fact that a current carrying coil placed in the magnetic field experiences a torque. This torque rotates the coil.

(2) **Construction :** It consists of the following components figure.

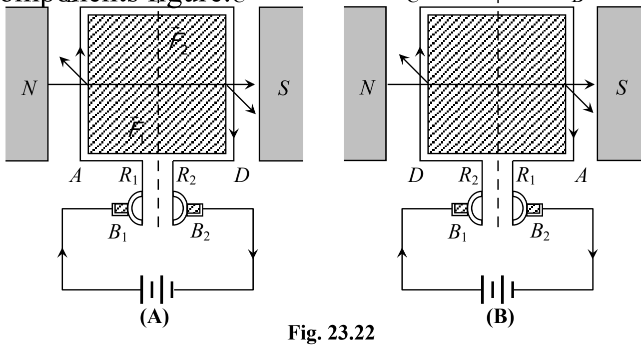


Fig. 23.22

$ABCD$ = Armature coil, S_1, S_2 = split ring comutators

B_1, B_2 = Carbon brushes, N, S = Strong magnetic poles

(3) **Working :** Force on any arm of the coil is given by $\vec{F} = i(\vec{l} \times \vec{B})$ in fig., force on AB will be perpendicular to plane of the paper and pointing inwards. Force on CD will be equal and opposite. So coil rotates in clockwise sense when viewed from top in fig. The current in AB reverses due to

commutation keeping the force on AB and CD in such a direction that the coil continues to rotate in the same direction.

(4) **Back emf in motor :** Due to the rotation of armature coil in magnetic field a back emf is induced in the circuit. Which is given by $\boldsymbol{e = E - iR}$.

Back emf directly depends upon the angular velocity ω of armature and magnetic field B. But for constant magnetic field B, value of back emf e is given by $e \propto \omega$ or $\boldsymbol{e = k\omega}$ $(e = NBA\omega\sin\omega t)$

(5) **Current in the motor :** $i = \frac{E-e}{R} = \frac{E-k\omega}{R}$;

When motor is just switched on *i.e.* $\omega = 0$ so $e = 0$ hence $i = \frac{E}{R} = \text{maximum}$ and at full speed, ω is maximum so back emf e is maximum and i is minimum. Thus, maximum current is drawn when the motor is just switched on which decreases when motor attains the speed.

(6) **Motor starter :** At the time of start a large current flows through the motor which may burn out it. Hence a starter is used for starting a dc motor safely. Its function is to introduce a suitable resistance in the circuit at the time of starting of the motor. This resistance decreases gradually and reduces to zero when the motor runs at full sped.

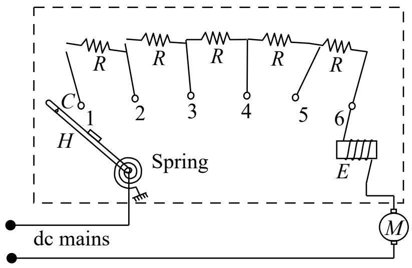


Fig. 23.23

The value of starting resistance is maximum at time $t = 0$ and its value is controlled by spring and electromagnetic system and is made to zero when the motor attains its safe speed.

(7) **Mechanical power and Efficiency of dc motor :**

Efficiency $\eta = \frac{P_{mechanical}}{P_{supplied}} = \frac{P_{out}}{P_{in}} = \frac{e}{E} = \frac{\text{Back e.m.f.}}{\text{Supply voltage}}$

(8) **Uses of dc motors :** They are used in electric locomotives, electric ears, rolling mills, electric cranes, electric lifts, dc drills, fans and blowers, centrifugal pumps and air compressors, *etc*.

ac Generator/Alternator/Dynamo

An electrical machine used to convert mechanical energy into electrical energy is known as ac generator/alternator.

(1) **Principle :** It works on the principle of electromagnetic induction *i.e.*, when a coil is rotated in uniform magnetic field, an induced emf is produced in it.

(2) **Construction :** The main components of ac generator are

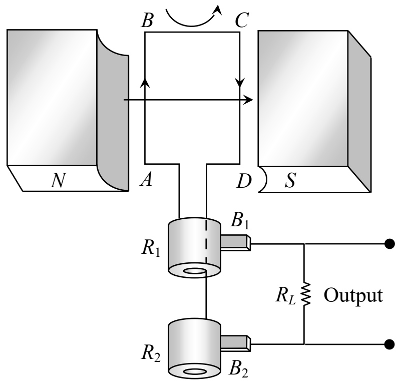


Fig. 23.24

(i) **Armature :** Armature coil ($ABCD$) consists of large number of turns of insulated copper wire wound over a soft iron core.

