FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Friday 05th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. Let f: [–1, 2] → R be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where [t] denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is :

(1) 6 (2) 3

(3) 4 (4) 5

Ans. (3)

Sol. Doubtful points : –1, 0, 1, $\sqrt{2}$, $\sqrt{3}$, 2

at x = $\sqrt{2}$, $\sqrt{3}$

$$f(x) = \underset{\text{Cont.}}{\left(2x^2 + x - [x]\right)} + \underset{\text{Cont.}}{\left[x^2\right]} = \text{Discount}$$

at x = –1 :

$$\left.\begin{aligned} \text{RHL} \Rightarrow f(x) &= (2 - 1 - (-1)) + 0 = 2 \\ f(-1) &= 2 - 1 - (-1) + 1 = 3 \end{aligned}\right\} \text{Dis.}$$

at x = 2 :

$$\left.\begin{aligned} \text{LHL} \Rightarrow f(x) &= 8 + 2 - 1 + 3 = 12 \\ f(2) &= 8 + 2 - 2 + 4 = 12 \end{aligned}\right\} \text{Cont.}$$

at x = 0 :

$$\left.\begin{aligned} \text{LHL} &\Rightarrow 0 + 0 - (-1) + 0 = 1 \\ f(0) &= 0 \end{aligned}\right\} \text{Dis.}$$

at x = 1

$$\left.\begin{aligned} \text{LHL} &\Rightarrow 2 + 1 - 0 + 0 = 3 \\ f(1) &= 3 - 1 + 1 = 3 \\ \text{RHL} &\Rightarrow 2 + 1 - 1 + 1 = 3 \end{aligned}\right\} \text{Cont.}$$

2. The differential equation of the family of circles passing the origin and having center at the line y = x is :

(1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$

(2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$

(3) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$

(4) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

Ans. (3)

Sol. $C \equiv x^2 + y^2 + gx + gy = 0$ .....(1)

$2x + 2yy' + g + gy' = 0$

$$g = -\left(\frac{2x + 2yy'}{1 + y'}\right)$$

Put in (1)

$$x^2 + y^2 - \left(\frac{2x + 2yy'}{1 + y'}\right)(x + y) = 0$$

$(x^2 - y^2 - 2xy)y' = x^2 - y^2 + 2xy$

3. Let $S_1 = \{z \in C : |z| \le 5\}$,

$$S_2 = \left\{z \in C : \text{Im}\left(\frac{z + 1 - \sqrt{3}\,i}{1 - \sqrt{3}\,i}\right) \ge 0\right\} \text{ and}$$

$S_3 = \{z \in C : \text{Re}(z) \ge 0\}$. Then

(1) $\frac{125\pi}{6}$ (2) $\frac{125\pi}{24}$

(3) $\frac{125\pi}{4}$ (4) $\frac{125\pi}{12}$

Ans. (4)

Sol. $S_1 : x^2 + y^2 \le 25$ .....(1)

$$S_2 : \text{Im of } \frac{z + \left(1 - \sqrt{3}\,i\right)}{\left(1 - \sqrt{3}\,i\right)} \ge 0$$

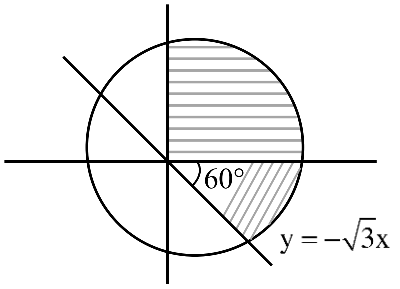
$$\text{Im of}\left(\frac{x + iy}{1 - \sqrt{3}\,i} + 1\right) \ge 0$$

$$\text{Im of}\left(\frac{(x + iy)\left(1 + \sqrt{3}\,i\right)}{4}\right) \ge 0$$

$\Rightarrow \sqrt{3}x + y \ge 0$ ......(2)

$S_3 : x \ge 0$ ......(3)

$$\text{Area} = \frac{5}{12}\left(\pi(5)^2\right)$$



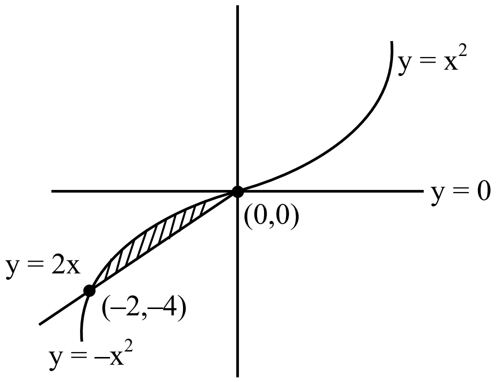
4. The area enclosed between the curves y = x|x| and y = x – |x| is :

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$

(3) 1 (4) $\frac{4}{3}$

Ans. (4)

Sol.



$$A=\int_{-2}^{0}-x^2-2x \;=\frac{4}{3}$$

5. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50th word is :

(1) OBBHJ (2) HBBJO

(3) OBBJH (4) JBBOH

Ans. (3)

Sol. B B H J O

$\boxed{B}$_____ $4!=24$

$\boxed{H}$_____ $\frac{4!}{2!}=12$

$\boxed{J}$_____ $\frac{4!}{2!}=12$

O B B H J

O B B J H $\rightarrow$ 50th rank

6. Let $\vec{a}=2\hat{i}+5\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{c}$ be three vectors such that $(\vec{c}+\hat{i})\times(\vec{a}+\vec{b}+\hat{i})=\vec{a}\times(\vec{c}+\hat{i})$. $\vec{a}.\vec{c}=-29$, then $\vec{c}.(-2\hat{i}+\hat{j}+\hat{k})$ is equal to :

(1) 10 (2) 5

(3) 15 (4) 12

Ans. (2)

Sol. Let's assume $\vec{v}=\vec{a}+\vec{b}+\hat{i}$

$$=5\hat{i}+3\hat{j}+\hat{k}$$

and $\vec{c}+\hat{i}=\vec{p}$

So,

$\vec{p}\times\vec{v}=\vec{a}\times\vec{p}$

$\vec{p}\times\vec{v}+\vec{p}\times\vec{a}=\vec{0}$

$\vec{p}\times(\vec{v}+\vec{a})=\vec{0}$

$\Rightarrow \vec{p}=\lambda(\vec{v}+\vec{a})$

$\vec{c}+i=\lambda(7\hat{i}+8\hat{j})$

$\overline{a}.\overline{c}+\overline{a}.\hat{i}=\lambda\overline{a}.(7\hat{i}+8\hat{j})$

$-29+2=\lambda(14+40)$

$\lambda=-\frac{1}{2}$

$$\vec{c}.(-2\hat{i}+\hat{j}+\hat{k})+\hat{i}.(-2\hat{i}+\hat{j}+\hat{k})=\lambda(7\hat{i}+8\hat{j}).(-2\hat{i}+\hat{j}+\hat{k})$$

$$=-\frac{1}{2}(-14+8)+2=5$$

7. Consider three vectors $\vec{a},\vec{b},\vec{c}$. Let $|\vec{a}|=2,|\vec{b}|=3$ and $\vec{a}=\vec{b}\times\vec{c}$. If $\alpha\in\left[0,\frac{\pi}{3}\right]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c}-\vec{a}|^2$ is equal to :

(1) 110 (2) 105

(3) 124 (4) 121

Ans. (3)

Sol. $|\vec{c}-\vec{a}| = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{c}$

$= |\vec{c}|^2 + 4 - 0$

$\because\ \vec{a} = \vec{b}\times\vec{c}$

$|\vec{a}| = |\vec{b}\times\vec{c}|$

$2 = 3|\vec{c}|\sin\alpha$

$|\vec{c}| = \frac{2}{3}\text{cosec}\alpha \qquad \alpha \in \left[0, \frac{\pi}{3}\right]$

$|\vec{c}|_{\min} = \frac{2}{3}\times\frac{2}{\sqrt{3}} \qquad \text{cosec}\alpha \in \left[\frac{2}{\sqrt{3}}, \infty\right)$

$\Rightarrow 27|\vec{c}-\vec{a}|^2_{\min} = 27\left(\frac{16}{27}+4\right) = 124$

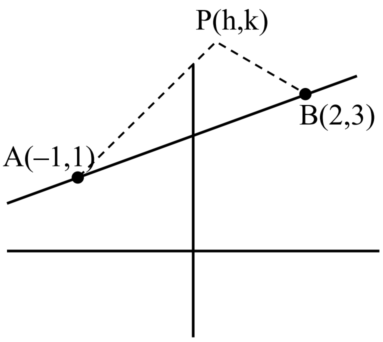
8. Let A(–1, 1) and B(2, 3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is ax + by = 15, then 5a + 2b is :

(1) $-\frac{12}{5}$ (2) $-\frac{6}{5}$

(3) 4 (4) 6

Ans. (1)

Sol.



$$\frac{1}{2}\begin{vmatrix} h & k & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 10$$

$-2x + 3y = 25$

$-\frac{6}{5}x + \frac{9}{5}y = 15$

$a = -\frac{6}{5}, b = \frac{9}{5}$

$5a = -6, 2b = \frac{18}{5}$

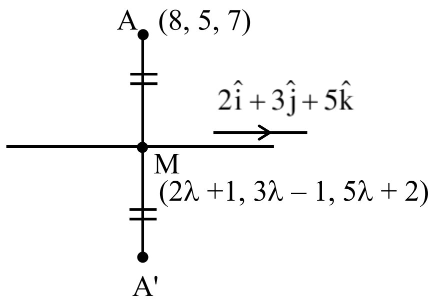
9. Let (α, β, γ) be the point (8, 5, 7) in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then α + β + γ is equal to

(1) 16 (2) 18

(3) 14 (4) 20

Ans. (3)

Sol.