(ii) **Strong field magnet :** A strong permanent magnet or an electromagnet whose poles (N and S) are cylindrical in shape in a field magnet. The armature coil rotates between the pole pieces of the field magnet. The uniform magnetic field provided by the field magnet is perpendicular to the axis of rotation of the coil.

(iii) **Slip rings :** The two ends of the armature coil are connected to two brass slip rings R_1 and R_2. These rings rotate along with the armature coil.

(iv) **Brushes :** Two carbon brushes (B_1 and B_2), are pressed against the slip rings. The brushes are fixed while slip rings rotate along with the armature. These brushes are connected to the load through which the output is obtained.

(3) **Working :** When the armature coil $ABCD$ rotates in the magnetic field provided by the strong field magnet, it cuts the magnetic lines of force. Thus

the magnetic flux linked with the coil changes and hence induced emf is set up in the coil. The direction of the induced emf or the current in the coil is determined by the Fleming's right hand rule.

The current flows out through the brush B_1 in one direction of half of the revolution and through the brush B_2 in the next half revolution in the reverse direction. This process is repeated. Therefore, emf produced is of alternating nature.

$e = -\frac{Nd\phi}{dt} = NBA\omega \sin\omega t = e_0 \sin\omega t$ where $e_0 = NBA\omega$

$i = \frac{e}{R} = \frac{e_0}{R}\sin\omega t = i_0 \sin\omega t$ $R \rightarrow$ Resistance of the circuit

dc Generator

If the current produced by the generator is direct current, then the generator is called dc generator.

dc generator consists of (i) Armature (coil) (ii) Magnet (iii) Commutator (iv) Brushes

In dc generator commutator is used in place of slip rings. The commutator rotates along with the coil so that in every cycle when direction of 'e' reverses, the commutator also reverses or makes contact with the other brush so that in the external load the current remains in the some direction giving dc

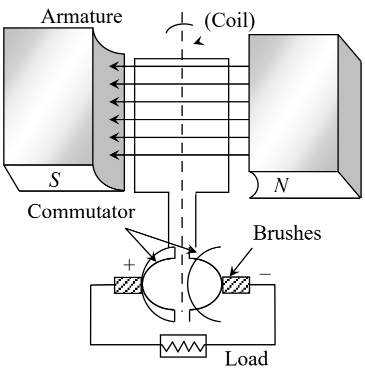


Fig. 23.25

Transformer

It is a device which raises or lowers the voltage in ac circuits through mutual induction.

It consists of two coils wound on the same core. The alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary.

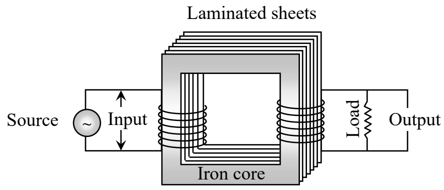


Fig. 23.26

(1) Transformer works on ac only and never on dc.

(2) It can increase or decrease either voltage or current but not both simultaneously.

(3) Transformer does not change the frequency of input ac.

(4) There is no electrical connection between the winding but they are linked magnetically.

(5) Effective resistance between primary and secondary winding is infinite.

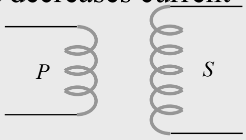
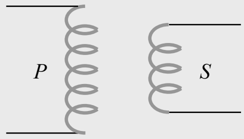
(6) The flux per turn of each coil must be same *i.e.* $\phi_S = \phi_P$; $-\frac{d\phi_S}{dt} = -\frac{d\phi_P}{dt}$.

(7) If N_P = number of turns in primary, N_S = number of turns in secondary, V_P = applied (input) voltage to primary, V_S = Voltage across secondary (load voltage or output), e_P = induced emf in primary ; e_S = induced emf in secondary, ϕ = flux linked with primary as well as secondary, i_P = current in primary; i_S = current in secondary (or load current)

As in an ideal transformer there is no loss of power *i.e.* $P_{out} = P_{in}$ so $V_S i_S = V_P i_P$ and $V_P \approx e_P$, $V_S \approx e_S$.