$\overrightarrow{AM}.\left(2\hat{i}+3\hat{j}+5\hat{k}\right) = 0$

$(2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$

$38\lambda = 57$

$\lambda = \frac{3}{2}$

$M\left(4, \frac{7}{2}, \frac{19}{2}\right)$

A'(0,2,12)

10. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[3]{5}}\right)^{12}$, x ≠ 0, is $\alpha\times 2^8\times\sqrt[5]{3}$, then 25α is equal to :

(1) 639 (2) 724

(3) 693 (4) 742

Ans. (3)

Sol. $T_{r+1} = {}^{12}C_r\left(\frac{3^{1/5}}{x}\right)^{12-r}\left(\frac{2x}{5^{1/3}}\right)^r$

$$T_{r+1} = \frac{{}^{12}C_r(3)^{\frac{12-r}{5}}(2)^r(x)^{2r-12}}{(5)^{r/3}}$$

r = 6

$$T_7 = \frac{{}^{12}C_6(3)^{6/5}(2)^6}{5^2} = \left(\frac{9\times 11\times 7}{25}\right)2^8.3^{1/5}$$

25α = 693

11. Let f, g : R → R be defined as : $f(x) = |x - 1|$ and $g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0 \end{cases}$. Then the function $f(g(x))$ is

(1) neither one-one nor onto.

(2) one-one but not onto.

(3) both one-one and onto.

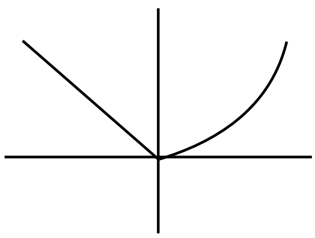
(4) onto but not one-one.

Ans. (1)

Sol. $f(g(x)) = |g(x) - 1|$

$$fog\begin{bmatrix} |e^x - 1| & x \geq 0 \\ |x+1-1| & x \leq 0 \end{bmatrix}$$

$$fog\begin{bmatrix} e^x - 1 & x \geq 0 \\ -x & x \leq 0 \end{bmatrix}$$



12. Let the circle $C_1 : x^2 + y^2 - 2(x + y) + 1 = 0$ and C_2 be a circle having centre at (–1, 0) and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :

(1) 2 (2) 1

(3) 6 (4) 4

Ans. (1)

Sol. $S_1 : x^2 + y^2 - 2x - 2y + 1 = 0$

$S_2 : x^2 + y^2 + 2x - 3 = 0$

Common chord = $S_1 - S_2 = 0$

$-4x - 2y + 4 = 0$

$2x + y = 2 \Rightarrow P(0, 2)$

$d^2_{(c,p)} = (1-0)^2 + (2-1)^2 = 2$

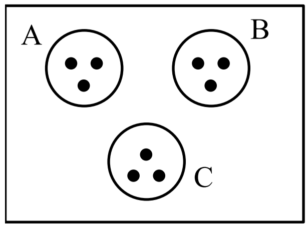
13. Let the set S = {2, 4, 8, 16, ..., 512} be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to :

(1) 1680 (2) 1520

(3) 1710 (4) 1640

Ans. (1)

Sol.



$$\frac{9!}{(3!3!3!)3!} \times 3!$$

14. The values of m, n, for which the system of equations

$x + y + z = 4,$

$2x + 5y + 5z = 17,$

$x + 2y + mz = n$

has infinitely many solutions, satisfy the equation :

(1) $m^2 + n^2 - m - n = 46$

(2) $m^2 + n^2 + m + n = 64$

(3) $m^2 + n^2 + mn = 68$

(4) $m^2 + n^2 - mn = 39$

Ans. (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 5 \\ 1 & 2 & m \end{vmatrix} = 0 \Rightarrow m = 2$

$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & 17 \\ 1 & 2 & n \end{vmatrix} = 0 \Rightarrow n = 7$

15. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set {1, 2, 3, 4, 5, 6}. If the probability of this equation having one real root bigger than the other is p, then 216p equals :

(1) 57 (2) 38

(3) 19 (4) 76

Ans. (2)

Sol. $D > 0$

$b^2 > 4ac$

$b = 3 : (a, c) = (1, 1)(1, 2)(2,1)$

$b = 4 : (a, c) = (1, 1)(1, 2)(2,1)(1,3)(3,1)$

$b = 5 : (a, c) = (1,1)(1,2)(2,1)\ (1,3)(3,1)(1,4)(4,1)$
$(1,5)(5,1)(1,6)(6,1)(2,3)(3,2)(2,2)$

$b = 6 : (a, c) = (1,1)(1,2)(2,1)\ (1,3)(3,1)(1,4)(4,1)$
$(1,5)(5,1)(1,6)(6,1)(2,3)(3,2)(2,4)(4,2)(2,2)$

fav. cases = 38

Prob. : $\dfrac{38}{6\times6\times6}$

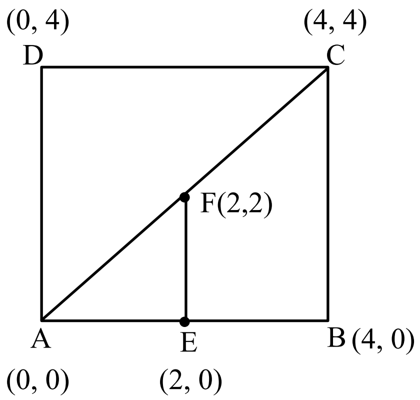
16. Let ABCD and AEFG be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies :

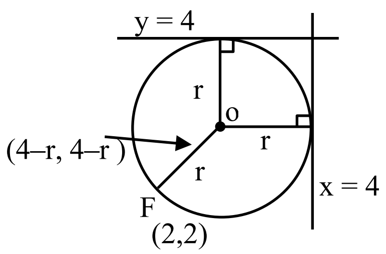
(1) $r = 1$ (2) $r^2 - 8r + 8 = 0$

(3) $2r^2 - 4r + 1 = 0$ (4) $2r^2 - 8r + 7 = 0$

Ans. (2)

Sol.





$OF^2 = r^2$

$(2 - r)^2 + (2 - r)^2 = r^2$

$r^2 - 8r + 8 = 0$

17. Let $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n > 0. If $\int_0^1 (1-x^{10})^{20}dx = a\times\beta(b,c)$, then 100(a + b + x) equals ______.

(1) 1021 (2) 1120

(3) 2012 (4) 2120

Ans. (4)

Sol. $I = \int_0^1 1.\left(1-x^{10}\right)^{20} dx$

$x^{10} = t$

$x = t^{1/10}$

$dx = \dfrac{1}{10}(t)^{-9/10} dt$

$I = \int_0^1 (1-t)^{20} \dfrac{1}{10}(t)^{-9/10} dt$

$I = \dfrac{1}{10}\int_0^1 t^{-9/10}\left(1-t\right)^{20} dt$

$a = \dfrac{1}{10}$ $b = \dfrac{1}{10}$ $c = 21$

18. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$.

If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then det(AB) is equal to :

(1) 343 (2) 125

(3) 64 (4) 216

Ans. (4)

Sol. Equating co-factor fo A_{21}

$(2\alpha^2 - 3\alpha) = \alpha$

$\alpha = 0, 2$ (accept)

Now, $2\alpha^2 - \alpha\beta = 3\alpha$

$\alpha = 2$ $\beta = 1$

$|AB| = |A\ \text{cof}\ (A)| = |A|^3$

$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix} = 6 - 2(9) + 3(6) = 6$

19. If $y(\theta)=\dfrac{2\cos\theta+\cos2\theta}{\cos3\theta+4\cos2\theta+5\cos\theta+2}$,

then at $\theta=\dfrac{\pi}{2}$, y'' + y' + y is equal to:

(1) $\dfrac{3}{2}$ (2) 1

(3) $\dfrac{1}{2}$ (4) 2

Ans. (4)

Sol. $y=\dfrac{2\cos\theta+2\cos^2\theta-1}{4\cos^3\theta-3\cos\theta+8\cos^2\theta-4+5\cos\theta+2}$

$$y=\frac{\left(2\cos^2\theta+2\cos\theta-1\right)}{\left(2\cos^2\theta+2\cos\theta-1\right)\left(2\cos\theta+2\right)}$$

$$y=\frac{1}{2}\left(\frac{1}{1+\cos\theta}\right)$$

$\Rightarrow \theta=\dfrac{\pi}{2}$ $\quad y=\dfrac{1}{2}$

$$y'=\frac{1}{2}\left(\frac{-1}{\left(1+\cos\theta\right)^2}\times\left(-\sin\theta\right)\right)$$

$\Rightarrow \theta=\dfrac{\pi}{2}$ $\quad y=\dfrac{1}{2}$

$$y''=\frac{1}{2}\left[\frac{\cos\theta\left(1+\cos\theta\right)^2-\sin\theta(2)(1+\cos\theta)(-\sin\theta)}{(1+\cos\theta)^4}\right]$$

$\Rightarrow \theta=\dfrac{\pi}{2}$ $\qquad y=1$

20. For $x \ge 0$, the least value of K, for which $4^{1+x}+4^{1-x}$, $\dfrac{K}{2}$, $16^x + 16^{-x}$ are three consecutive terms of an A.P. is equal to :

(1) 10 (2) 4

(3) 8 (4) 16

Ans. (1)

Sol. $k=4\underbrace{\left(4^x+\dfrac{1}{4^x}\right)}_{\ge 2}+\underbrace{\left(4^{2x}+\dfrac{1}{4^{2x}}\right)}_{\ge 2}$

$k \ge 10$

SECTION-B

21. Let the mean and the standard deviation of the probability distribution

X	α	1	0	–3
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to _____.

Ans. (5)

Sol. $\dfrac{1}{3}+k+\dfrac{1}{6}+\dfrac{1}{4}=1 \qquad \Rightarrow k=\dfrac{1}{4}$

$$\mu=\frac{\alpha}{3}+\frac{1}{4}-\frac{3}{4}$$

$$\boxed{\mu=\frac{\alpha}{3}-\frac{1}{2}}$$

$$\sigma=\sqrt{\left(\alpha^2\frac{1}{3}+\frac{1}{4}+9\frac{1}{4}\right)-\left(\frac{\alpha}{3}-\frac{1}{2}\right)^2}$$

$$\sigma=\sqrt{\frac{2\alpha^2}{9}+\frac{\alpha}{3}+\frac{9}{4}}$$

$\sigma=\mu+2$

$$\sigma^2=\left(\mu+2\right)^2 \Rightarrow \frac{2\alpha^2}{9}+\frac{\alpha}{3}+\frac{9}{4}=\frac{\alpha^2}{9}+\frac{9}{4}+\alpha$$

$$\frac{\alpha^2}{9}-\frac{2\alpha}{3}=0$$

$\alpha = 0$, (reject) or $\alpha = 6$

($\because$ $x=0$ is already given)

$\Rightarrow \sigma + \mu = 2\mu + 2$

$= 5$

22. Let $y = y(x)$ be the solution of the differential equation $\dfrac{dy}{dx}+\dfrac{2x}{\left(1+x^2\right)^2}y=xe^{\frac{1}{\left(1+x^2\right)}}$; $y(0) = 0$.

Then the area enclosed by the curve $f(x)=y(x)e^{-\frac{1}{\left(1+x^2\right)}}$ and the line $y - x = 4$ is ____.

Ans. (18)

Sol. IF $= e^{\int \frac{2x}{(1+x^2)^2}dx} = e^{\frac{-1}{1+x^2}}$

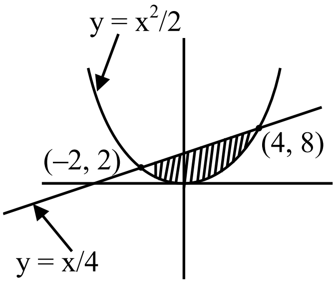
$$y \cdot e^{\frac{-1}{1+x^2}} = \int x \cdot e^{\frac{1}{1+x^2}} \cdot e^{\frac{-1}{1+x^2}} dx$$

$$y \cdot e^{\frac{-1}{1+x^2}} = \frac{x^2}{2} + c$$

$(0, 0) \Rightarrow \boxed{C = 0}$

$$y(x) = \frac{x^2}{2} e^{\frac{1}{1+x^2}}$$

$$f(x) = \frac{x^2}{2}$$



$$A = \int_{-2}^{4} (x+4) - \frac{x^2}{2} dx = 18$$

23. The number of solutions of $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x-1)^2 = 0$, where $-\pi \le x \le \pi$, is

Ans. (2)

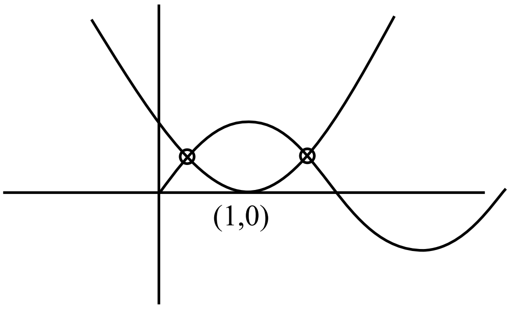
Sol. $\sin^2 x - (x^2 - 2x - 2)\sin x - 3(x-1)^2 = 0$

$\sin^2 x - (x-1)^2)\sin x - 3(x-1)^2 = 0$

roots : -3 $\quad (x-1)^2$

$\sin x = -3$ (reject) or $(x-1)^2$

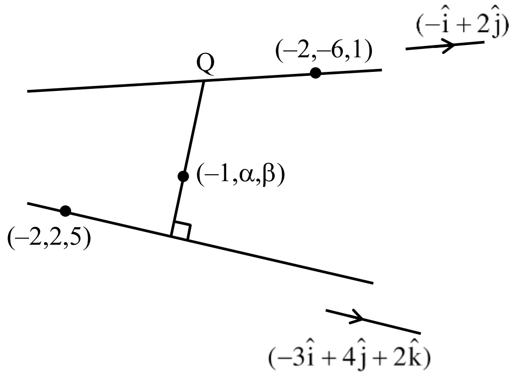
$\sin x = (x-1)^2$



24. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$. Then $(\alpha - \beta)^2$ is equal to _____.

Ans. (25)

Sol.



$P(-3\lambda - 2, 4\lambda + 2, 2\lambda + 5)$

$Q(-\mu - 2, 2\mu - 6, 1)$

DRS of PQ $= (3\lambda - \mu, 2\mu - 4\lambda - 8, -2\lambda - 4)$

$$\text{DRS of PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$$

$$= (4\hat{i} + 2\hat{j} + 2\hat{k})$$

OR

$(2, 1, 1)$

$$\frac{3\lambda - \mu}{2} = \frac{2\mu - 4\lambda - 8}{1} = \frac{-2\lambda - 4}{1}$$

$\Rightarrow \mu = \lambda + 2$ & $7\lambda = \mu - 8$

$\boxed{\lambda = -1}$ $\quad \boxed{\mu = 1}$

$Q : (-3, -4, 1)$

$$L_{PQ} = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$$

$$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha + 4}{1} = \frac{\beta - 1}{1}$$

$\Rightarrow \alpha = -3, \beta = 2$

$(\alpha - \beta)^2 = 25$

25. If

$$1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+....$$

upto ∞ = $2\left(\sqrt{\frac{b}{a}}+1\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with gcd(a, b) = 1, then 11a + 18b is equal to ______.

Ans. (76)

Sol. $S=1+\frac{x}{2\sqrt{3}}+\frac{x^2}{18}+\frac{x^3}{36\sqrt{3}}+\frac{x^4}{180}+....\infty$

Put $\frac{x}{\sqrt{3}}=t$, where $x=\sqrt{3}-\sqrt{2}$

$S=1+\frac{t}{2}+\frac{t^2}{6}+\frac{t^3}{12}+\frac{t^4}{20}+....$

$S=1+t\left(1-\frac{1}{2}\right)+t^2\left(\frac{1}{2}-\frac{1}{3}\right)+t^3\left(\frac{1}{3}-\frac{1}{4}\right)+t^4\left(\frac{1}{4}-\frac{1}{5}\right)$

$S=\left(1+t+\frac{t^2}{2}+\frac{t^3}{3}+\frac{t^3}{4}+...\right)-\left(\frac{t}{2}+\frac{t^2}{3}+\frac{t^3}{4}+\frac{t^4}{5}+...\right)$

$S=\left(t+\frac{t^2}{2}+...\right)-\frac{1}{t}\left(t+\frac{t^2}{2}+\frac{t^3}{3}+...\right)+2$

$S=2+\left(1-\frac{1}{t}\right)\left(-\log(1-t)\right)=\left(\frac{1}{t}-1\right)\log(1-t)+2$

$S=2+\left(\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}-1\right)\log\left(1-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)$

$S=2+\left(\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)\log e\frac{\sqrt{2}}{\sqrt{3}}$

$S=2+\frac{(\sqrt{6}+2)}{2}\log e\frac{2}{3}=2+\left(\sqrt{\frac{3}{2}}+1\right)\log e\frac{2}{3}$

a = 2, b = 3

11a + 18b = 11 × 2 +18 × 3 = 76

26. Let a > 0 be a root of the equation $2x^2 + x - 2 = 0$.

If $\lim_{x\to\frac{1}{a}}\frac{16\left(1-\cos(2+x-2x^2)\right)}{(1-ax^2)}=\alpha+\beta\sqrt{17}$, where $\alpha, \beta \in Z$ then $\alpha+\beta$ is equal to ______.

Ans. (170)

Sol. $2x^2+x-2=0 \begin{matrix}\nearrow a\\ \searrow b\end{matrix}$

$2x^2-x-2=0 \begin{matrix}\nearrow \frac{1}{a}\\ \searrow \frac{1}{b}\end{matrix}$

$\lim_{x\to\frac{1}{a}}16\cdot\frac{\left(1-\cos2\left(x-\frac{1}{a}\right)\left(x-\frac{1}{b}\right)\right)}{4\left(x-\frac{1}{b}\right)^2}\times\frac{4\left(x-\frac{1}{b}\right)^2}{a^2\left(x-\frac{1}{a}\right)^2}$

$=16\times\frac{2}{a^2}\left(\frac{1}{a}-\frac{1}{b}\right)^2$

$=\frac{32}{a^2}\left(\frac{17}{4}\right)=\frac{17.8}{a^2}=\frac{17\times8\times16}{\left(-1+\sqrt{117}\right)^2}$

$=\frac{136.16}{18.2\sqrt{7}}\times\frac{18+2\sqrt{7}}{18+2\sqrt{7}}$

$=\frac{136}{256}\left(18+2\sqrt{7}\right)\cdot16$

$=153+17\sqrt{17}=\alpha+\beta\sqrt{17}$

$\alpha+\beta = 153 + 17 = 170$

27. If $f(t)=\int_0^{\pi}\frac{2x\,dx}{1-\cos^2 t\sin^2 x}$, $0 < t < \pi$, then the value of $\int_0^{\frac{\pi}{2}}\frac{\pi^2 dt}{f(t)}$ equals _____.