Hence $\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{i_P}{i_S} = k$; k = Transformation ratio (or turn ratio)

Table 23.3 : Types of transformer

Step up transformer	**Step down transformer**
It increases voltage and decreases current  $V_S > V_P$	It decreases voltage and increases current  $V_S < V_P$

$N_S > N_P$	$N_S < N_P$
$E_S > E_P$	$E_S < E_P$
$i_S < i_P$	$i_S > i_P$
$R_S > R_P$	$R_S < R_P$
$t_S > t_P$	$t_S > t_P$
$k > 1$	$k < 1$

(8) **Efficiency of transformer (η) :** Efficiency is defined as the ratio of output power and input power

i.e. $\eta\% = \frac{P_{out}}{P_{in}} \times 100 = \frac{V_S i_S}{V_P i_P} \times 100$

For an ideal transformer $P_{out} = P_{in}$ so $\eta = 100\%$ (But efficiency of practical transformer lies between 70% – 90%)

For practical transformer $P_{in} = P_{out} + P_{losses}$

so $\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{(P_{out} + P_L)} \times 100 = \frac{(P_{in} - P_L)}{P_{in}} \times 100$

(9) **Losses in transformer :** In transformers some power is always lost due to, heating effect, flux leakage eddy currents, hysteresis and humming.

(i) ***Cu* loss (i^2R) :** When current flows through the transformer windings some power is wasted in the form of heat $(H = i^2Rt)$. To minimize this loss windings are made of thick *Cu* wires (To reduce resistance)

(ii) **Eddy current loss :** Some electrical power is wasted in the form of heat due to eddy currents, induced in core, to minimize this loss transformers core are laminated and silicon is added to the core material as it increases the resistivity. The material of the core is then called silicon-iron (steel).

(iii) **Hystersis loss :** The alternating current flowing through the coils magnetises and demagnetises the iron core again and again. Therefore, during each cycle of magnetisation, some energy is lost due to hysteresis. However, the loss of energy can be minimised by selecting the material of core, which has a narrow hysterisis loop. Therefore core of transformer is made of soft iron. Now a days it is made of "Permalloy" (*Fe*-22%, *Ni*-78%).

(iv) **Magnetic flux leakage :** Magnetic flux produced in the primary winding is not completely linked with secondary because few magnetic lines of force complete their path in air only. To minimize this loss secondary winding is kept inside the primary winding.

(v) **Humming losses :** Due to the passage of alternating current, the core of the transformer starts vibrating and produces humming sound. Thus, some part (may be very small) of the electrical energy is wasted in the form of humming sounds produced by the vibrating core of the transformer.

(10) **Uses of transformer :** A transformer is used in almost all ac operations *e.g.*

(i) In voltage regulators for TV, refrigerator, computer, air conditioner etc.

(ii) In the induction furnaces.

(iii) Step down transformer is used for welding purposes.

(iv) In the transmission of ac over long distance.

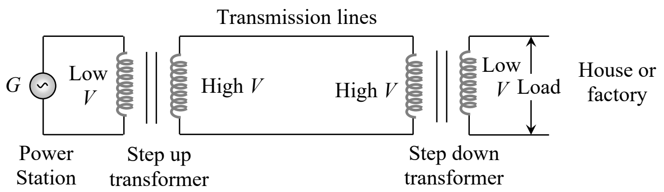


Fig. 23.27

(v) Step down and step up transformers are used in electrical power distribution.

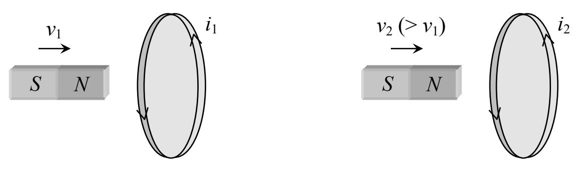
(vi) Audio frequency transformers are used in radiography, television, radio, telephone *etc*.

(vii) Radio frequency transformers are used in radio communication.

(viii) Transformers are also used in impedance matching.

Tips & Tricks

✍ If a bar magnet moves towards a fixed conducting coil, then due to the flux changes an



emf, current and charge induces in the coil. If speed of magnet increases then induced emf and induced current increases but induced charge remains same

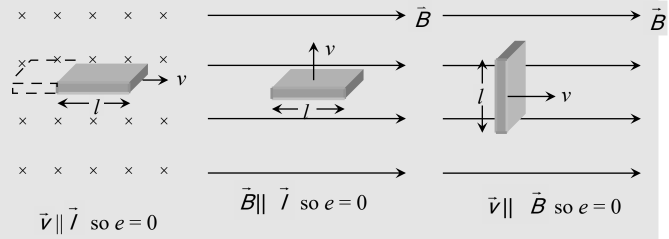
Induced parameter : e_1, i_1, q_1 $\qquad$ $e_2\,(> e_1), i_2(> i_1), q_2\,(= q_1)$

✍ Can ever electric lines of force be closed curve ? Yes, when produced by a changing magnetic field.