Ans. (1)

Sol. $f(t)=\int_0^{\pi}\frac{2x}{1-\cos^2 t\sin^2 x}dx$ .....(1)

$$= 2\int_0^{\pi} \frac{(\pi - x)dx}{1-\cos^2 t \sin^2 x} \quad ....(2)$$

$$2f(t) = 2\int_0^{\pi} \frac{\pi}{1-\cos^2 t \sin^2 x}dx$$

$$f(t) = \int_0^{\pi} \frac{\pi}{1-\cos^2 t \sin^2 x}dx$$

divide & by $\cos^2 x$

$$f(t) = \pi\int_0^{\pi} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \tan^2 x}$$

$$f(t) = 2\pi\int_0^{\pi/2} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \tan^2 x}$$

$\tan x = z$

$\sec^2 x dx = dz$

$$f(t) = 2\pi\int_0^{\infty} \frac{dz}{1+\sin^2 t \cdot z^2}$$

$$= \frac{\pi^2}{\sin t}$$

Then $\displaystyle\int_0^{\pi/2} \frac{\pi^2}{f(t)}dt$

$$= \int_0^{\pi/2} \sin t\, dt$$

$= 1$

28. Let the maximum and minimum values of $\left(\sqrt{8x - x^2 - 12} - 4\right)^2 + (x-7)^2$, $x \in R$ be M and m respectively. Then $M^2 - m^2$ is equal to _____.

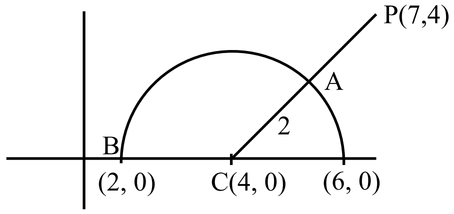
Ans. (1600)

Sol. $(x-7)^2 + (y-4)^2$

$y = \sqrt{8x - x^2 - 12}$

$y^2 = -(x-4)^2 + 16 - 12$

$(x-4)^2 + y^2 = 4$



$m = 9$

$M = 41$

$M^2 - m^2 = 41^2 - 9^2 = 1600$

29. Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x - 9)$ at the point P. The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is ______.

Ans. (10)

Sol. $y^2 = 4(x-9)$

slope of tangent $= \frac{-1}{2}$

Point of contact $P\left(9 + \frac{1}{\left(-\frac{1}{2}\right)^2}, \frac{2\times 1}{\frac{-1}{2}}\right)$

P(13, –4)

center of circle C(7, 4)

distance $CP = \sqrt{(13-7)^2 + (-4-4)^2}$

$= 10$

30. The number of real solutions of the equation $x|x+5| + 2|x+7| - 2 = 0$ is _____.

Ans. (3)

30. The number of real solutions of the equation $x|x+5| + 2|x+7| - 2 = 0$ is _____.

Allen Ans. (3)

Sol. **Case I :** $x \geq -5$

$x^2 + 5x + 2x + 12 = 0$

$x^2 + 7x + 12 = 0$

$x = -3, -4$

Case II : $-7 < x < -5$

$-x^2 - 5x + 2x + 14 - 2 = 0$

$-x^2 - 3x + 12 = 0$

$$x = \frac{-3 \pm \sqrt{9+48}}{2}$$

$$= \frac{-3 \pm \sqrt{57}}{2}$$

$$x = \frac{-3-\sqrt{57}}{2}, \frac{-3+\sqrt{57}}{2} \text{(rejected)}$$

Case III : $x \leq -7$

$-x^2 - 5x - 2x - 14 - 2 = 0$

$x^2 + 7x + 16 = 0$

$D = 49 - 64 < 0$

No solutions

No. of solutions = 3

SECTION-A

31. Given below are two statements :

Statement I : When the white light passed through a prism, the red light bends lesser than yellow and violet.

Statement II : The refractive indices are different for different wavelengths in dispersive medium. In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true.

(2) Statement I is true but Statement II is false.

(3) Both Statement I and Statement II are false.

(4) Statement I is false but Statement II is true.

Ans. **(1)**

Sol. As $\lambda_{red} > \lambda_{yellow} > \lambda_{violet}$

Light ray with longer wavelength bends less.

32. Which of the following statement is not true about stopping potential (V_0)?

(1) It depends on the nature of emitter material.

(2) It depends upon frequency of the incident light.

(3) It increases with increase in intensity of the incident light.

(4) It is 1/e times the maximum kinetic energy of electrons emitted.

Ans. **(3)**

Sol. $KE_{max} = h\nu - \phi_0 = eV$

33. The angular momentum of an electron in a hydrogen atom is proportional to : (Where r is the radius of orbit of electron)

(1) $\sqrt{r}$ (2) $\frac{1}{r}$

(3) r (4) $\frac{1}{\sqrt{r}}$

Ans. **(1)**

Sol. $F_C = \frac{mv^2}{r}$

$$\frac{Kq_1q_2}{r^2} = \frac{mv^2}{r}$$

$$mv^2r^2 = Kq_1q_2r$$

$$\frac{L^2}{m} = Kq_1q_2r$$

$$L \propto \sqrt{r}$$

34. A galvanometer of resistance 100 Ω when connected in series with 400 Ω measures a voltage of upto 10 V. The value of resistance required to convert the galvanometer into ammeter to read upto 10 A is $x \times 10^{-2}$ Ω. The value of x is :

(1) 2 (2) 800

(3) 20 (4) 200

Ans. **(3)**

Sol. $i_g = \frac{10}{400+100} = 20\times10^{-3}\,A$

For ammeter

Let shunt resistance = S

$i_gR = (i - i_g)S$

$20 \times 10^{-3} \times 100 = 10\,S$

$S = 20 \times 10^{-2}\,\Omega$

35. The vehicles carrying inflammable fluids usually have metallic chains touching the ground :

(1) To conduct excess charge due to air friction to ground and prevent sparking.

(2) To alert other vehicles.

(3) To protect tyres from catching dirt from ground.

(4) It is a custom.

Ans. **(1)**

Sol. Static charge is developed due to air friction. This can result in combustion. So, metallic chains is used to discharge excess charge.

36. If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e. mean free path) is represented by :

(1) $\frac{1}{\sqrt{2n\pi d^2}}$ (2) $\sqrt{2}n\pi d^2$

(3) $\frac{1}{\sqrt{2}n\pi d^2}$ (4) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$

Ans. **(3)**

Sol. n = number of molecule per unit volume

d = diameter of the molecule

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$ (By Theory)

37. A particle moves in x-y plane under the influence of a force $\vec{F}$ such that its linear momentum is $\vec{P}(t) = \hat{i}\cos(kt) - \hat{j}\sin(kt)$. If k is constant, the angle between $\vec{F}$ and $\vec{P}$ will be :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

Ans. **(1)**

Sol. $\vec{P} = \cos(kt)\hat{i} - \sin(kt)\hat{j}$; $|\vec{P}| = 1$

$\because \vec{P} = m\vec{v}$

$\therefore \hat{P} = \hat{v}$

$\Rightarrow \hat{v} = \cos(kt)\hat{i} - \sin(kt)\hat{j}$

$\hat{a} = \frac{-k\sin(kt)\hat{i} - k\cos(kt)\hat{j}}{k}$

$\Rightarrow \hat{a} = -\sin kt\hat{i} - \cos kt\hat{j}$

$\because \hat{F} = \hat{a} = -\sin kt\,\hat{i} - \cos kt\,\hat{j}$

$\cos\theta = \frac{\hat{F}.\hat{P}}{|\hat{F}||\hat{P}|} = -\frac{\sin kt\cos t + \sin kt\cos t}{1\times 1} = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

38. The electrostatic force $(\vec{F}_1)$ and magnetic force $(\vec{F}_2)$ acting on a charge q moving with velocity v can be written :

(1) $\vec{F}_1 = q\vec{V}.\vec{E}$, $\vec{F}_2 = q(\vec{B}.\vec{V})$

(2) $\vec{F}_1 = q\vec{B}$, $\vec{F}_2 = q(\vec{B}\times\vec{V})$

(3) $\vec{F}_1 = q\vec{E}$, $\vec{F}_2 = q(\vec{V}\times\vec{B})$

(4) $\vec{F}_1 = q\vec{E}$, $\vec{F}_2 = q(\vec{B}\times\vec{V})$

Ans. **(3)**

Sol. $\vec{F}_1 = q\vec{E}$ (Theory)

$\vec{F}_2 = q(\vec{V}\times\vec{B})$

39. A man carrying a monkey on his shoulder does cycling smoothly on a circular track of radius 9m and completes 120 revolutions in 3 minutes. The magnitude of centripetal acceleration of monkey is (in m/s^2) :

(1) zero (2) $16\pi^2\ ms^{-2}$

(3) $4\pi^2\ ms^{-2}$ (4) $57600\ \pi^2\ ms^{-2}$

Ans. **(2)**

Sol. Given : R = 9m,

120 revolution in 3 min

$\omega = \frac{120\ \text{Rev.}}{3\ \text{min.}} = \frac{120\times 2\pi\ \text{rad}}{3\times 60\ \text{sec}} = \frac{4\pi}{3}\ \text{rad/s}$

$a_{centripetal} = \omega^2 R = \left(\frac{4\pi}{3}\right)^2 \times 9 = 16\pi^2\ m/s^2$

40. A series LCR circuit is subjected to an AC signal of 200 V, 50 Hz. If the voltage across the inductor (L = 10 mH) is 31.4 V, then the current in this circuit is ________ :

(1) 68 A (2) 63 A

(3) 10 A (4) 10 mA

Ans. **(3)**

Sol. Voltage across inductor $V_L = IX_L$

$31.4 = I[L\omega]$

$31.4 = I[L(2\pi f)]$

$31.4 = I[10 \times 10^{-3}(2 \times 3.14) \times 50$

$\Rightarrow I = 10$ A

41. What is the dimensional formula of ab^{-1} in the equation $\left(P+\frac{a}{V^2}\right)(V-b)=RT$, where letters have their usual meaning.

(1) $[M^0L^3T^{-2}]$ (2) $[ML^2T^{-2}]$

(3) $[M^{-1}L^5T^3]$ (4) $[M^6L^7T^4]$

Ans. **(2)**

Sol. $\because$ [V] = [b]

$\therefore$ Dimension of b = $[L^3]$

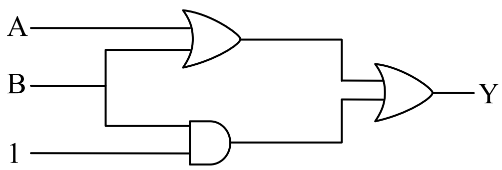
& $[P]=\left[\frac{a}{V^2}\right]$

$[a] = [PV^2] = [ML^{-1}T^{-2}][L^6]$

Dimension of a = $[ML^5T^{-2}]$

$$\therefore\ ab^{-1}=\frac{[ML^5T^{-2}]}{[L^3]}=[ML^2T^{-2}]$$

42. The output (Y) of logic circuit given below is 0 only when :

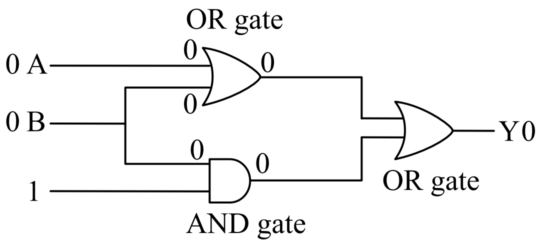


(1) A = 1, B = 0 (2) A = 0, B = 0

(3) A = 1, B = 1 (4) A = 0, B = 1

Ans. **(2)**

Sol.



43. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to :

(1) t^2 (2) $t^{2/3}$

(3) $t^{3/2}$ (4) t

Ans. **(3)**

Sol. P = costant $\Rightarrow$ FV = constant

$$\Rightarrow m\frac{dV}{dt}V = \text{constant}$$

$$\int_0^V VdV=(C)\int_0^t dt$$

$$\left(\frac{V^2}{2}\right)=Ct$$

$V\propto t^{1/2}$

$$\frac{ds}{dt}\propto t^{1/2}$$

$$\int_0^S ds=K\int_0^t t^{1/2}dt$$

$$S=K\times\frac{2}{3}t^{3/2}$$

$S\propto t^{3/2}$

$\therefore$ displacement is proportional to $(t)^{3/2}$

44. Match **List-I** with **List-II** :-

	List-I **EM-Wave**		**List-II** **Wavelength Range**
(A)	Infra-red	(I)	$< 10^{-3}$ nm
(B)	Ultraviolet	(II)	400 nm to 1 nm
(C)	X-rays	(III)	1 mm to 700 nm
(D)	Gamma rays	(IV)	1 nm to 10^{-3} nm

Choose the correct answer from the options given below :

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(4) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

Ans. **(2)**

Sol. Infrared is the least energetic thus having biggest wavelength (λ) & gamma rays are most energetic thus having smallest wavelength (λ).

45. During an adiabatic process, if the pressure of a gas is found to be proportional to the cube of its absolute temperature, then the ratio of $\frac{C_P}{C_V}$ for the gas is :

(1) $\frac{5}{3}$ (2) $\frac{9}{7}$

(3) $\frac{3}{2}$ (4) $\frac{7}{5}$

Ans. **(3)**

Sol. $P \propto T^3$

PT^{-3} = constant

$\because \frac{PV}{T} = nR$ = constant from ideal gas equation

(P) $(PV)^{-3}$ = constant

$P^{-2} V^{-3}$ = cosntant ...(1)

$\because$ Process equation for adiabatic process is

PV^y = constant ...(2)

Comparing equation (1) and (2)

$$\frac{C_P}{C_V} = y = \frac{3}{2}$$

46. Match **List-I** with **List-II** :

	List-I		**List-II**
(A)	A force that restores an elastic body of unit area to its original state	(I)	Bulk modulus
(B)	Two equal and opposite forces parallel to opposite faces	(II)	Young's modulus
(C)	Forces perpendicular everywhere to the surface per unit area same everywhere	(III)	Stress
(D)	Two equal and opposite forces perpendicular to opposite faces	(IV)	Shear modulus

Choose the correct answer from the options given below :

(1) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(4) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Ans. **(3)**

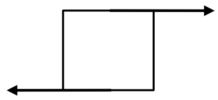
Sol. **(A)** stress $= \frac{F_{restoring}}{A}$

If A = 1

Stress = $F_{restoring}$

(A)-(III)

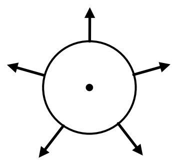
(B)



$\Rightarrow$ Shear stress

(B)-(IV)

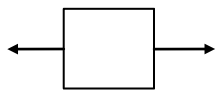
(C)



$\Rightarrow$ Volumetric stress

(C)-(I)

(D)



$\Rightarrow$ Longitudinal stress

(D)-(II)

47. A vernier callipers has 20 divisions on the vernier scale, which coincides with 19^{th} division on the main scale. The least count of the instrument is 0.1 mm. One main scale division is equal to ____mm.

(1) 1 (2) 0.5

(3) 2 (4) 5

Ans. **(3)**

Sol. 20 VSD = 19 MSD

$1VSD = \frac{19}{20} MSD$

L.C. = 1 MSD – 1 VSD

$0.1 \text{ mm} = 1MSD - \frac{19}{20} MSD$

$0.1 = \frac{1}{20} MSD$

1 MSD = 2mm

48. A heavy box of mass 50 kg is moving on a horizontal surface. If co-efficient of kinetic friction between the box and horizontal surface is 0.3 then force of kinetic friction is :

(1) 14.7 N

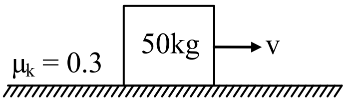
(2) 147 N

(3) 1.47 N

(4) 1470 N

Ans. (2)

Sol.



$F_k = \mu_k N = 0.3 \times 50 \times 9.8 = 147$ N

49. A satellite revolving around a planet in stationary orbit has time period 6 hours. The mass of planet is one-fourth the mass of earth. The radius orbit of planet is : (Given = Radius of geo-stationary orbit for earth is 4.2×10^4 km)

(1) 1.4×10^4 km

(2) 8.4×10^4 km

(3) 1.68×10^5 km

(4) 1.05×10^4 km

Ans. (4)

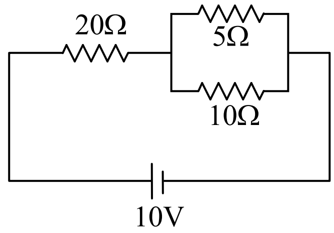
Sol. $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}\left(\frac{M_2}{M_1}\right)^{1/2}$$

$$\frac{6}{24} = \frac{(r_1)^{3/2}}{(4.2\times10^4)^{3/2}}\left(\frac{M}{M/4}\right)^{1/2}$$

$r_1 = 1.05 \times 10^4$ km

50. The ratio of heat dissipated per second through the resistance 5 Ω and 10 Ω in the circuit given below is :

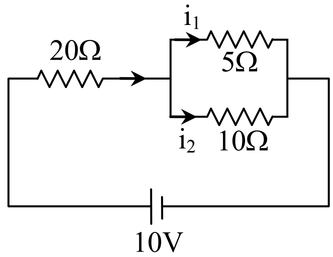


(1) 1 : 2 (2) 2 : 1

(3) 4 : 1 (4) 1 : 1

Ans. (2)

Sol.



$$\frac{i_1}{i_2} = \frac{10}{5} = \frac{2}{1}$$

$$\frac{P_1}{P_2} = \frac{i_1^2 R_1}{i_2^2 R_2} = \left(\frac{2}{1}\right)^2 \times \frac{5}{10} = \frac{2}{1}$$

SECTION-B

51. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 'm' number of turns. It carries a current of 5A. If the magnitude of the magnetic field inside the solenoid is 6.28×10^{-3} T, then the value of m is :

Ans. (500)

Sol. $\mu_0 n i = B$ n = number of turns per unit length

$$\mu_0\left(\frac{m}{\ell}\right)i = B$$

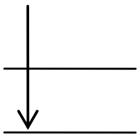
$$m = \frac{B.\ell}{\mu_0 i} = \frac{6.28\times10^{-3}\times0.5}{12.56\times10^{-7}\times5}$$

m = 500

52. The shortest wavelength of the spectral lines in the Lyman series of hydrogen spectrum is 915 Å. The longest wavelength of spectral lines in the Balmer series will be _______ Å.

Ans. (6588)

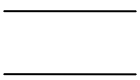
Sol. Lyman Series



Shortest, $\frac{hc}{\lambda} = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\lambda\downarrow E\uparrow$; $\frac{hc}{\lambda_0} = -13.6(1)$

Balmer Series :



$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{hc}{\lambda_1} = -13.6\times\left(\frac{5}{36}\right)$$

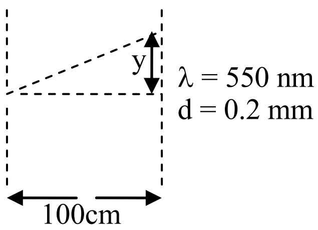
$$\Rightarrow \frac{-13.6\lambda_0}{\lambda_1} = -13.6\times\frac{5}{36}$$

$$\lambda_1 = \frac{\lambda_0\times 36}{5} = \frac{915\times 36}{5} = 6588$$

53. In a single slit experiment, a parallel beam of green light of wavelength 550 nm passes through a slit of width 0.20 mm. The transmitted light is collected on a screen 100 cm away. The distance of first order minima from the central maximum will be x × 10^{-5} m. The value of x is :

Ans. (275)

Sol.