✍ No flux cutting $\longrightarrow$ No EMI

✍ Vector form of motional emf : $e = (\vec{v} \times \vec{B}).\vec{l}$

✍ In motional emf $\vec{B}, \vec{v}$ and $\vec{l}$ are three vectors. If any two vector are parallel – No flux cutting.



✍ A piece of metal and a piece of non-metal are dropped from the same height near the surface of the earth. The non-metallic piece will reach the ground first because there will be no induced current in it.

✍ If an aeroplane is landing down or taking off and its wings are in the east-west direction, then the potential difference or emf will be induced across the wings. If an aeroplane is landing down or taking off and its wings are in the north-south direction, then no potential difference or emf will be induced.

✍ When a conducting rod moving horizontally on equator of earth no emf induces because there is no vertical component of earth's magnetic field. But at poles B_V is maximum so maximum flux cutting hence emf induces.

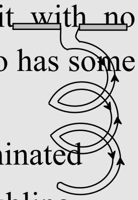
✍ When a conducting rod falling freely in earth's magnetic field such that it's length lies along East - West direction then induced emf continuously increases w.r.t. time and induced current flows from West - East.

✍ 1 *henry* = 10^9 *emu* of inductance or 10^9 *ab-henry*.

✍ Inductance at the ends of a solenoid is half of it's the inductance at the centre. $\left(L_{end} = \frac{1}{2} L_{centre}\right)$.

✍ A thin long wire made up of material of high resistivity behaves predominantly as a resistance. But it has some amount of inductance as well as capacitance in it. It is thus difficult to obtain pure resistor. Similarly it is difficult to obtain pure capacitor as well as pure inductor.

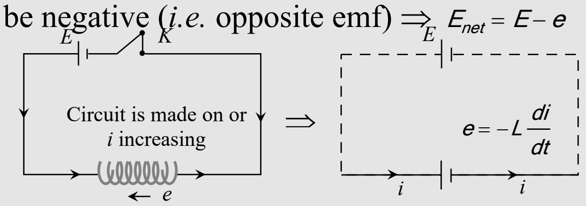
✍ Due to inherent presence of self inductance in all electrical circuits, a resistive circuit with no capacitive or inductive element in it, also has some inductance associated with it.



The effect of self-inductance can be eliminated as in the coils of a resistance box by doubling back the coil on itself.

✍ It is not possible to have mutual inductance without self inductance but it may or may not be possible self inductance without mutual inductance.

✍ If main current through a coil increases (i↑) so $\frac{di}{dt}$ will be positive (+*ve*), hence induced emf *e* will be negative (*i.e.* opposite emf) $\Rightarrow E_{net} = E - e$

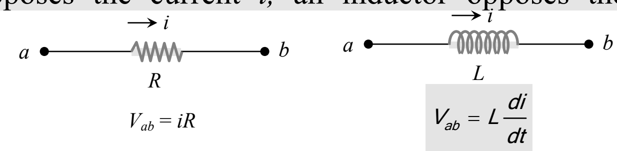


✍ Sometimes at sudden opening of key, because of high inductance of circuit a high momentarily induced emf produced and a sparking occurs at key position. To avoid sparking a capacitor is connected across the key.

✍ Sometimes at sudden opening of key, because of high inductance of circuit a high momentarily induced emf produced and a sparking occurs at key position. To avoid sparking a capacitor is connected across the key.

✍ One can have resistance with or without inductance but one can't have inductance without having resistance.

✍ The circuit behaviour of an inductor is quite different from that of a resistor. while a resistor opposes the current *i*, an inductor opposes the



change $\frac{di}{dt}$ in the circuit.

✍ In *RL*-circuit with dc source the time taken by the current to reach half of the maximum value is called half life time and it is given by $T = 0.693\ \frac{L}{R}$.

✍ dc motor is a highly versatile energy conversion device. It can meet the demand of loads requiring high starting torque, high accelerating and decelerating torque.

✍ When a source of emf is connected across the two ends of the primary winding alone or across the two ends of secondary winding alone, ohm's law can be applied. But in the transformer as a whole, ohm's law should not be applied because primary winding and secondary winding are not connected electrically.

✍ Even when secondary circuit of the transformer is open it also draws some current called no load primary current for supplying no load *Cu* and iron loses.

✍ Transformer has highest possible efficiency out of all the electrical machines.