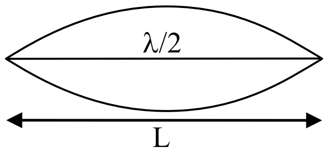


$$y = \frac{\lambda D}{d} = \frac{550\times10^{-9}\times100\times10^{-2}}{0.2\times10^{-3}} = 275$$

54. A sonometer wire of resonating length 90 cm has a fundamental frequency of 400 Hz when kept under some tension. The resonating length of the wire with fundamental frequency of 600 Hz under same tension _______ cm.

Ans. (60)

Sol.



$f_0 = 400$ Hz ; $v = \sqrt{\frac{T}{\mu}}$ = constant

$\frac{\lambda}{2} = L$; $v = f_0\lambda$

$\frac{v}{2f_0} = L \Rightarrow v = 2Lf_0$

$$L' = \frac{v}{2f'} = \frac{2Lf_0}{2f'}$$

$$= \frac{Lf_0}{f'} = \frac{90\times400}{600} = 60$$

55. A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $\frac{x}{5}$. The value of x is ________.

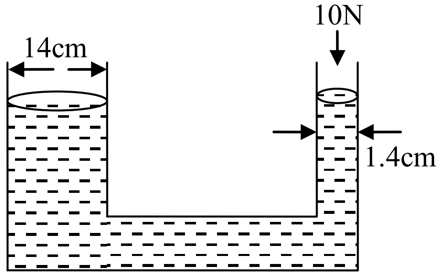
Ans. (2)

Sol. $$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2} = \frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}mR^2\right)\omega^2}{\left(\frac{1}{2}\right)\left(\frac{2}{3}mR^2\right)\omega^2 + \frac{1}{2}m(R\omega)^2}$$

$$= \frac{\frac{2}{3}}{\frac{2}{3}+1} = \frac{2}{5}$$

x = 2

56. A hydraulic press containing water has two arms with diameters as mentioned in the figure. A force of 10 N is applied on the surface of water in the thinner arm. The force required to be applied on the surface of water in the thicker arm to maintain equilibrium of water is _______ N.



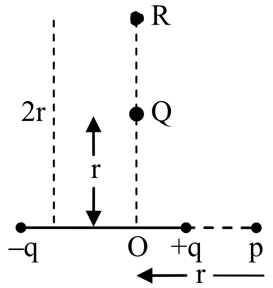
Ans. **(1000 N)**

Sol. $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$$\frac{F_1}{\pi(7)^2} = \frac{10}{\pi \times (0.7)^2}$$

$F_1 = 1000$ N

57. The electric field at point p due to an electric dipole is E. The electric field at point R on equitorial line will be $\frac{E}{x}$. The value of x :



Ans. **(16)**

Sol. $E_P = \frac{2KP}{r^3} = E$

$$E_R = \frac{KP}{(2r)^3} = \frac{E}{16}$$

x = 16

58. The maximum height reached by a projectile is 64 m. If the initial velocity is halved, the new maximum height of the projectile is ______ m.

Ans. (16)

Sol. $H_{max} = \frac{u^2 \sin^2 \theta}{2g}$

$$\frac{H_{1max}}{H_{2max}} = \frac{u_1^2}{u_2^2}$$

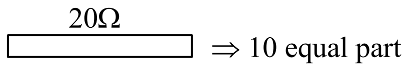
$$\frac{64}{H_{2\,max}} = \frac{u^2}{(u/2)^2}$$

$H_{2max} = 16$ m

59. A wire of resistance 20 Ω is divided into 10 equal parts. A combination of two parts are connected in parallel and so on. Now resulting pairs of parallel combination are connected in series. The equivalent resistance of final combination is _______ Ω.

Ans. (5)

Sol.



Each part has resistance = 2Ω

2 parts are connected in parallel so, R = 1Ω

Now, there will be 5 parts each of resistance 1Ω, they are connected in series.

$R_{eq} = 5R$, $R_{eq} = 5\Omega$

60. The current in an inductor is given by I = (3t + 8) where t is in second. The magnitude of induced emf produced in the inductor is 12 mV. The self-inductance of the inductor _______ mH.

Ans. (4)

Sol. I = 3t + 8

ε = 12 mV

$$|\varepsilon| = L\left|\frac{dI}{dt}\right|$$

12 = L × 3

L = 4 mH

SECTION-A

61. Match **List - I** with **List - II**.

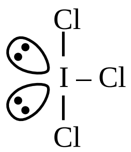
List - I	**List - II**
(A) ICI	(I) T -Shape
(B) ICI_3	(II) Square pyramidal
(C) CIF_5	(III) Pentagonal bipyramidal
(D) IF_7	(IV) Linear

Choose the **correct** answer from the options given below:

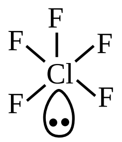
(1) (A)–(I), (B)–(IV), C–(III), D–(II)
(2) (A)–(I), (B)–(III), C–(II), D–(IV)
(3) (A)–(IV), (B)–(I), C–(II), D–(III)
(4) (A)–(IV), (B)–(III), C–(II), D–(I)

Ans. (3)

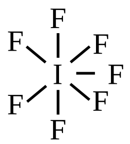
Sol. A. I – Cl (iv) linear

B. 

(I) T-shape

C. 

(II) Square pyramidal

D. 

(III) Pentagonal bipyramidal

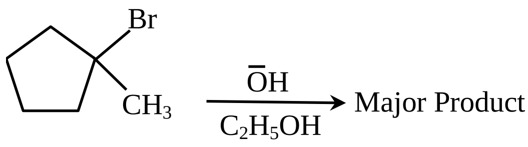
62. While preparing crystals of Mohr's salt, dil. H_2SO_4 is added to a mixture of ferrous sulphate and ammonium sulphate, before dissolving this mixture in water, dil. H_2SO_4 is added here to:

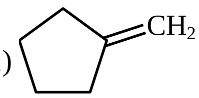
(1) prevent the hydrolysis of ferrous sulphate
(2) prevent the hydrolysis of ammonium sulphate
(3) make the medium strongly acidic
(4) increase the rate of formation of crystals

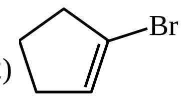
Ans. (1)

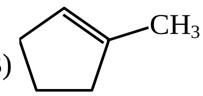
Sol. Fe^{+2} ions undergoes hydrolysis, therefore while preparing aqueous solution of ferrous sulphate and ammonium sulphate in water dilute sulphuric acid is added to prevent hydrolysis of ferrous sulphate.

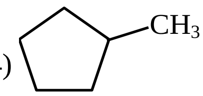
63. Identify the major product in the following reaction.



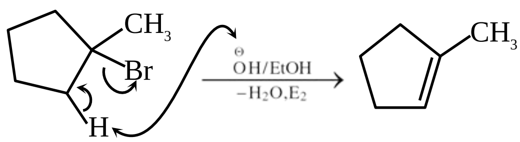
(1) 

(2) 

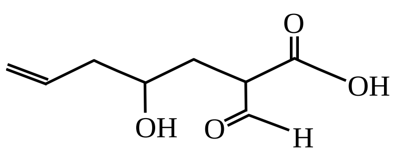
(3) 

(4) 

Ans. (3)

Sol. 

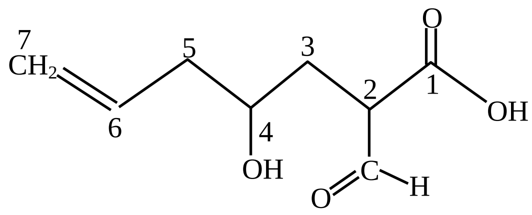
64. The correct nomenclature for the following compound is:



(1) 2–carboxy–4–hydroxyhept–6–enal
(2) 2–carboxy–4–hydroxyhept–7–enal
(3) 2–formyl–4–hydroxyhept–6–enoic acid
(4) 2–formyl–4–hydroxyhept–7–enoic acid

Ans. (3)

Sol.



2-formly-4-hydroxyhept-6-enoic acid

65. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : NH_3 and NF_3 molecule have pyramidal shape with a lone pair of electrons on nitrogen atom. The resultant dipole moment of NH_3 is greater than that of NF_3.

Reason (R) : In NH_3, the orbital dipole due to lone pair is in the same direction as the resultant dipole moment of the N–H bonds. F is the most electronegative element.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

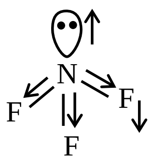
(2) **(A)** is false but **(R)** is true

(3) **(A)** is true but **(R)** is false

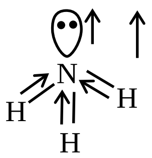
(4) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of **(A)**

Ans. **(1)**

Sol.



Resultant dipole moment = 0.80×10^{-30} Cm



Resultant dipole moment = 4.90×10^{-30} cm

66. Given below are two statements:

Statement I : On passing $HCl_{(g)}$ through a saturated solution of $BaCl_2$, at room temperature white turbidity appears.

Statement II : When HCl gas is passed through a saturated solution of NaCl, sodium chloride is precipitated due to common ion effect.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is incorrect but **Statement II** is correct

(4) Both **Statement I** and **Statement II** are correct

Ans. **(1)**

Sol. $BaCl_2$, NaCl are soluble but on adding HCl(g) to $BaCl_2$, NaCl solutions, Sodium or Barium chlorides may precipitate out, as a consequence of the law of mass action.

67. The metal atom present in the complex MABXL (where A, B, X and L are unidentate ligands and M is metal) involves sp^3 hybridization. The number of geometrical isomers exhibited by the complex is:

(1) 4 (2) 0

(3) 2 (4) 3

Ans. **(2)**

Sol. Tetrahedral complex does not show geometrical isomerism.

68. Match **List - I** with **List - II**.

	List - I (Pair of Compounds)		**List - II (Isomerism)**
(A)	n-propanol and Isopropanol	(I)	Metamerism
(B)	Methoxypropane and ethoxyethane	(II)	Chain Isomerism
(C)	Propanone and propanal	(III)	Position Isomerism
(D)	Neopentane and Isopentane	(IV)	Functional Isomerism

(1) (A)–(II), (B)–(I), (C)–(IV), (D)–(III)

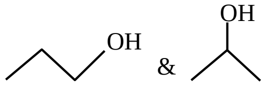
(2) (A)–(III), (B)–(I), (C)–(II), (D)–(IV)

(3) (A)–(I), (B)–(III), (C)–(IV), (D)–(II)

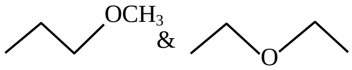
(4) (A)–(III), (B)–(I), (C)–(IV), (D)–(II)

Ans. **(4)**

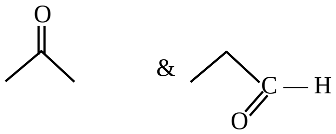
Sol.



⇒ Position isomers



⇒ Metamers



⇒ Functional isomers

⇒ Chain isomers

neopentane isopentane

69. The quantity of silver deposited when one coulomb charge is passed through $AgNO_3$ solution:

(1) 0.1 g atom of silver

(2) 1 chemical equivalent of silver

(3) 1 g of silver

(4) 1 electrochemical equivalent of silver

Ans. (4)

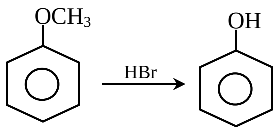
Sol. W = ZIt

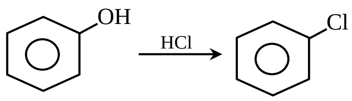
W = ZQ

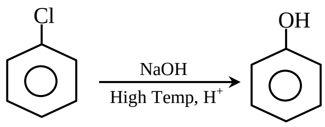
$Q = \frac{W}{Z}$

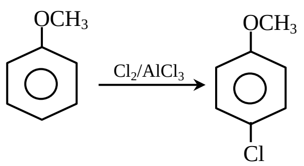
W = ZQ = (electrochemical equivalent)

70. Which one of the following reactions is NOT possible?

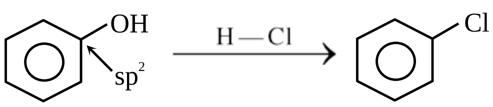
(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. 

Not Possible

71. Given below are two statements :

Statement I : The metallic radius of Na is 1.86 A° and the ionic radius of Na^+ is lesser than 1.86 A°.

Statement II : Ions are always smaller in size than the corresponding elements.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is correct but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is incorrect but **Statement II** is true

Ans. (1)

Sol. $r_{Na} > r_{Na^+}$

So, Statement (I) is correct but size of anions are greater than size of neutral atoms.

So statement (II) is incorrect.

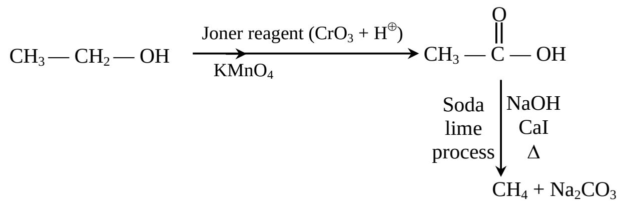
72. $CH_3CH_2–OH \xrightarrow[\text{(ii) } KMnO_4 \text{ (iii)NaOH, CaO,}\Delta]{\text{(i) Jone's Reagent}} P$

Consider the above reaction sequence and identify the major product P.

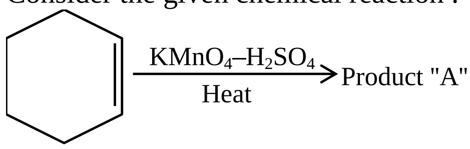
(1) Methane (2) Methanal

(3) Methoxymethane (4) Methanoic acid

Ans. (1)

Sol. 

73. Consider the given chemical reaction :

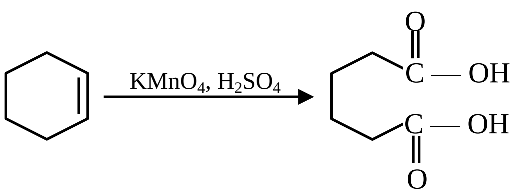


Product "A" is :

(1) picric acid (2) oxalic acid

(3) acetic acid (4) adipic acid

Ans. (4)

Sol. 

74. For the electro chemical cell
$M|M^{2+}||X|X^{2-}$

If $E^0_{\left(M^{2+}/M\right)} = 0.46\,V$ and $E^0_{\left(X/X^{2-}\right)} = 0.34V$.

Which of the following is **correct** ?

(1) $E_{cell} = -0.80$ V

(2) $M + X \rightarrow M^2 + X^{2-}$ is a spontaneous reaction

(3) $M^{2+} + X^{2-} \rightarrow M + X$ is a spontaneous reaction

(4) $E_{cell} = 0.80$ V

Ans. **(3)**

Sol. $M \mid M^{+2} \parallel X / X^{2-}$

$E^o_{cell} = E^o_{M/M^{+2}} + E^o_{X/X^{-2}}$

$= -0.46 + 0.34 = -0.12V$

As E^o_{cell} is negative so anode becomes cathode and cathode become anode. Spontaneous reaction will be

$M^{+2} + X^{2-} \longrightarrow M + X$

75. The number of moles of methane required to produce 11g CO_2(g) after complete combustion is:
(Given molar mass of methane in g mol^{-1} : 16)

(1) 0.75 (2) 0.25

(3) 0.35 (4) 0.5

Ans. **(2)**

Sol. $C_nH_{2n+2} + \frac{3n+1}{2}O_2 \longrightarrow nCO_2 + (n+1)H_2O$

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

4gm 11gm

0.25 mole 0.25 mole

0.25 mole CH_4 gives 0.25 mole (or 11gm) CO_2

76. The number of complexes from the following with no electrons in the t_2 orbital is ________.
$TiCl_4$, $[MnO_4]^-$, $[FeO_4]^{2-}$, $[FeCl_4]^-$, $[CoCl_4]^{2-}$

(1) 3 (2) 1

(3) 4 (4) 2

Ans. **(1)**

Sol. $TiCl_4 \Rightarrow Ti^{+4}$ $e^0t_2^0$

$MnO_4^- \Rightarrow Mn^{+7}$ $e^0t_2^0$

$FeO_4^{2-} \Rightarrow Fe^{+6}$ $e^2t_2^0$

$FeCl_4^{2-} \Rightarrow Fe^{+2}$ $e^3t_2^3$

$CoCl_4^{2-} \Rightarrow Co^{+2}$ $e^4t_2^3$

77. The number of ions from the following that have the ability to liberate hydrogen from a dilute acid is ________. Ti^{2+}, Cr^{2+} and V^{2+}

(1) 0 (2) 2

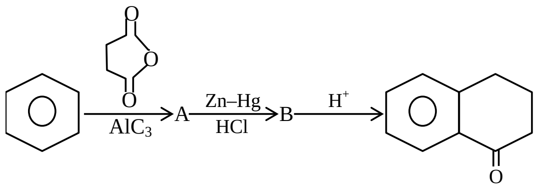
(3) 3 (4) 1

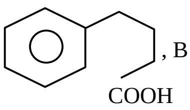
Ans. **(3)**

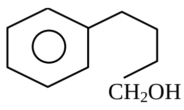
Sol. The ions Ti^{+2}, V^{+2} Cr^{+2} are strong reducing agents and will liberate hydrogen from a dilute acid, eg.

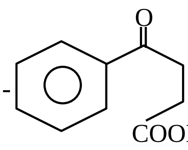
$$2Cr^{+2}_{(aq.)} + 2H^+_{(aq.)} \longrightarrow 2Cr^{+3}_{(aq.)} + H_2(g)$$

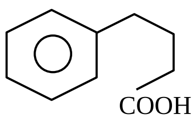
78. Identify A and B in the given chemical reaction sequence : -

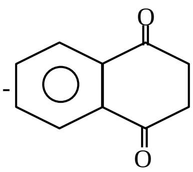


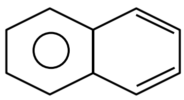
(1) A - 

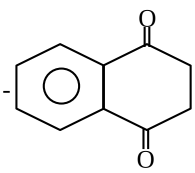
, B - 

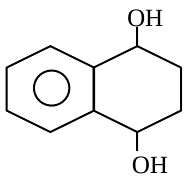
(2) A - 

, B - 

(3) A - 

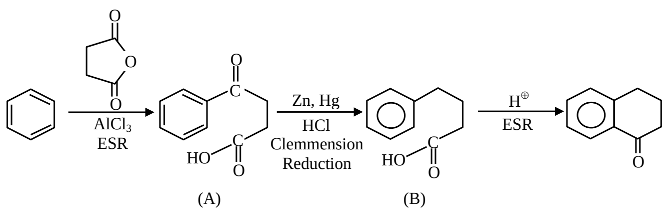
, B - 

(4) A - 

, B - 

Ans. **(2)**

Sol.



79. The correct statements from the following are :

(A) The decreasing order of atomic radii of group 13 elements is Tl > In > Ga > Al > B.

(B) Down the group 13 electronegativity decreases from top to bottom.

(C) Al dissolves in dil. HCl and liberate H_2 but conc. HNO_3 renders Al passive by forming a protective oxide layer on the surface.

(D) All elements of group 13 exhibits highly stable +1 oxidation state.

(E) Hybridisation of Al in $[Al(H_2O)_6]^{3+}$ ion is sp^3d^2.

Choose the **correct** answer from the options given below :

(1) (C) and (E) only

(2) (A), (C) and (E) only

(3) (A), (B), (C) and (E) only

(4) (A) and (C) only

Ans. (1)

Sol. A. size order $T\ell > In > Al > Ga > B$

B. Electronegativity order $B > Al < Ga < In < T\ell$

D. B, Al are more stable in +3 oxidation state

So, only C, E statements are correct.

80. Coagulation of egg, on heating is because of :

(1) Denaturation of protein occurs

(2) The secondary structure of protein remains unchanged

(3) Breaking of the peptide linkage in the primary structure of protein occurs

(4) Biological property of protein remains unchanged

Ans. (1)

Sol. Coagulation of egg give primary structure of protein, which is known as denaturation of protein

SECTION-B

81. Combustion of 1 mole of benzene is expressed at

$$C_6H_6(1) + \frac{15}{2}O_2(g) \rightarrow CO_2(g) + 3H_2O(1).$$

The standard enthalpy of combustion of 2 mol of benzene is – 'x' kJ.

x = _______.

(1) standard Enthalpy of formation of 1 mol of $C_6H_6(1)$, for the reaction $6C(graphite) + 3H_2(g) \rightarrow C_6H_6(1)$ is 48.5 kJ mol^{-1}.

(2) Standard Enthalpy of formation of 1 mol of $CO_2(g)$, for the reaction $C(graphite) + O_{2(g)} \rightarrow CO_2(g)$ is –393.5 kJ mol^{-1}.

(3) Standard and Enthalpy of formation of 1 mol of $H_2O(1)$, for the reaction $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(1)$ is –286 kJ mol^{-1}.

Ans. (6535)

Sol. $6C(graphite)+3H_2(g) \rightarrow C_6H_6(\ell)$; $\Delta H = 48.5$ kJ/mol

$C(graphite)+O_2(g) \rightarrow CO_2(g)$; $\Delta H = -393.5$ kJ/mol

$H_2^{(g)} + \frac{1}{2}(g) \longrightarrow H_2O(\ell)$; $\Delta H = -286$ kJ/mol

equation –(1) × 1 + (2) × 6 + (3) × 3

– 48.5 –6 × 393.5 – 3 × 286

= – 3267.5 kJ for 1 mol

= – 6535 kJ for 2 mol

Ans. 6535 kJ

82. The fusion of chromite ore with sodium carbonate in the presence of air leads to the formation of products A and B along with the evolution of CO_2. The sum of spin-only magnetic moment values of A and B is ___ B.M. (Nearest integer)

(Given atomic number : C : 6, Na : 11, O : 8, Fe : 26, Cr : 24]

Ans. (6)

Sol. $4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

A = Na_2CrO_4, B = Fe_2O_3

Spin only magnetic moment

For Na_2CrO_4 $\mu_B = 0$

For Fe_2O_3 $\mu_B = 5.9$

sum = 5.9

83. X of enthanamine was subjected to reaction with $NaNO_2$/HCl followed by hydrolysis to liberate N_2 and HCl. The HCl generated was completely neutralised by 0.2 moles of NaOH. X is _____ g.

Ans. (9)

Sol. $CH_3—CH_2—NH_2 \xrightarrow{NaNO_2 + HCl} CH_3—CH_2—\overset{\oplus}{N_2}\overset{\ominus}{Cl}$

0.2 mole

MW of ethanamine = 45

45 × 0.2 = 9 gm

$\downarrow$ HOH

$CH_3—CH_2—OH + N_2 + HCl$

(g)

0.2 mole

84. In an atom, total number of electrons having quantum numbers n = 4, $|m_l| = 1$ and $m_s = -\frac{1}{2}$ is

Ans. (6)

Sol. n = 4

ℓ	m_ℓ
0	0
1	–1, 0, +1
2	–2, –1, 0, +1, +2, +3

So number of orbital associated with

n = 4, $|m_\ell| = 1$ are 6

Now each orbital contain one e^- with $m_s = -\frac{1}{2}$

85. Using the given figure, the ratio of R_f values of sample A and sample C is $x \times 10^{-2}$. Value of x is ___________.

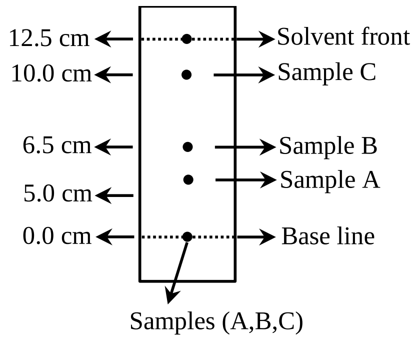


Fig : Paper chromatography of Samples

Ans. (50)

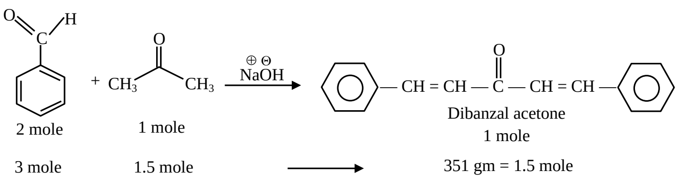
Sol. R_f of A = $\frac{5}{12.5}$ R_f of C = $\frac{10}{12.5}$

Ratio = $\frac{R_{f(A)}}{R_{f(C)}} = \frac{1}{2}$ = 0.5 or 50×10^{-2}

86. In the Claisen-Schmidt reaction to prepare 351 g of dibenzalacetone using 87 g of acetone, the amount of benzaldehyde required is __________g. (Nearest integer)

Ans. (318)

Sol. Claisen Schmidt reaction



mw of benzaldehyde = 106

106 × 3 = 318 gm. Benzaldehyde is required to give 1.5 mole (or 351 gm) product

87. Consider the following single step reaction in gas phase at constant temperature.

$2A_{(g)} + B_{(g)} \rightarrow C_{(g)}$

The initial rate of the reaction is recorded as r_1 when the reaction starts with 1.5 atm pressure of A and 0.7 atm pressure of B. After some time, the rate r_2 is recorded when the pressure of C becomes 0.5 atm. The ratio $r_1 : r_2$ is ________ $\times 10^{-1}$. (Nearest integer)

Ans. (315)

Sol. 2A(g) + B(g) $\longrightarrow$ C(g)

	2A(g)	B(g)	C(g)
r_1	1.5 atm	0.7 atm	
r_2	0.5 atm	0.2 atm	0.5 atm

$\because r = K[P_A]^2[P_B]$

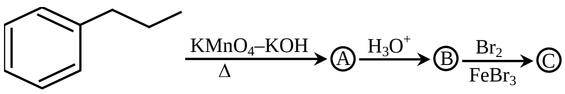
$r_1 = K[1.5]^2[0.7]$

$r_2 = K[0.5]^2[0.2]$

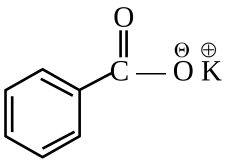
$\frac{r_1}{r_2} = 9 \times \frac{7}{2} = 31.5 = 315 \times 10^{-1}$

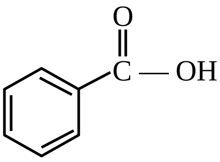
Ans. 315

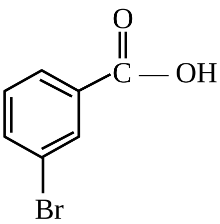
88. The product © in the following sequence of reactions has _________ π bonds.



Ans. (4)

Sol. A = 

B = 

C = 

π bonds = 4

89. Considering acetic acid dissociates in water, its dissociation constant is 6.25×10^{-5}. If 5 mL of acetic acid is dissolved in 1 litre water, the solution will freeze at $-x \times 10^{-2}$ °C, provided pure water freezes at 0 °C.

x = _________. (Nearest integer)

Given : $(K_f)_{water} = 1.86$ K kg mol^{-1}.

density of acetic acid is 1.2 g mol^{-1}

molar mass of water = 18 g mol^{-1}.

molar mass of acetic acid = 60 g mol^{-1}.

density of water = 1 g cm^{-3}

Acetic acid dissociates as

$$CH_3COOH \rightleftharpoons CH_3COO^{\ominus} + H^{\oplus}$$

Ans. (19)

Sol. Mass of $CH_3COOH = V \times d$

$= 5$ ml $\times 1.2$ g/ml

$= 6$ gm

$$n_{CH_3COOH} = \frac{6}{60} = 0.1 \text{mol}$$

$$m_{CH_3COOH} \approx M_{CH_3COOH} = \frac{0.1}{1} = 0.1 \text{M}$$

$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

C

$C - C\alpha$ $\quad C\alpha \quad C\alpha$

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

$1 - \alpha \approx 1 \Rightarrow K_a = C\alpha^2$

$$\alpha = \sqrt{\frac{Ka}{C}} = \sqrt{\frac{6.25\times10^{-5}}{0.1}} = 25 \times 10^{-3}$$

V.f. (i) $= 1 + \alpha(n - 1) = 1 + \alpha(2 - 1) = 1 + \alpha$

$= 1 + 25 \times 10^{-3} = 1.025$

$\Delta T_f = iK_f m$

$= (1.025)(1.86)(0.1)$

$= 0.19$

$= 19 \times 10^{-2}$

90. Number of compounds from the following with zero dipole moment is ___________.

HF, H_2, H_2S, CO_2, NH_3, BF_3, CH_4, $CHCl_3$, SiF_4, H_2O, BeF_2

Ans. (6)

Sol. H_2, CO_2, BF_3, CH_4, SiF_4 , BeF_2

are symm. molecule so dipole moment is zero